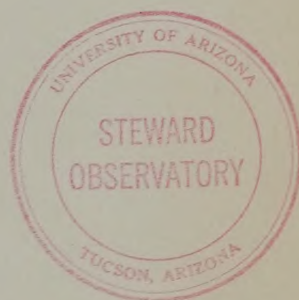


UNIVERSITY OF CALIFORNIA PUBLICATIONS
ASTRONOMY

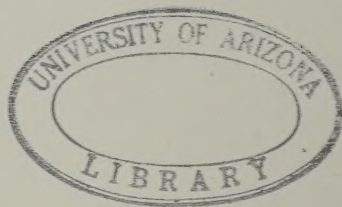
LICK OBSERVATORY BULLETINS

VOLUME XIII
NUMBERS 385 TO 401
1927-1928



UNIVERSITY OF CALIFORNIA PRESS
BERKELEY

8
9



CONTENTS

	PAGE
NUMBER 385.—The Light-Variations of the Satellites of <i>Jupiter</i> and Their Application to Measures of the Solar Constant. Joel Stebbins.....	1
NUMBER 386.—The Spectroscopic Binary β <i>Doradus</i> . Dorothy Applegate.....	12
NUMBER 387.—Observations of <i>Mars</i> at the Opposition of 1924. Robert J. Trumpler.....	19
NUMBER 388.—Elements and Ephemeris of Comet <i>d</i> 1927 (Stearns). H. Thiele.....	46
Preliminary Elements of Object <i>Comas Solà</i> (1927 AA). Robert S. Richardson, P. A. McNally, S. J. and Elizabeth E. Sternberg.....	47
Elements and Ephemeris of Object <i>Comas Solà</i> (1927 AA). P. A. McNally, S. J., and Elizabeth E. Sternberg....	48
Note on the Orbit of the Spectroscopic Binary 112 <i>Herculis</i> . W. F. Meyer.....	49
NUMBER 389.—Photographs of <i>Mars</i> and of <i>Jupiter</i> taken by Light of Different Colors During 1926. William H. Wright.....	50
NUMBER 390.—The Space Distribution of the Stars of Different Spectral Classes as Determined from Studies in the <i>Cygnus</i> Region of the Milky Way. Allan D. Maxwell.....	68
NUMBER 391.—Meridian Circle Observations of Eros Stars. R. H. Tucker.....	92
NUMBER 392.—Meridian Circle Observations of Eros Stars, Second List. R. H. Tucker.....	104
NUMBER 393.—On the Radial Velocity Curve of S <i>Sagittae</i> . T. S. Jacobsen.....	112
NUMBER 394.—Elements and Ephemeris of Comet <i>j</i> 1927 (Schwassmann-Wachmann). L. Berman and F. L. Whipple.....	117
NUMBER 395.—Elements and Ephemeris of Comet <i>k</i> 1927 (Skjellerup). N. U. Mayall, H. G. Miles, and F. L. Whipple.....	120
NUMBER 396.—The Spectra of the Carbon Stars. C. D. Shane.....	123
NUMBER 397.—Observations made with a Pair of Five-foot Cameras on the Light-deflections in the Sun's Gravitational Field at the Total Solar Eclipse of September 21, 1922. W. W. Campbell and R. J. Trumpler.....	130
NUMBER 398.—Motion of Matter in the Tail of Comet 1908 III (Morehouse). N. T. Bobrovnikoff.....	161
NUMBER 399.—Elements and Ephemeris of Comet <i>a</i> 1928 (Reinmuth). L. Berman and F. L. Whipple.....	173
NUMBER 400.—The Radial Velocity Orbit of χ <i>Draconis</i> . R. T. Crawford.....	176
NUMBER 401.—Further Photometric Measures of <i>Jupiter's</i> Satellites and <i>Uranus</i> , with Tests of the Solar Constant. Joel Stebbins and T. S. Jacobsen.....	180

PLATES

PLATE I.—FIGS. 1-12.—Photographs of <i>Mars</i>	22
PLATE II.—FIGS. 1-12.—Photographs of <i>Mars</i>	30
PLATE III.—FIGS. 1-2.—Maps of <i>Mars</i>	40
PLATE IV.—FIGS. 1-3.—Drawings of the South Polar Cap of <i>Mars</i> and Its Surroundings.....	41
PLATE V.—Drawings of the South Polar Cap and Its Surroundings.....	44
PLATES VI, VII, VIII.—Photographs of <i>Mars</i>	58
PLATE IX.—Photographs of <i>Jupiter</i> , 1926.....	58
PLATE X.—Spectra: (a) Two-fold Enlargement of a Plate of Selected Area 88E. (b) Typical Giant and Dwarf Spectra of Class K0.....	80
PLATE XI.—Spectra of HD 182040, Class R0; HD 16115, Class R3; HD 19557, Class R5; HD 25408, Class R8; HD 223075, Class N0; <i>Y Canum Venaticorum</i> , Class N3; <i>V Aquilae</i> , Class N6.....	126

LICK OBSERVATORY BULLETIN

NUMBER 385

THE LIGHT-VARIATIONS OF THE SATELLITES OF JUPITER AND THEIR
APPLICATION TO MEASURES OF THE SOLAR CONSTANT

BY

JOEL STEBBINS

During a sojourn of a month at the Lick Observatory in August and September, 1926, I began a series of measures of the satellites of *Jupiter* with the photo-electric photometer attached to the 12-inch refractor. In these observations I had the very efficient assistance of Messrs. T. S. Jacobsen and N. W. Storer, who after my departure from Mount Hamilton in the middle of September kindly secured additional measures which they have placed at my disposal. The object of the work was to determine whether the satellites could be compared photometrically with nearby stars for tests of the solar variation. Any light changes in the Sun would of course be reflected from all four satellites, and reference to a group of comparison stars would eliminate most of the effect of absorption in the Earth's atmosphere. The idea of testing the constancy of the Sun's radiation by observations of the planets and asteroids is of course an old one, but not to my knowledge has there been any careful study of the satellites of *Jupiter* for this purpose. The planets *Mars*, *Jupiter*, and *Saturn* have been observed photometrically by Guthnick,¹ but of these *Saturn* seems to be the only one which itself is fairly constant in light. With several satellites of *Jupiter* observable, any simultaneous changes in the light of all of them could be attributed to a solar variation. The four bright satellites are of the fifth and sixth magnitude, and ordinarily it should not be difficult to find suitable comparison stars for measurement with the photo-electric photometer.

When I arrived at Mount Hamilton, the photometer had not been in active use for some time, and the first

problem was to put it into good working order. The main mechanical features of the instrument are of the first class, and only a few modifications were necessary in the installation. First of all, the wiring was put into metallic conduit separate from all other circuits; also the battery was enclosed in a metal box. One may experiment with open wiring for some time without difficulty, but my experience has been that it is best to put the whole electrical system inside of a grounded metal covering, and then no trouble from stray electric charges will arise.

At first there seemed to be a large contact potential of the grounding key, but after some experimenting this turned out to be due to a charge on the fine natural amber support of the contact plate. In the dry summer air at Mount Hamilton, the conditions are perfect for any object holding a charge, so this insulating piece of amber after once picking up a charge would hold it indefinitely. The difficulty was removed by wrapping tin foil about most of the amber surface. Also another stray charge was found on the string of the grounding mechanism, which entered the cell box through a hard rubber bushing. The friction of the string on the rubber was producing a charge, which would leak off in several seconds, but it would induce a large effect on the cell and electrometer system. The substitution of a metal wire for the string eliminated this trouble. These may seem like small items, but the successful performance of an electrostatic instrument like the photo-electric photometer depends upon attention to just these details.

Another matter was the question of a good ground connection. As the 110-volt A. C. lighting circuit of the observatory has the neutral side connected to the

¹ A. N., 206, 157, 1918; 208, 105, 1919; 212, 39, 1920.

water pipes, I preferred not to mix up with that system, and we simply connected to the mounting of the 12-inch telescope which seemed to have enough capacity to be used instead of the earth.

Although the fine conditions on the mountain may continue for many days and nights, it is necessary to force dry air into the cell box in damp weather. A new drying apparatus was installed, consisting of a calcium-chloride "tower," two wash bottles with sulfuric acid, and a bulb of glass wool. With this arrangement a very slow bubbling of air into the instrument from the tank seemed to give perfect insulation of the cell and connections.

For potential we used fresh radio "B" batteries, 310 volts on the cell, and 45 volts on each side of the electrometer. The best battery that I know of for this purpose is made up of modified standard Carhart cells, but repeated comparative tests have shown that ordinary dry cells give a steady enough voltage, provided the potential is not forced up too near the glowing point of the photo-electric cell.

The sensitivity attained with the installation in 1926 may be measured by the photo-electric effect from a star of magnitude 1.0 and spectrum A0, which was about 2.5 volts per second as compared with 0.10 volts per second in 1915.² This increase of twenty-five fold has been due to the work of several persons. A number of mechanical features are from the ideas of Dr. Elmer Dershem, but the fundamental improvement is that of the quartz photo-electric cell developed by Professor Jakob Kunz of the University of Illinois.

After my arrival on August 11 we made a few observations the same evening, though it required several days to get the photometer going. Regular observations were begun on August 16, but it was not until August 23 or 24 that everything was finally on a fairly permanent basis. Although this delay of a week or more seemed pretty long at the time, it was a reasonably prompt solution of the difficulties involved.

The program for a set of measures on the satellites of *Jupiter* was soon adopted as follows. Readings, usually six, were taken on each of the available satellites, then on the four comparison stars, then again on the satellites. A second set would repeat the measures in reverse order, and a third set would repeat the first. The main problem was to eliminate the effect of the sky background, lighted up by the planet itself. The diaphragm at the focus of the telescope objective has an aperture of 90", or about twice the apparent diameter of *Jupiter*. It was found feasible to observe the satellites down to a distance of about 100" from the planet's center, but this required great care in the setting, and constant checking on the guiding of the instrument. After each group of readings on a satel-

lite, the telescope was turned off in declination, first to one side and then to the other, and sky readings were taken with the satellite just out of the field. In the extreme case of a satellite near the planet, and our own bright Moon near *Jupiter*, the sky effect would amount to one-fourth or even one-third of the total. In these circumstances more time was spent in measuring the sky than the satellites.

Very often two satellites would be in the field at once, and their combined light could be measured, but unless they were quite close together so that the sky correction could be easily determined, the observation was not attempted. This frequent disturbance of the measures of one satellite by the proximity of another, or of the planet itself, necessarily reduces the total number of observations that can be made to advantage. In the beginning it was not attempted to observe Satellite I, but after some practice a few measures were made of it near elongation.

A diaphragm smaller than 90" diameter might be used without encroaching upon the beam of light from a celestial object, but this would require the repeated withdrawal and re-insertion of a slide, and accurate following by the driving clock. The mounting of the 12-inch refractor is some forty years old or more, and while it performs perhaps as well as it ever did, the clamp and slow motion in right ascension are worked by ropes, and there is a certain clumsiness in all manipulations of the telescope. A modern mounting with clamps and slow motions at the eye-end, setting-dial for right ascension, and electric dome control, would reduce the observing time to about one-half what it is with the present arrangements.

But there remains the California sky. What a luxury it is to take some measures of an object, to turn off and not come back to it for half an hour or more, and then to find the second set of readings agreeing within one or two per cent of the first ones! Moreover, these conditions last all night, and the next night, and again the next. During my stay of thirty-three nights on the mountain, thirty were clear, and though during two or three of these there were passing thin clouds, all of the thirty nights would have been called first-class on the standards of the Mississippi Valley.

As *Jupiter* was moving slowly in the sky during the period of observation, it was possible to select four comparison stars which would practically eliminate the atmospheric extinction. The data from the Henry Draper Catalogue are as shown in Table I.

Considerable attention has been paid to the correction for differential atmospheric extinction. It is sufficient to use the reduction to no atmosphere to be of the form $0^m20 f \sec z$, where z is the apparent zenith distance, 0^m20 the approximate visual extinction at the

² *Lick Obs. Bull.*, 8, 186, 1916.

TABLE I
COMPARISON STARS

		R.A. 1900	Decl. 1900	Visual magnitude	Photo- electric magnitude	Spectrum	
(1)	ϵ Capricorni	21 ^h 16 ^m 7 ^s	-17° 16'	4 ^m 30	5 ^m 00	K0	With sector 1 ^m 5
(2)	γ Capricorni	21 34.6	-17 07	3.80	5.50	Fp	
(3)	δ Capricorni	21 36.1	-14 29	5.28	5.83	G5	
(4)	μ Capricorni	21 47.8	-14 01	5.18	5.38	F0	

zenith, and f a factor which varies with the spectral type of the star, and with changing transparency of the air. Because of the uniform excellence of the sky at Mount Hamilton, and the relatively small variation in zenith distance during the measures, the transparency was assumed to be the same on all nights.

The variation of f with spectral type can be determined roughly from the regular measures of the comparison stars, in particular stars (1) and (4) which are farthest apart in position, and are of spectra K0 and F0 respectively. From the variation of the difference of magnitude between these two stars with changing hour angle there was found,

Star	Spectrum	Extinction factor f	Transmission at Zenith
(4)	F0	0.95	0.839
(1)	K0	0.82	0.859

These values of the transmission may be compared with Abbot's values of the zenith transmission at Mount Wilson,³ 0.830 for 4500A and 0.873 for 5000A respectively. The maximum sensitivity of a quartz-

potassium cell for constant energy along the spectrum is about at 4600A. The satellites were assumed to be of spectrum G, like the Sun, and a factor $f=0.88$ was used for them, although our Moon is known to have a color-index about equivalent to K0.

Except for the principle of the thing, it would have been sufficient simply to ignore the extinction, because of the favorable grouping of the comparison stars. The differential extinction between *Jupiter* and the mean of the four amounted to a hundredth of a magnitude on only two or three occasions, the average correction for the entire series being only 0^m 004.

In Table II are the observations of the satellites and comparison stars. The day begins at Greenwich midnight. The angle α is the jovicentric elongation of the Earth from the Sun, or what we may call the solar phase of the satellites. The magnitudes of the satellites and comparison stars are referred to the mean of the latter, adopted to be 5.46, and have already been corrected for atmospheric extinction. Except for this small correction, the magnitudes are given exactly as they came out in the reductions.

TABLE II
JOURNAL OF OBSERVATIONS

1926 G. C. T.	α	Satellites				Comparison Stars				Remarks
		I	II	III	IV	(1)	(2)	(3)	(4)	
Aug. 16.318	0 ^m 24			(5 ^m 127)	(6 ^m 247)	5 ^m 147	5 ^m 491	5 ^m 773	5 ^m 431	Tests, reject.
16.342	0.24			(5.147)	(6.197)	5.133	5.461	5.793	5.453	Tests, reject.
17.343	0.40		5 ^m 977	5.214	6.216	5.125	5.483	5.786	5.444	
17.407	0.41		5.935	5.173	6.208	5.087	5.499	5.797	5.455	
18.365	0.60		5.713			5.087	5.495	5.829	5.428	
20.303	1.00		5.926	5.126	6.171	5.126	5.487	5.771	5.458	
20.337	1.01		5.947	5.101	6.183	5.134	5.487	5.781	5.438	
21.315	1.22			5.160	6.237	5.112	5.477	5.790	5.463	
21.339	1.22			5.179	6.233	5.103	5.478	5.801	5.459	
22.406	1.46		(5.777)		(6.284)	5.098	5.434	5.834	5.475	Trouble with photometer, reject.
23.316	1.65					5.092	5.488	5.802	5.458	Bright Moon.
24.286	1.86		6.042	5.259	6.328	5.110	5.497	5.779	5.454	
24.317	1.87		6.056	5.267	6.309	5.123	5.467	5.788	5.462	
24.382	1.88		6.013	5.229	6.279	5.141	5.472	5.774	5.454	
25.306	2.08			5.223	6.302	5.117	5.475	5.782	5.466	
25.331	2.08			5.208	6.324	5.118	5.490	5.781	5.450	
25.397	2.10		5.790	5.231	6.311	5.120	5.469	5.789	5.464	
26.360	2.31					5.132	5.462	5.786	5.459	Fog interfered.
Aug. 26.390	2.31		5.829	5.173		5.117	5.474	5.784	5.466	

³ *The Sun*, p. 297.

TABLE II—(Continued)

1926 G. C. T.	a	Satellites				Comparison Stars				Remarks
		I	II	III	IV	(1)	(2)	(3)	(4)	
Aug. 27.261	2°49		5 ^m 959	5 ^m 166	6 ^m 348	5 ^m 130	5 ^m 477	5 ^m 785	5 ^m 448	
27.289	2.50		5.978	5.166	6.350	5.129	5.479	5.784	5.450	
27.330	2.51		5.990	5.175	6.343	5.128	5.472	5.785	5.453	
28.275	2.71			5.226	6.364	5.129	5.472	5.793	5.447	
28.300	2.71			5.229	6.367	5.126	5.475	5.782	5.456	
28.344	2.72			5.236	6.363	5.128	5.490	5.779	5.442	
29.331	2.93		5.810		6.388	5.140	5.483	5.779	5.437	
29.371	2.94		5.808		6.390	5.121	5.477	5.793	5.440	
31.264	3.34		6.075	5.330	6.428	5.116	5.479	5.789	5.456	
31.294	3.34		6.079	5.321	6.434	5.130	5.471	5.789	5.448	
31.343	3.35		6.110	5.337	6.451	5.127	5.483	5.779	5.450	
Sept. 1.264	3.55			5.285	6.460	5.129	5.479	5.785	5.448	
1.290	3.55			5.295	6.465	5.133	5.483	5.790	5.432	
1.331	3.56			5.311	6.457	5.114	5.481	5.798	5.445	
2.253	3.75		5.832		6.466	5.123	5.492	5.779	5.445	
2.309	3.76		5.846		6.477	5.116	5.477	5.799	5.448	II incomplete.
2.357	3.77			(5.280)						From II+III, reject.
3.242	3.95		5.985	5.201		5.111	5.486	5.781	5.462	
3.271	3.95		5.994	5.205		5.134	5.475	5.783	5.451	
3.313	3.96		5.989	5.204		5.117	5.478	5.800	5.444	
4.261	4.15		6.048	5.270	6.452	5.123	5.493	5.782	5.444	
4.278	4.16		6.058							II incomplete.
4.301	4.16			5.274	6.448	5.129	5.482	5.781	5.448	
4.342	4.17			5.275	6.458	5.100	5.491	5.794	5.454	
5.253	4.35		5.836		6.438	5.120	5.475	5.785	5.460	
5.290	4.36	5.858	5.845		6.441	5.124	5.477	5.790	5.450	
5.335	4.37	5.824	5.815		6.439	5.141	5.490	5.765	5.443	
6.233	4.55	5.947		5.375	6.436	5.132	5.478	5.783	5.446	
6.265	4.55	5.969		5.367	6.448	5.129	5.497	5.783	5.433	
6.309	4.56	6.002		5.361	6.425	5.112	5.468	5.800	5.459	
7.235	4.74	5.817	6.115	5.371	6.452	5.109	5.479	5.796	5.454	
7.269	4.75	5.830	6.121	5.367	6.458	5.128	5.479	5.777	5.457	
7.331	4.76	5.810	6.146	5.394	6.472	5.113	5.477	5.800	5.448	
8.232	4.93	6.046		5.335	6.443	5.113	5.487	5.787	5.452	
8.263	4.94	6.049		5.360	6.454	5.122	5.479	5.791	5.449	
8.314	4.95	6.044		5.355	6.473	5.108	5.478	5.800	5.452	
9.222	5.12	5.828								I incomplete.
9.237	5.13		5.867		6.495	5.129	5.489	5.789	5.431	
9.275	5.13		5.865		6.490	5.127	5.472	5.781	5.460	
9.294	5.14			5.290						From I+III.
9.317	5.14		5.875		6.503	5.132	5.479	5.781	5.447	
9.340	5.14			5.286						From IV.
10.270	5.32		6.007	5.251	6.541	5.139	5.489	5.765	5.447	
10.315	5.33		6.016	5.250	6.527	5.102	5.497	5.781	5.458	
11.235	5.50		6.136	5.303		5.107	5.471	5.806	5.450	From II+IV.
11.267	5.51		6.104	5.304		5.115	5.474	5.800	5.450	From II+IV.
11.303	5.52		6.097	5.315		5.127	5.478	5.788	5.446	From II+IV.
13.253	5.89	5.964	(5.891)	5.430	6.576	5.112	5.506	5.776	5.447	II incomplete, reject.
13.311	5.90	5.997		5.417	6.557	5.124	5.481	5.802	5.433	
14.217	6.06	5.876	6.157	5.489	6.577	5.127	5.471	5.792	5.448	III half weight.
14.266	6.07	5.886	6.135	5.425	6.598	5.142	5.476	5.792	5.430	
21.288	7.28			5.485		5.086	5.486	5.807	5.461	
22.214	7.42	6.095		5.464	6.624	5.089	5.491	5.794	5.468	
22.259	7.43	6.123		5.463	6.615	5.135	5.486	5.775	5.443	
24.218	7.73	6.165		5.352	6.619	5.105	5.463	5.825	5.448	
24.260	7.74	(6.129)		(5.288)	(6.564)	5.154	5.468	5.788	5.429	Poor, rejected.
25.227	7.88		6.266	5.378	6.647	5.108	5.481	5.794	5.456	
Sept. 25.292	7.89		6.259	5.383	6.660	5.127	5.476	5.817	5.420	

TABLE II—(Continued)

1926 G. C. T.	α	Satellites				Comparison Stars				Remarks
		I	II	III	IV	(1)	(2)	(3)	(4)	
Sept. 28.197	8°32		6 ^m 129	5 ^m 513		5 ^m 126	5 ^m 508	5 ^m 783	5 ^m 426	
28.253	8.33		6.187	5.546		5.155	5.484	5.764	5.435	
30.201	8.60	6.009	5.975		6.699	5.096	5.457	5.820	5.469	
30.256	8.60	5.996	5.991		6.757	5.100	5.487	5.789	5.463	
Oct. 5.216	9.24		6.205	5.587	6.864	5.119	5.486	5.798	5.437	
6.232	9.36			5.613	6.821	5.085	5.480	5.820	5.454	
8.198	9.57	6.257		5.479	6.781	5.117	5.467	5.803	5.455	
18.178	10.49	6.145	6.155		7.001	5.098	5.473	5.831	5.437	
20.178	10.63		6.427	5.726	6.974	5.114	5.453	5.819	5.453	
23.163	10.82		6.359	5.579		5.078	5.465	5.819	5.477	
26.201	10.99			5.742	6.882	5.105	5.483	5.791	5.460	
27.168	11.04		6.476	5.763	6.897	5.116	5.467	5.802	5.456	
Dec. 11.028	9.96	(6.585)			7.297			5.812	5.440	I poor, rejected.
20.019	9.23			5.927	7.252	5.101	5.467	5.819	5.454	

For comparison of results on different dates, the next step is to reduce all magnitudes to standard or mean opposition. This is accomplished by the formula:

$$\text{Reduction to mean opposition} = 5 \log r_0 \Delta_0 / r \Delta,$$

where r is the heliocentric and Δ the geocentric distance of *Jupiter*, and $r_0 = 5.2028$, $\Delta_0 = 4.2028$. A table of this correction was made, of which the following is the skeleton:

TABLE III
REDUCTION TO MEAN OPPOSITION

1926 Aug. 14	+0 ^m 159
22	+ .158
30	+ .147
Sept. 7	+ .125
15	+ .096
23	+ .058
Oct. 1	+ .014
9	— .034
17	— .087
25	— .141
Nov. 2	— .196

In Table IV the results for each satellite are brought together; the date in the first column identifies the corresponding observation in Table II. The second column contains the orbital phase computed from the time of superior conjunction of the satellite with *Jupiter*, as given in the *American Ephemeris*. Each magnitude in the third column is obtained from the one in Table II by applying the proper correction from Table III. The fourth column contains the correction for the solar phase α , which will be discussed later. The reduced magnitudes in the fifth column are derived from the two preceding columns. The residuals in the sixth column are based upon the final light-curves, and in the seventh column these residuals are averaged for each date.

TABLE IV
REDUCED MAGNITUDES
SATELLITE I

1926	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Sept. 5	51°7	5 ^m 989	—0 ^m 068	5 ^m 921	+0 ^m 013	
5	60.8	5.955	— .068	5.887	— .012	0.000
6	243.6	6.075	— .071	6.004	.000	
6	250.1	6.097	— .071	6.026	.000	
6	259.1	6.130	— .071	6.059	+ .008	+ .003
7	87.6	5.942	— .073	5.869	— .007	
7	94.5	5.955	— .074	5.881	+ .010	
7	107.1	5.934	— .074	5.860	— .005	— .001
8	290.6	6.167	— .076	6.091	+ .001	
8	296.9	6.170	— .077	6.093	+ .002	
8	307.3	6.165	— .077	6.088	— .001	+ .001
9	132.2	5.946	— .080	5.866	+ .002	+ .002
13	232.8	6.067	— .091	5.976	.000	
13	244.6	6.100	— .091	6.009	.000	.000
14	69.0	5.975	— .094	5.881	— .011	
14	79.0	5.985	— .094	5.891	+ .008	— .002
22	257.0	6.157	— .115	6.042	— .002	
22	266.1	6.185	— .115	6.070	+ .004	+ .001
24	304.8	6.217	— .120	6.097	+ .007	+ .007
24	313.4	6.181	— .120	6.061	(— .027)	
30	82.6	6.028	— .133	5.895	+ .016	
30	93.8	6.014	— .133	5.881	+ .009	+ .012
Oct. 8	270.0	6.228	— .148	6.080	+ .008	+ .008
18	140.7	6.051	— .162	5.889	+ .021	+ .021
Dec. 11	292.9	6.125	— .154	5.971	(— .119)	

SATELLITE II

1926	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Aug. 17	309°8	6 ^m 137	—0 ^m 013	6 ^m 124	0 ^m 000	
17	316.3	6.095	— .013	6.082	— .022	—0 ^m 011
18	53.6	5.873	— .019	5.854	— .016	— .016
20	250.3	6.085	— .032	6.053	— .021	
20	253.7	6.106	— .032	6.074	— .009	— .015
22	103.8	5.935	— .045	5.890	(+ .034)	
24	294.6	6.198	— .057	6.141	— .001	
24	297.8	6.212	— .057	6.155	+ .014	
24	304.4	6.169	— .057	6.112	— .022	— .003

TABLE IV—(Continued)

SATELLITE II

1926	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Aug. 25	47°4	5 ^m 944	—0 ^m 064	5 ^m 880	0 ^m 000	0 ^m 000
26	148.2	5.982	— .069	5.913	+ .014	+ .014
27	236.6	6.111	— .074	6.037	.000	
27	239.4	6.130	— .075	6.055	+ .007	
27	243.6	6.142	— .075	6.067	+ .009	+ .005
29	86.7	5.958	— .086	5.872	+ .023	
29	90.7	5.956	— .086	5.870	+ .021	+ .022
31	282.9	6.219	— .096	6.123	— .015	
31	285.9	6.223	— .096	6.127	— .013	
31	290.9	6.254	— .096	6.158	+ .016	— .004
Sept. 2	124.7	5.971	— .106	5.865	— .001	
2	130.4	5.985	— .106	5.879	+ .005	+ .002
3	225.1	6.122	— .111	6.011	+ .010	
3	228.0	6.130	— .111	6.019	+ .007	
3	232.3	6.125	— .111	6.014	— .010	+ .002
4	328.5	6.182	— .115	6.067	.000	
4	330.2	6.192	— .115	6.077	+ .015	+ .005
5	69.2	5.967	— .119	5.848	— .008	
5	73.0	5.976	— .120	5.856	+ .003	
5	77.5	5.946	— .120	5.826	— .025	— .010
7	270.3	6.240	— .128	6.112	— .008	
7	273.8	6.246	— .128	6.118	— .008	
7	280.0	6.270	— .128	6.142	+ .006	— .003
9	113.4	5.985	— .135	5.850	— .006	
9	117.3	5.983	— .135	5.848	— .011	
9	121.6	5.993	— .136	5.857	— .006	— .008
10	218.3	6.121	— .139	5.982	— .006	
10	222.9	6.130	— .139	5.991	— .005	— .006
11	316.2	6.247	— .142	6.105	.000	
11	319.5	6.214	— .142	6.072	— .022	
11	323.1	6.207	— .142	6.065	— .015	— .012
13	161.0	5.994	— .149	5.845	(— .071)	
14	258.8	6.256	— .152	6.104	+ .008	
14	263.7	6.234	— .152	6.082	— .024	— .008
25	295.4	6.313	— .177	6.136	— .006	
25	302.3	6.306	— .177	6.129	— .007	— .006
28	237.0	6.160	— .181	5.979	— .060	
28	242.6	6.217	— .181	6.036	— .020	— .040
30	80.1	5.994	— .183	5.811	— .039	
30	85.7	6.009	— .183	5.826	— .023	— .031
Oct. 5	228.7	6.194	— .189	6.005	— .007	— .007
18	102.7	6.061	— .196	5.865	+ .014	+ .014
20	305.4	6.319	— .197	6.122	— .010	— .010
23	247.8	6.231	— .197	6.034	— .035	— .035
27	313.8	6.321	— .198	6.123	+ .012	+ .012

SATELLITE III

1926	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Aug. 16	244°1	5 ^m 287	—0 ^m 008	5 ^m 279	(—0 ^m 093)	
16	245.3	5.307	— .008	5.299	(— .073)	(—0 ^m 083)
17	295.8	5.384	— .014	5.370	+ .012	
17	299.1	5.333	— .014	5.319	— .036	— .012
20	85.2	5.285	— .034	5.251	+ .013	
20	86.9	5.260	— .034	5.226	— .014	.000
21	136.2	5.318	— .042	5.276	— .022	
21	137.4	5.337	— .042	5.295	— .003	— .012
24	286.1	5.415	— .061	5.354	— .010	
24	287.7	5.423	— .061	5.362	— .001	

TABLE IV—(Continued)

SATELLITE III

1926	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Aug. 24	290°9	5 ^m 385	—0 ^m 061	5 ^m 324	—0 ^m 038	—0 ^m 016
25	337.6	5.378	— .067	5.311	— .001	
25	338.8	5.362	— .067	5.295	— .016	
25	342.2	5.385	— .068	5.317	+ .009	— .003
26	32.2	5.326	— .074	5.252	+ .014	+ .014
27	76.2	5.318	— .079	5.239	+ .010	
27	77.6	5.318	— .079	5.239	+ .008	
27	79.6	5.327	— .080	5.247	+ .015	+ .011
28	127.3	5.376	— .086	5.290	+ .002	
28	128.6	5.379	— .086	5.293	+ .003	
28	130.8	5.386	— .086	5.300	+ .008	+ .004
31	278.1	5.474	— .103	5.371	+ .003	
31	279.6	5.465	— .103	5.362	— .006	
31	282.1	5.481	— .103	5.378	+ .012	+ .003
Sept. 1	328.5	5.427	— .108	5.319	— .007	
1	329.8	5.437	— .108	5.329	+ .005	
1	331.9	5.452	— .109	5.343	+ .023	+ .007
2	23.4	5.419	— .114	5.305	(+ .053)	
3	68.3	5.338	— .119	5.219	— .003	
3	69.7	5.341	— .119	5.222	— .001	
3	71.9	5.340	— .119	5.221	— .003	— .002
4	119.7	5.404	— .124	5.280	+ .002	
4	121.7	5.408	— .124	5.284	+ .004	
4	123.8	5.409	— .124	5.285	+ .001	+ .002
6	219.1	5.503	— .133	5.370	+ .006	
6	220.7	5.495	— .133	5.362	— .003	
6	223.0	5.489	— .133	5.356	— .010	— .002
7	269.6	5.496	— .138	5.358	— .014	
7	271.4	5.492	— .138	5.354	— .017	
7	274.5	5.518	— .138	5.380	+ .010	— .007
8	319.9	5.456	— .142	5.314	— .022	
8	321.5	5.481	— .143	5.338	+ .004	
8	324.1	5.476	— .143	5.333	+ .003	— .005
9	12.5	5.408	— .147	5.261	— .009	
9	14.6	5.404	— .147	5.257	— .009	— .009
10	62.7	5.365	— .151	5.214	— .006	
10	64.9	5.364	— .151	5.213	— .008	— .007
11	111.3	5.414	— .155	5.259	— .010	
11	112.9	5.414	— .155	5.259	— .011	
11	114.7	5.425	— .155	5.270	— .003	— .008
13	213.0	5.533	— .164	5.369	+ .007	
13	216.0	5.520	— .164	5.356	— .007	.000
14	261.6	5.588	— .167	5.421	+ .048	
14	264.1	5.524	— .167	5.357	— .016	+ .005
21	258.0	5.552	— .191	5.361	— .013	— 0.13
22	304.7	5.526	— .193	5.333	— .019	
22	307.0	5.525	— .193	5.332	— .017	— .018
24	45.6	5.404	— .198	5.206	— .020	— .020
24	47.7	5.340	— .198	5.142	(— .080)	
25	96.4	5.425	— .201	5.224	— .026	
25	99.7	5.430	— .201	5.229	— .026	— .026
28	246.1	5.544	— .208	5.336	— .037	
28	248.9	5.576	— .208	5.368	— .005	— .021
Oct. 5	239.6	5.576	— .221	5.355	— .016	— .016
6	290.7	5.596	— .223	5.373	+ .011	+ .011
8	29.6	5.450	— .225	5.225	— .015	— .015
20	272.4	5.618	— .237	5.381	+ .010	+ .010
23	62.4	5.451	— .239	5.212	— .008	— .008
26	215.2	5.594	— .241	5.353	— .010	— .010
27	263.8	5.608	— .241	5.367	— .006	— .006
Dec. 20	86.9	5.433	— .221	5.212	— .028	— .028

TABLE IV—(Continued)

SATELLITE IV

	1926	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Aug. 16	145°6	6 ^m 407	—0 ^m 015	6 ^m 392	(—0 ^m 003)		
16	146.1	6.357	— .015	6.342	(— .053)	(—0 ^m 028)	
17	167.9	6.376	— .025	6.351	— .028		
17	169.3	6.368	— .026	6.342	— .036	— .032	
20	232.1	6.330	— .063	6.267	— .046		
20	232.8	6.342	— .064	6.278	— .035	— .040	
21	254.1	6.395	— .076	6.319	+ .012		
21	254.6	6.391	— .076	6.315	+ .008	+ .010	
22	277.7	6.442	— .090	6.352	(+ .041)		
24	318.5	6.484	— .114	6.370	+ .038		
24	319.2	6.465	— .114	6.351	+ .019		
24	320.6	6.435	— .115	6.320	— .012	+ .015	
25	340.7	6.457	— .126	6.331	— .008		
25	341.2	6.478	— .126	6.352	+ .012		
25	342.6	6.465	— .128	6.337	— .003	.000	
27	23.0	6.500	— .149	6.351	+ .003		
27	23.7	6.502	— .150	6.352	+ .004		
27	24.5	6.495	— .151	6.344	— .004	+ .001	
28	45.0	6.514	— .162	6.352	.000		
28	45.6	6.517	— .162	6.355	+ .003		
28	46.5	6.513	— .162	6.351	— .001	+ .001	
29	67.9	6.536	— .174	6.362	.000		
29	68.8	6.538	— .175	6.363	.000	.000	
31	109.8	6.572	— .196	6.376	— .012		
31	110.5	6.578	— .196	6.382	— .006		
31	111.6	6.595	— .196	6.399	+ .010	— .003	
Sept. 1	131.5	6.602	— .207	6.395	— .002		
1	132.1	6.607	— .207	6.400	+ .003		
1	133.0	6.598	— .207	6.391	— .006	— .002	
2	153.0	6.605	— .217	6.388	— .004		
2	154.2	6.616	— .218	6.398	+ .006	+ .001	
4	196.5	6.586	— .237	6.349	.000		
4	197.4	6.582	— .238	6.344	— .005		
4	198.3	6.592	— .238	6.354	+ .008	+ .001	
5	218.0	6.569	— .247	6.322	— .002		
5	218.8	6.572	— .247	6.325	+ .002		
5	219.8	6.570	— .248	6.322	.000	.000	
6	239.3	6.564	— .256	6.308	— .002		
6	240.0	6.576	— .256	6.320	+ .011		
6	240.9	6.553	— .257	6.296	— .012	— .001	
7	261.0	6.577	— .266	6.311	+ .004		
7	261.7	6.583	— .266	6.317	+ .010		
7	263.1	6.596	— .266	6.330	+ .023	+ .012	
8	282.6	6.564	— .275	6.289	— .024		
8	283.3	6.575	— .275	6.300	— .013		
8	284.4	6.594	— .275	6.319	+ .005	— .011	
9	304.4	6.613	— .284	6.329	+ .004		
9	305.2	6.608	— .284	6.324	— .001		
9	306.1	6.621	— .284	6.337	+ .011	+ .005	
10	326.8	6.655	— .293	6.362	+ .027		
10	327.8	6.641	— .293	6.348	+ .012	+ .020	
13	31.4	6.679	— .317	6.362	+ .013		
13	32.7	6.660	— .318	6.342	— .007	+ .003	
14	52.3	6.676	— .324	6.352	— .002		
14	53.4	6.697	— .325	6.372	+ .018	+ .008	
22	225.4	6.686	— .377	6.309	— .009		
22	226.4	6.677	— .377	6.300	— .017	— .013	
24	268.8	6.671	— .388	6.283	— .025	— .025	
24	269.7	6.616	— .389	6.227	(— .082)		
25	290.6	6.694	— .394	6.300	— .017		
25	292.0	6.707	— .394	6.313	— .005	— .011	

TABLE IV—(Continued)

SATELLITE IV

	1926	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Sept. 30	38°2	6 ^m 718	—0 ^m 418	6 ^m 300	—0 ^m 050		
30	39.4	6.775	— .418	6.357	+ .007	— .022	
Oct. 5	146.8	6.853	— .438	6.415	+ .020	+ .020	
6	168.8	6.804	— .441	6.363	— .016	— .016	
8	211.4	6.752	— .448	6.304	— .026	— .026	
18	66.4	6.907	— .472	6.435	+ .073	+ .073	
20	109.5	6.866	— .476	6.390	+ .004	+ .004	
26	239.6	6.734	— .485	6.249	— .060	— .060	
27	260.5	6.742	— .486	6.256	— .051	— .051	
Dec. 11	142.7	6.851	— .459	6.392	— .004	— .004	
20	335.1	6.758	— .438	6.320	— .018	— .018	

In the discussion of the observational data in tables II and IV, we begin with the comparison stars. Dividing the measures into two series, and taking the general means, there is found for the photo-electric magnitudes:

TABLE V

MAGNITUDES OF COMPARISON STARS

	1926	(1)	(2)	(3)	(4)
Aug. 16-Sept. 14.....	5 ^m 119	5 ^m 480	5 ^m 789	5 ^m 451	
P.E. one observation.....	±.009	±.007	±.008	±.007	
Sept. 21-Oct. 27.....	5.110	5.478	5.802	5.450	
P.E. one observation.....	±.014	±.010	±.013	±.011	

The average probable errors for the two groups are $\pm 0^m008$ and $\pm 0^m012$, and hence for the mean of four stars in a set are $\pm 0^m004$ and $\pm 0^m006$ respectively. These probable errors were derived from all the observations, good, poor, and indifferent, and as the comparison stars show absolutely no indication of variability, the mean of four of them may be considered as a satisfactory standard of reference. The mean residuals of the comparison stars for each date are shown in fig. 2.

The opposition of *Jupiter* occurred on August 15, and it was a week or more later before the photometer was in the best condition; then the full Moon came along close to *Jupiter* and made observations difficult. For these reasons the results from August 16 to 23 are considered as of less weight than those from August 24 to September 14, though the transition from fair to good measures was presumably gradual. After September 14 the observations were taken only at odd times, and they have been used to determine the general trend of the light of the satellites with increasing solar phase.

The corrected magnitudes for the satellites in the third column of table IV all exhibit the same phenomenon—a light-variation in the orbital period superimposed upon a brightening up at the full phase of opposition, similar to that of our Moon. These two

effects may be disentangled by successive approximations. A rough guess is made as to the variation of a satellite with solar phase α , all observations are then corrected for this variation, and the resulting magnitudes are plotted according to the orbital phase. The light-curve in the orbital period is then drawn, and the residuals give a second determination of the solar-phase effect. The original observations are then corrected

anew for the solar-phase effect, a second orbital light-curve is determined, and so on. As the orbital variation is represented more easily by a free-hand curve than by any simple formula, there is no advantage in a least-squares solution at any step. Elementary considerations show that the light received by a satellite from *Jupiter* is negligible compared with that received from the Sun.

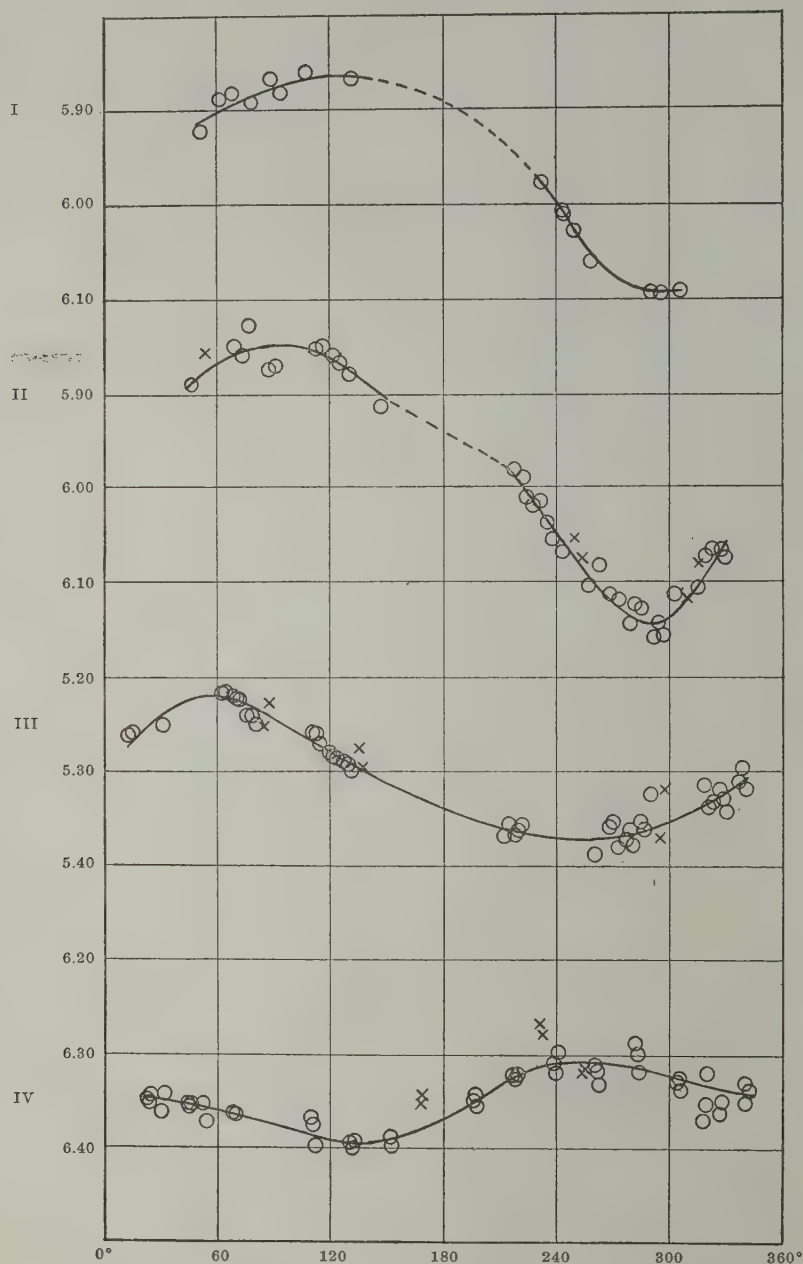


Fig. 1. Variations of the Satellites of *Jupiter* with Orbital Phase.

It has been found by Mueller⁴ that the visual magnitudes of superior planets and asteroids may be represented near opposition as linear functions of the solar phase. For the present measures of *Jupiter's* satellites, however, a second-degree term can be added which expresses the variation as

$$m = m_0 + ba + ca^2,$$

where m is the magnitude at solar phase a expressed in degrees, b and c are empirical constants, and m_0 is the magnitude at opposition.

After some trials the following expressions were adopted for the different satellites. The measures of satellite I were not begun until three weeks after opposition, so a linear relation is sufficient for that case.

$$\begin{aligned} \text{Satellite I, } m &= m_0 + 0.0155 a \\ \text{II, } m &= m_0 + 0.0337 a - 0.00143 a^2 \\ \text{III, } m &= m_0 + 0.0347 a - 0.00116 a^2 \\ \text{IV, } m &= m_0 + 0.065 a - 0.0019 a^2 \end{aligned} \quad (\text{A})$$

In the fourth column of table IV the reduction for a is the sum of the two terms in (A), but with reversed sign. The magnitudes in the fifth column are thus reduced to zero solar phase and are ready for the determination of the orbital variations. The light-curves for the satellites are given in fig. 1, where it is seen that each has the sort of variation which we might expect from a body of non-uniform surface brightness keeping one face toward the primary. The results for satellite I agree better than was expected from the proximity to *Jupiter*; II and III give satisfactory curves; but with its longer period the variation of IV is not so well determined. The period and range for each satellite may be summarized as follows:

TABLE VI

Satellite	Period	Range	Phase of maximum	Phase of minimum
I	1.77	0 ^m 229	122°	295°(?)
II	3.55	0.294	90	292
III	7.17	0.155	58	255
IV	16.75	0.090	255	135

It may be noted that I, II, and III are in a general way brighter on the front side in their motion, while IV is brighter on the rear side.

We now return to the variation with solar phase, which is shown in fig. 2. The computed curves are simply the graphs of the fourth column of table IV, while the plotted magnitudes are derived by subtracting these reversed sums from the mean residuals in the fifth column of that table. As already noted, the measures beginning with about August 24, or $a=2^\circ$, were of the best quality, and it was this middle

series which was used in deriving the orbital light-curves. Although the first observations are not of great weight, each of the satellites II, III, and IV, shows a greater increase of light near opposition than is represented by the second-degree expression in a .

In table VII are brought together the residuals by nights for satellites II, III, and IV. These discordances include the errors in the original measures, the errors in the correction for atmospheric extinction, the errors in the assumed variation of the satellites with the orbital and solar phase, and finally the variations of the Sun itself, all referred to the mean of the four comparison stars. Any change in the solar radiation would be reflected in all of the satellites simultaneously. The residuals of the first week are systematically negative, as mentioned before, so there remain some twenty nights beginning with August 24. From these there is found an average residual of ± 0.004 magnitude, which indicates no change in the Sun which could be detected during this short interval of three weeks. The latter measures from September 21 to October 27 are not sufficient to show anything but the general character of the variation of the satellites, and the two observations in December are of still less weight.

TABLE VII

MEAN RESIDUALS BY NIGHTS						Remarks
1926	II	III	IV	Mean		
Aug. 17	- ^m 011	- ^m 012	- ^m 032	- ^m 018		
18	- 16			- 16		
20	- 15	0	- 40	- 18		All poor
21		- 12	+ 10	- 1		
Aug. 24	- 3	- 16	+ 15	- 1		
25	0	- 3	0	- 1		
26	+ 14	+ 14		+ 14		Poor, one set only
27	+ 5	+ 11	+ 1	+ 6		
28		+ 4	0	+ 2		
29	+ 22		0	+ 11		
31	- 4	+ 3	- 3	- 1		
Sept. 1		+ 7	- 2	+ 2		
2	+ 2		+ 1	+ 2		
3	+ 2	- 2		0		
4	+ 8	+ 2	+ 1	+ 4		
5	- 10		0	- 5		
6		- 2	- 1	- 2		
7	- 3	- 7	+ 12	+ 1		
8		- 5	- 11	- 8		
9	- 8	- 9	+ 5	- 4		
10	- 6	- 7	+ 20	+ 2		
11	- 12	- 8		- 10		
13		0	+ 3	+ 2		
14	- 8	+ 5	+ 8	+ 2		

The average deviation for a night, ± 0.004 magnitude, found in the best part of the present work, represents about what might be expected under good conditions in a long series for tests of the solar constant

⁴ *Potsdam Publ.*, 8, 366, 1893.

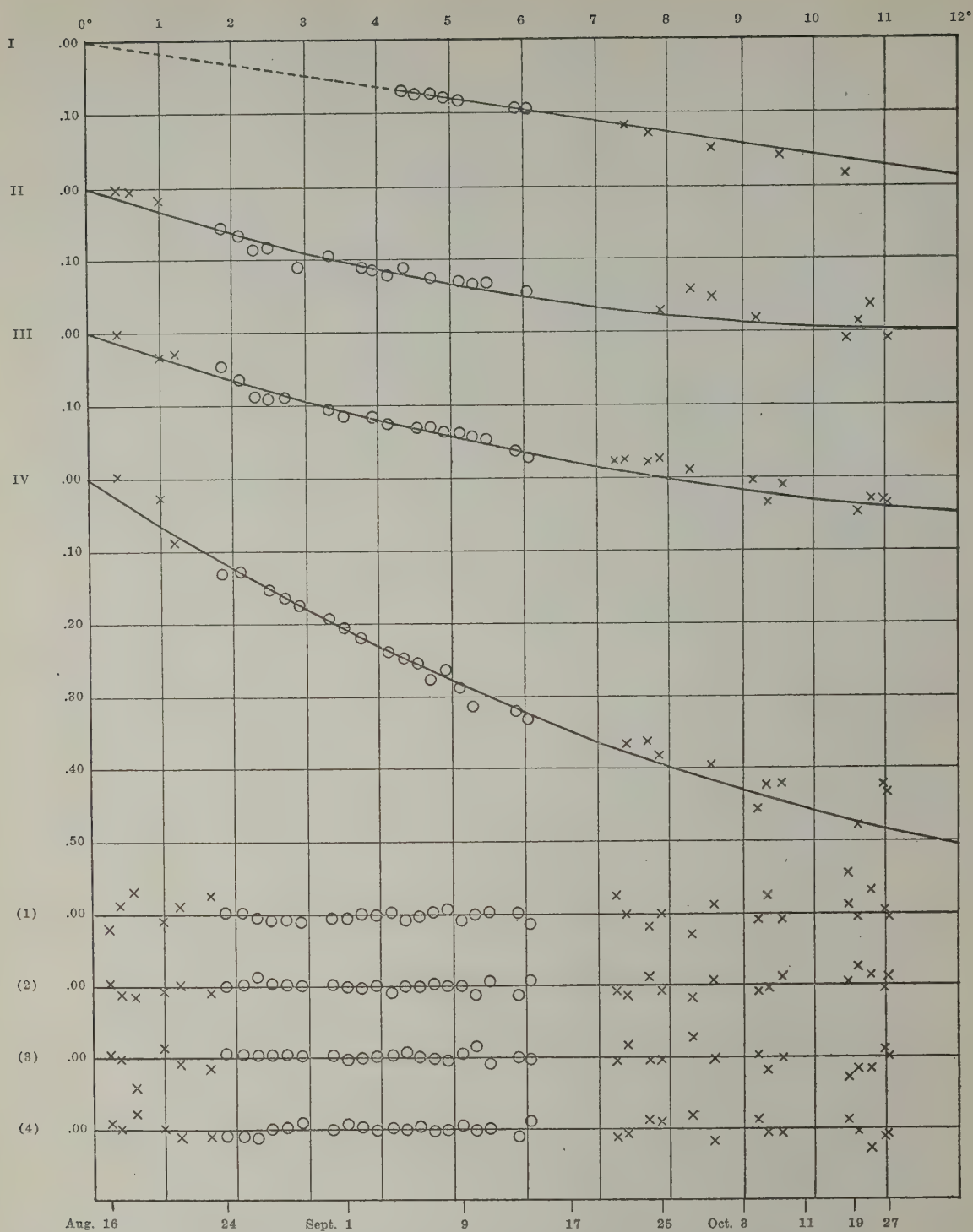


Fig. 2. Variations of the Satellites of *Jupiter* with Solar Phase; Comparison Stars.

by this method. With average errors of say four-tenths of one per cent, a solar variation of one or two per cent inside of a week ought not to escape detection, and these results could be used for confirmation of the short-period variations found by the Smithsonian observers. We should expect that the solar variation at 4600A, the region of maximum sensitivity of the photo-electric cell, would vary more than the total radiation which makes up the solar constant.

In the 1926 opposition, *Jupiter* was pretty far south, the declination being -17° . Succeeding oppositions will be more and more favorable for northern latitudes, but this advantage will be offset in part by the dates running into the winter season. In 1927 there will be ample opportunity to get a long series before opposition, which comes on September 22.

We now turn to a comparison with the previous photometric observations of the satellites of *Jupiter*. The results of visual observers, extending over half a century, have been brought together by Guthnick in *Astronomische Nachrichten*, **198**, 233, 1914. The wriggly curves there exhibited for the satellites of *Jupiter* and the inner satellites of *Saturn* are not particularly convincing, but the variation of *Iapetus* with a range of 1.5 magnitudes is within reach of visual photometers. The inadequacy of visual observations is evidenced by their failure to show the conspicuous solar-phase effect, ranging from 0.2 to 0.5 magnitude for different satellites. Even when only differential measures on the satellites themselves are made, satellite IV should show a discrepancy of 0.3 magnitude from the others, because of its greater solar-phase coefficient. The solar-phase variation of the planet *Jupiter* has been determined photo-electrically by Guthnick (see table VIII), with of course satisfactory precision.

The present conclusion that the four bright satellites have light-variations in conformity with each keeping one side toward the planet is in complete disagreement with the observations of Professor W. H. Pickering on the forms of the satellites. In his recent paper⁵ on the third satellite he finds an ellipticity of 1.05 or more and a semi-period of rotation 3.5 hours, which would give a corresponding light-variation of five per cent. All I can say is that the light-variations observable with the photo-electric photometer are much smaller than anything that my own eye would be capable of in detecting elliptical forms of planetary disks, of whatever size.

We now compare the solar phase-effect of these satellites with the same phenomenon for other bodies. Following Mueller it has been customary to define the "phase-coefficient" as the change in magnitude per degree change in solar phase, but because of the terms

in a^2 the present results are not strictly comparable with those where a linear relation was assumed. A rough comparison may be made by assuming the phase-coefficient to be one-tenth of the sum of the a and a^2 terms for 10° . When this is done, we have the following:

TABLE VIII

	Phase-Coefficient magnitude per degree	Authority
Jupiter I	0.016	This paper.
II	.019	
III	.023	
IV	.046	
Jupiter	.015	Guthnick, <i>Astronomische Nachrichten</i> , 206, 157, 1918.
Mercury	.037	Mueller, 1893.
Venus	.013	Mueller, 1893.
Mars	.015	Mueller, 1893.
Moon	.022	Russell, <i>Astrophysical Journal</i> , 43 , 114, 1916.
Mean of 30 asteroids	.030	Mueller, <i>Photometrie der Gestirne</i> , p. 379.

The coefficients for *Jupiter* and his satellites are photo-electric; the others are visual. It will be noted that the phase-effect for the inner three satellites does not much exceed that for *Jupiter*, being slightly less than that of the Moon, while the fourth satellite flashes out much more; its coefficient would rank high even for an asteroid.

SUMMARY

The present series of photo-electric measures of the four bright satellites of *Jupiter* shows that each one is variable in its period of revolution, and also exhibits a flashing up at the full phase. These phenomena are readily explained by the assumption that the periods of rotation and revolution of each satellite are the same and that each has a rough, irregularly spotted surface like the Moon.

Comparison of the satellites with a group of four stars gives a test of the constancy of the solar radiation with the effect of the Earth's atmosphere practically eliminated. During three weeks of continuous observation, the mean residual for a night was ± 0.004 magnitude, or four-tenths of one per cent, an accordance which shows that this is a promising method for tests of the Sun.

I am much indebted to Dr. Aitken and other members of the observatory staff for courtesies extended during the course of this work.

MADISON, WISCONSIN,
January, 1927.
Issued March 10, 1927.

⁵ *Publ. A. S. P.*, **37**, 191, 1925.

LICK OBSERVATORY BULLETIN

NUMBER 386

THE SPECTROSCOPIC BINARY β DORADUS

BY

DOROTHY APPLGATE

The variable radial velocity of β Doradus ($\alpha = 5^h 32^m 7^s$; $\delta = -62^\circ 33'$) was discovered by Palmer¹ from his measures of three-prism spectrograms secured at Santiago, Chile, in 1903 and 1904. Additional three-prism observations were made during the years 1903–1910 and from 1913–1918 36 two-prism observations were taken and made use of by R. E. Wilson² who published an orbit in 1921. By using the three-prism observations to supplement those made with the two-prism, he found the period to be 9.842 days and showed that his observations could not be represented by simple elliptic motion. He also noted that the spectrum and orbital elements resemble those of Cepheids. In 1920 a new series of three-prism spectrograms was taken for Wilson who intended to continue his investigation; the pressure of other work, however, prevented his doing so and the spectrograms were sent to Mt. Hamilton where the present study of them was undertaken last year. In the meantime, in 1924, Lunt pub-

lished an orbit³ for this star based upon 44 observations obtained with the four-prism spectrograph at the Cape Observatory in 1909–1913 and 1921–1922 and the six Chile radial velocities of 1903–1904. Using a period of 9.84072 days he derived a set of orbital elements on the assumption of simple elliptic motion. It is to be noted, however, that his observations in the portion of the velocity curve directly preceding the maximum velocity of approach are not well represented by the computed curve, and afford evidence of an irregularity similar to that found by Wilson. In order to bring the four Cape observations secured in 1909–1913 with the long camera into agreement with the short camera ones of 1921–1922, Lunt found it necessary to apply to them a systematic correction of 2.5 km./sec. Likewise a correction of -5 km. was applied to the early Santiago observations in order to adjust them to his curve. He concluded from this that the velocity of the system was probably variable.

¹ *L. O. B.* 3, 3, 1904. ² *Pop. Astr.* 29, 84, 1921.³ *An. Cape Obs.* 10, 29G, 1924.

TABLE 1

Date	J. D.	Phase	V km./sec.	Meas. by	O-C km./sec.		Remarks
1903 Sept. 20	6378.86	4.43	+ 1.10 + 1.19 - 0.36	M P Ag	+0.5	Hn a Car.	
Dec. 21	6470.73	7.72	+16.07 +16.64	P Ag	+1.8		
04 Jan. 22	6502.64	0.26	+28.16 +30.25 +27.05	P M Ag	+2.9	Hn a Car.	
Sept. 12	6736.88	8.14	+18.4	P&W	-0.2		w $\frac{1}{2}$
Nov. 14	6799.70	2.06	+ 1.4	P&W	-0.6		w $\frac{1}{2}$
Dec. 9	6824.75	7.43	+11.8	P&W	+0.2		w $\frac{1}{2}$

TABLE 1—(Continued)

Date	J. D.	Phase	V km./sec.	Meas. by	O-C km./sec.		Remarks
1906 Dec. 14	7559.77	4.31	+ 0.9	Pk	+0.4		w 1/2
			+ 0.6	Ag			
07 Jan. 11	7587.68	2.69	+ 1.95	Cs	+0.1	Hn a Car.	
			- 0.66	Ag			
Nov. 4	7884.75	4.50	- 5.51	Pk	-3.6		
			- 3.02	M		Hn a Car.	
			- 3.66	M			
			- 3.69	Ag			
Nov. 15	7895.71	5.60	- 4.05	Pk	-0.3		
			- 5.31	Ag			
08 Jan. 10	7951.69	2.55	+ 1.22	Pk	+0.8		
			+ 1.49	Ag			
10 Oct. 19	8964.78	1.93	+ 1.05	Pk	-1.6		
			+ 1.74	Ag			
13 Nov. 24	0096.64	1.92	+ 2.2	Hu	-0.5	Hn a Car.	II
			+ 0.4	R.W.		Hn δ Sag.	
15 Sept. 22	0763.86	9.75	+26.6	Sc	+0.8	Hn a Car.	II
Oct. 11	0782.68	8.98	+29.6	Sc	+4.8	Hn a Car.	II
			+27.2	Hu		Hn a Car.	
16 Oct. 4	1141.78	3.84	+ 1.5	Sc	+1.2	Hn a Car.	II
			+ 1.0	Hu		Hn a Car.	
Oct. 8	1145.82	7.88	+15.6	Sc	+0.2	Hn a Car.	II
Oct. 25	1162.73	5.11	- 7.3	Sc	-1.3	Hn a Car.	II
			- 7.3	Hu		Hn a Car.	
Nov. 14	1182.82	5.52	- 2.6	Sc	+3.0	Hn a Car.	II
17 Feb. 21	1281.58	5.86	- 3.6	Sc	+0.7	Hn a Car.	II
	23	1283.60	7.88	Sc	-2.0	Hn a Car.	II
	25	1285.57	0.01	Sc	-0.8	Hn a Car.	II
			+25.3	Hu		Hn a Car.	
27	1287.54	1.98	+ 1.3	Sc	-0.3	Hn a Car.	II
Mar. 1	1289.55	3.99	- 1.1	Sc	-0.9	Hn a Car.	II
	2	1290.52	4.96	Sc	+1.6	Hn a Car.	II
	4	1292.56	7.00	Sc	-0.3	Hn a Car.	II
	6	1294.54	8.98	Sc	-1.0	Hn a Car.	II
			+23.9	Hu		Hn a Car.	
	7	1295.54	0.14	Sc	+0.9	Hn a Car.	II
			+26.6	Hu		Hn a Car.	
	9	1297.57	2.17	Sc	-1.7	Hn a Car.	II
	11	1299.55	4.15	Sc	+2.5	Hn a Car.	II
Sept. 19	1491.80	9.40	+27.4	R.W.	+2.1	Hn a Car.	II
24	1496.89	4.65	- 4.3	R.W.	-0.7	Hn a Car.	II
			- 2.7	Hu		Hn a Car.	
Oct. 16	1518.72	6.80	+ 2.7	Hu	-1.0	Hn a Car.	II
	19	1521.82	0.06	Hu	+0.5	Hn a Car.	II
	22	1524.69	2.93	Hu	+3.0	Hn a Car.	II
	25	1527.71	5.95	Hu	-1.7	Hn a Car.	II
	26	1528.66	6.90	Hu	+1.2	Hn a Car.	II
18 Jan. 11	1605.64	5.14	- 3.2	R.W.	+2.6	Hn a Car.	II
	13	1607.57	7.07	R.W.	-1.9	Hn a Car.	II
	14	1608.64	8.14	R.W.	-1.5	Hn a Car.	II
	15	1609.59	9.1	R.W.	+1.6	Hn a Car.	II
	16	1610.63	0.3	R.W.	0.0	Hn a Car.	II
	17	1611.62	1.3	R.W.	-1.3	Hn a Car.	II
	18	1612.67	2.3	R.W.	-0.8	Hn a Car.	II
	20	1614.55	4.2	R.W.	+0.6	Hn a Car.	II
	21	1615.55	5.2	Hu	-1.7	Hn a Car.	II
			- 8.2	R.W.		Hn a Car.	
	22	1616.56	6.2	R.W.	+1.9	Hn a Car.	II
	23	1617.56	7.2	R.W.	-3.8	Hn a Car.	II
20 Mar. 27	2411.55	4.0	- 0.93	Hu		Hn Sky	
			+ 0.29	P	+0.1	Hn β Dor.	
			+ 0.90	P		Hn Sky	
			- 0.53	Hu		Hn a Car.	

TABLE 1—(Continued)

Date	J. D.	Phase	v km./sec.	Meas. by	O—C km./sec.		Remarks
1920	Mar. 28	2412.58	5 ^d 0	— 4.40	Hu		Hn Sky
				— 7.63	P		Hn β Dor.
				— 5.41	P	+0.7	Hn Sky
	Mar. 29	2413.51	6.0	— 5.09	Hu		Hn α Car.
				— 2.83	Hu		Hn Sky
				— 3.90	P	—0.4	Hn β Dor.
	Mar. 30	2414.60	7.0	— 4.11	Hu		Hn α Car.
				+ 5.72	Hu		Hn Sky
				+ 6.27	P	+0.2	Hn β Dor.
	Apr. 3	2418.58	1.2	+ 5.87	Hu		Hn α Car.
				+10.52	Hu		Hn Sky
				+ 9.87	P	—0.5	Hn β Dor.
	Apr. 4	2419.52	2.1	+ 9.42	Hu		Hn α Car.
				+ 0.84	Hu		Hn Sky
				+ 0.22	P	—0.2	Hn β Dor.
	Apr. 5	2420.52	3.1	+ 0.59	Hu		Hn α Car.
				— 0.74	Hu		Hn Sky
				— 0.45	P	—0.1	Hn β Dor.
	Apr. 6	2421.51	4.1	— 0.10	Hu		Hn α Car.
				— 0.19	Hu		Hn Sky
				— 1.35	P	—0.3	Hn β Dor.
	Apr. 7	2422.54	5.1	— 1.18	Hu		Hn α Car.
				— 4.90	P		m of 2
				— 3.23	Hu		m of 2
	Apr. 8	2423.52	6.1	— 5.46	Hu	+0.7	Hn Sky
				+ 1.47	Hu		Hn 7954
				— 5.60	P		Hn Sky
	Apr. 9	2424.52	7.1	— 5.68	Ag		m of 3
				— 1.25	Hu		
				— 2.96	Hu	+0.6	Hn α Car.
	Apr. 10	2425.52	8.1	— 1.77	P		m of 2
				— 2.53	P		Hn β Dor.
				+ 7.96	Hu		
	Apr. 11	2426.50	9.1	+ 7.12	P		Hn β Dor.
				+ 7.24	Hu	+1.0	Hn α Car.
				+ 7.98	Ag		
	Apr. 12	2427.50	0.3	+17.44	Hu		
				+17.04	P		Hn β Dor.
				+14.70	Hu	—0.7	Hn α Car.
	Apr. 20	2435.51	8.3	+16.98	Ag		
				+22.27	Hu		
				+23.86	P	—2.0	Hn β Dor.
	Apr. 21	2436.59	9.4	+22.30	Hu		Hn α Car.
				+22.65	Hu		
				+24.18	P	—0.9	Hn β Dor.
	Apr. 22	2437.53	0.5	+23.60	Hu		Hn α Car.
				+19.07	Hu		
				+19.38	P	+0.7	Hn β Dor.
	Apr. 23	2438.52	1.5	+19.40	Hu		Hn α Car.
				+25.24	Hu		
				+25.96	P	+0.3	Hn β Dor.
	Apr. 22	2437.53	0.5	+25.61	Hu		Hn α Car.
				+23.80	Hu		
				+23.78	P	+2.0	Hn β Dor.
	Apr. 23	2438.52	1.5	+23.10	Hu		Hn α Car.
				+ 6.13	Hu		
				+ 6.30	P		Hn β Dor.
	Apr. 23	2438.52	1.5	+ 7.53	Hu	—0.1	Hn α Car.
				+ 6.00	Ag		

In Table 1 is given a list of all the observations of β Doradus made at Santiago. The first column contains the date; the second the Julian day; the third the phase, using a period of 9.8417 days and reckoning from Julian day 2422427.20. In the fourth column is given the observed velocity after the usual correction has been applied to reduce it to the Lick Observatory system of radial velocities. These corrections take care of personal equations and systematic errors introduced by different standard comparison plates employed with the Hartmann comparator. In the fifth column the measurer is indicated. The following abbreviations have been used for the different observers: M, Moore;

star's variation and the last and first halves respectively of the two adjacent periods. As an aid in determining the period more accurately the 1903-1910 observations made with the same spectrograph were used.

After this observational curve had been determined by the three-prism spectrograms it was desired to investigate its agreement with the other available observations of the star. Accordingly there are plotted in the figure in addition to the 1920 three-prism observations: the two other sets from Santiago—12 1903-1910 3-prism and 36 1913-1918 2-prism; and two sets published by Lunt—4 1909-1913 long camera and 40 1921-1922 short camera.

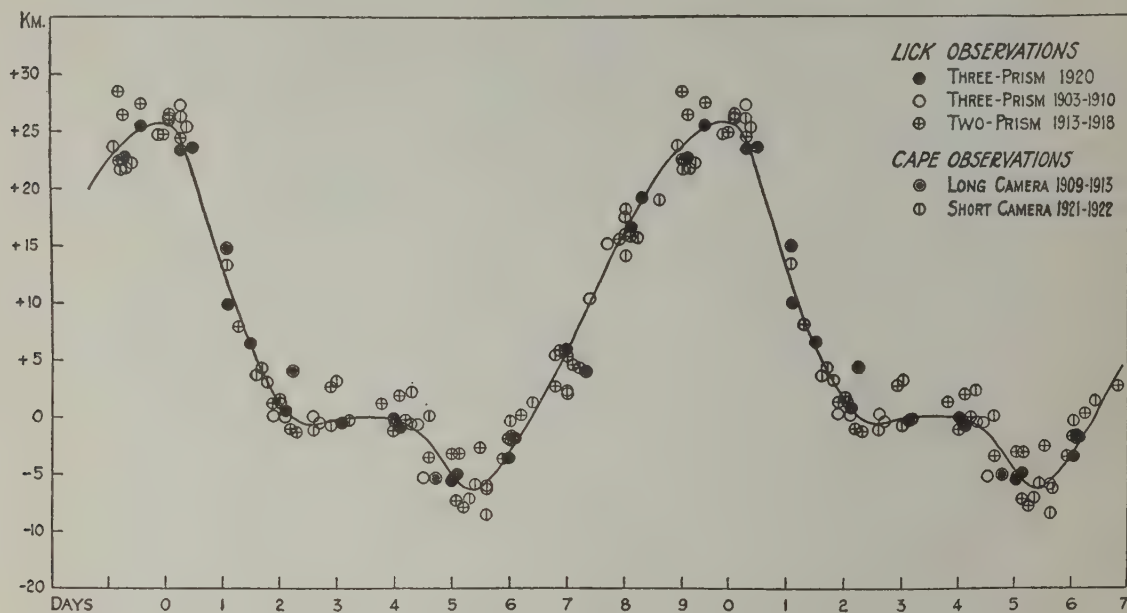


Figure 1. Velocity curve of β Doradus.

P, Palmer; W, Wright; Pk, Paddock; Cs, Curtis; Hu, Huffer; R.W., Wilson; Sc, Scott; Ag, Applegate. In the next column are given the residuals read from the observational curve in the sense, observational curve minus observed value with the systematic correction. The measures were made in the usual way with a micrometer microscope except as indicated in the seventh column, where the abbreviation Hn signifies that the plate was measured on the Hartmann spectro-comparator, against a standard plate of the star noted. Where not otherwise indicated in the column under "Remarks" the observation was made with the three-prism spectrograph.

The observational curve, Fig. 1, represents the 18 observations made during March and April, 1920, with the three-prism spectrograph of the Chile Station. These observations cover one complete period of the

It was found necessary to add a correction of -1.6 km./sec. to the measures at hand of the 1903-1910 three-prism observations after the usual corrections had been applied in order to bring them into systematic agreement with the 1920 observations. In order to determine whether this discrepancy was real or the result of different methods of measurement or of instrumental adjustments I measured the best eight plates. The correction that would have to be applied to my measures was first determined by measures of a number of *Venus* and *Mars* plates taken during the same years with the same instrument. These corrected measures stood systematically 0.7 km. above the observational curve. This difference being in the same sense as that found for the old measures indicates that the discrepancy is real but the number of observations and the magnitudes of the residuals do not render

the evidence conclusive. To the new and old measures combined, a correction of -1.18 km. had to be applied in order to bring them into agreement with the 1920 curve.

The 1913-1918 two-prism observations fit the 1920 curve with no mean residual.

To Lunt's observed values it was found necessary to apply a correction of $+3.2$ km. for the long camera observations and one of $+1.8$ km. for the short camera.

The evidence for a change in the velocity of the system seems small. The residual -1.2 km. representing the 1920 observational curve minus the 1903-1910 observations is quite definite so far as the limited number of observations is concerned. It is not a large residual however, and cannot be given too much weight. The two-prism observations offer no evidence since, when the usual corrections have been applied, they fit the 1920 curve with no systematic difference. These observations do seem to indicate a slight change in the form of the curve, but in view of the large probable error given by the two-prism values little or no weight should be given to this indication. A comparison of the radial velocities of a large number of stars determined at the Chile Station with values obtained for the same stars at the Cape observatory shows that in general the results of the two observatories are in satisfactory agreement. However, evidence of a change of a few kilometers in the velocity of the system based upon observations made at two different observatories should not be given too much weight, at least until a thorough study has been made of the systematic differences between velocities obtained at the two observatories.

There seems to be no evidence of a change in the period.

Obviously, the observed velocity curve of β Doradus cannot be represented by simple elliptic motion. The problem of determining an orbit demands that a secondary oscillation be superimposed. The determination of this secondary cannot be very certain since its period can be either one-half or one-third the period of the primary. Wilson used a secondary oscillation of one-half the period of the primary. On the basis of the present observations, however, smaller residuals were obtained when a secondary of one-third the period of the primary was superimposed, as was found to be true in the case of ζ Geminorum.⁴ On account of this uncertainty in the derivation of the orbital elements and the doubtful value of such an orbit if the star is a Cepheid, it was considered worth while only to determine approximate elements. The elements thus determined, using as a secondary a sine curve of one-third the period are:

⁴ *Ap. J.* **13**, 90, 1901.

	PRIMARY	SECONDARY
P	9.8417 days	3.28
V	$+7.9$ km.	$+0.2$
K	$+15.4$ km.	$+2.9$
e	0.3	0.0
ω	-3°	$76^\circ \pm$
T	2422426.61 J. D.	2422428.82
$a \sin i$	$1,700,000 \pm$ km.	130,000

As a matter of interest approximate elements were also derived for an orbit of the type suggested by Curtis⁵ with the velocity curve extended down to a minimum that ignores the irregularity and considers it due to some effect aside from orbital motion:

P	9.8417 days
V	$+5.8$ km.
K	$+18.5$ km.
e	$.18 \pm$
ω	$40^\circ \pm$
T	2422427.66 J. D.
$a \sin i$	$2,400,000 \pm$ km.

The velocity of the system obtained by measuring areas under the observational curve is $+7.4$ km./sec.

It will be noted that these elements are quite similar to those of Cepheid variables. The period is practically 10 days, the eccentricity and $a \sin i$ are small, and ω is in the first quadrant.

As pointed out by Wilson, the spectrum of β Doradus also has certain peculiarities in common with Cepheids. The similarities may be grouped as follows:

(1) The spectrum is of class F5 which is a typical Cepheid type.

(2) There is a shift of the maximum intensity of the continuous spectrum toward the violet at minimum velocity, which Albrecht noted to be true of Cepheids. It may be of interest to remark in this connection that on three dates, at phases 7.1, 8.1, and 9.1, the observer noted in the observing book that the star appeared red.

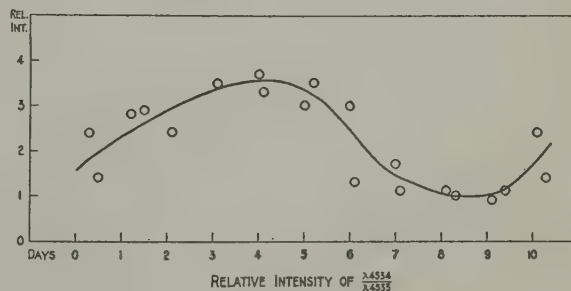


Figure 2.

(3) The enhanced lines are strengthened at minimum velocity. A number of lines was observed to vary in this manner but only one pair, $\lambda\lambda 4534$ and

⁵ *Proc. Nat. Ac. Sc.* **9**, 187, 1923.

4533A showed sufficient variation to be estimated without great difficulty and uncertainty. The relative intensity of this pair was estimated on each of the 1920 three-prism plates and plotted with phase in Fig. 2. The scale on which these intensities were estimated is purely relative, the ordinate representing intensity of $\lambda 4534$. $\lambda 4534$ is the spark line and $\lambda 4533$ intensity of $\lambda 4533$.

the arc. The curve shows that the spark line is of equal intensity with the arc line at maximum velocity, which in Cepheids corresponds to minimum light, and over three times as intense at minimum velocity, corresponding to maximum light. Henroteau observed this pair of lines⁶ in a number of Cepheids and found them varying in this way.

(4) There is some evidence that the spectral class varies during the period. The variation seems very small and I succeeded only in judging its direction, using Albrecht's⁷ criterion. Albrecht investigated and tabulated the change with spectral class in the wave lengths of a large number of lines. Eight of these lines were measured on each of the four spectrograms of β Doradus used in the study (to be described later) of lines of different level, and the direction of change of wave length throughout the period indicated for each line was compared with its variation with spectral class as found by Albrecht. All the lines but one observed in this manner indicated later type at maximum velocity and earlier at minimum.

Adams and Joy⁸ have found a much greater range of spectral type as indicated by hydrogen lines in Cepheids than that obtained from the general spectrum; therefore $H\gamma$, the only H line included in the range of the three-prism plates, was observed on the 1920 spectrograms. The difference in intensity from maximum to minimum was not very great and was impossible to estimate except qualitatively; but it was quite certain, showing strongest at minimum velocity and weakest at maximum. The line is diffuse, but sharper at both maximum and minimum, and blends with several other lines at intermediate phases. On none of the plates is it as strong as it is in the Sun.

At the suggestion of Dr. R. H. Curtiss, a study was made of the velocities given by lines originating at different heights in the star's atmosphere. The study was not comprehensive, only four plates being measured for the purpose: one taken near maximum velocity, one near minimum, and two at intermediate phases.

Every one of the four plates was measured twice direct and twice reversed, one setting being made at each measure. The wave lengths of all lines that could

be measured with any degree of accuracy on each plate were determined. For identification of the lines Rowland's "Table of Preliminary Wave-lengths" was used, and for determining their levels, Mitchell's table of chromospheric wave-lengths published in *Ap. J.* **38**, 407, 1913, since it may be assumed that the order of the levels at which different lines originate in the atmosphere of this star is indicated by the heights of the same lines in the solar chromosphere.

The identification in a great many cases was rather uncertain due to the difficulty in assigning the proper weights to the different components of blends. However, on any one plate these uncertainties ought almost to cancel and, as will be shown, the residuals were used for deriving Table 4 in such a way as to lose them entirely.

The lines were studied with reference to the residual velocities of four different level groups: low level, containing lines of 350 to 500 km. chromospheric height; intermediate, 550 to 750 km.; high, 800 to 1000 km.; and a group of highly enhanced lines of chromospheric height ranging from 800 to 2500 km. The residuals as given in Table 2 represent the velocities observed on each plate for the different levels minus the mean velocity given by the low level lines on that plate. Table 3 is similar to Table 2 but made in a different way. For this table only differences between residuals given by the same line on different plates were used, the plate taken at phase 6.1 being used as standard and the values for the table being finally based on the mean residuals for the different levels on that plate. In this manner errors of identification were eliminated, but for them were substituted, in the values for each plate, the errors to which the values for the standard plate were subject. In Table 4, column 1 contains the phase of each plate measured; column 2 the total number of lines measured on each plate; and columns 4, 5, 6, and 7 the number of lines of each level measured on each plate.

TABLE 2
RESIDUAL VELOCITIES

Phase	Low Level	Medium	High	Enhanced
1.2	0.00	+1.37	+3.18	-0.63
5.1	0.00	+0.64	+1.18	-0.92
6.1	0.00	-0.82	+3.61	-4.16
9.4	0.00	+2.36	+1.98	-3.42

TABLE 3
RESIDUAL VELOCITIES

Phase	Low Level	Medium	High	Enhanced
1.2	0.00	+1.27	+2.59	-1.42
5.1	0.00	-0.87	+2.10	-3.05
6.1	0.00	-0.82	+3.61	-4.16
9.4	0.00	+1.35		-1.83

⁶ *J. Roy. Astr. Soc. Can.* **19**, 81, 1925.

⁷ *Ap. J.* **54**, 161, 1921.

⁸ *Proc. Nat. Ac. Sc.* **4**, 129, 1918.

TABLE 4
NUMBER OF LINES

Phase	Total	Low Level	Medium	High	Enhanced
1.2	98	66	18	8	6
5.1	83	50	21	7	5
6.1	77	60	8	4	5
9.4	49	36	10	1	2

For comparing the velocities given by different levels on one plate, Table 2 is probably more reliable than Table 3 since for it more lines were used in as much as lines measured only on one plate were available, and because in Table 3 the relative values for the different levels on each plate are affected in the same way by the inaccuracies of the standard plate. For studying the variation of one level from phase to phase Table 3 is probably better since here the errors of identification were lost and the whole computation was concerned with the relative values from phase to phase.

It will be seen by these tables that lines originating at high levels show increasingly greater velocities of recession than do those of low level until we come to the very high enhanced lines. Then the difference, high minus low, becomes negative instead of positive. There were only six of these lines used, however. If two of them, which are blends and have the enhanced components to the violet, are removed the difference becomes smaller but does not change sign. This result is contrary to that found by Curtiss and others. Curtiss⁹ did not find the velocities given by high level lines differing consistently in either sense from those given by low level lines; only back and forth with phase; and did not find his very high enhanced lines to vary in a different sense from the other high level lines.

Tracing the variation of any one of the high levels from phase to phase we find the residual, high minus low, greatest at maximum velocity and least at minimum. If we assume that in this star, as in stars known to be Cepheids, maximum velocity of recession corresponds to minimum light and minimum velocity to maximum light, then this variation is in the opposite sense to that found by Curtiss and anticipated by St. John and Adams.¹⁰ Instead of having the "downward drift" of the high level lines more pronounced at maximum temperature we have it more pronounced at minimum. St. John and Adams found that in the stars of different temperatures which they studied the difference, high minus low, was always positive although

it varied with temperature, the difference being greater at higher temperatures. Unlike the results for β *Doradus*, they found the difference, arc minus spark, to be consistent with the high minus low results.

SUMMARY

The results of this investigation may be summarized briefly as follows:

1. The velocity curve of β *Doradus* is not representative of simple elliptic motion but is like those of ζ *Geminorum* and some other Cepheids.
2. The elements of an orbit for this star are similar to those of Cepheids.
3. The spectrum is similar to that of Cepheids in several ways: notably in the change throughout the period of the intensities of certain lines and in the shift of the maximum intensity of the continuous background to the violet at the time of minimum velocity.
4. The spectral class is typical of Cepheids.
5. There seems to be a definite difference between residual velocities as given by lines originating at different levels and between those of the same level at different phases of the period. The direction of this difference is opposite to that found by Curtiss in the case of W *Sagittarii*.

In view of these phenomena β *Doradus* becomes interesting as being a star which shows Cepheid characteristics without having the light variation typical of all other Cepheids. It seems remarkable that if there is any variation it should not have been detected since the star is of 3.8 visual magnitude. There may be, however, a small variation. In fact, a glance at the spectrograms shows that those taken at minimum velocity are, practically without exception, better exposed than those taken near maximum velocity, some of the latter being quite underexposed. Assuming that the time of exposure was more or less the same for all the plates, and that conditions were more or less the same, this would seem to indicate that the star's light is really variable. At least, the possibility that this may be the case might justify an investigation.

My thanks are due to Dr. J. H. Moore for his help and advice during the course of this work.

February, 1927.

Issued March 14, 1927.

⁹ *Publ. Ast. Soc. Pac.* **38**, 148, 1926.

¹⁰ *Ap. J.* **60**, 43, 1924.

LICK OBSERVATORY BULLETIN

NUMBER 387

OBSERVATIONS OF MARS AT THE OPPOSITION OF 1924

BY

ROBERT J. TRUMPLER

The general program of the Lick Observatory for observations of *Mars* at the opposition of 1924 consisted of two parts. The first, a color survey of the planet through photography by light of different colors, was undertaken by Mr. Wright with the Crossley reflector and his results have already been reported upon.¹ The second part, which forms the subject of the present publication, is based on photographic and visual observations with the 36-inch refractor and is accordingly restricted to the small region of the spectrum within which this telescope functions effectively.

At first the program was to a certain extent experimental in character, in order to compare the relative merits of the photographic and visual method of observation. When many of the photographs were found to be of good definition showing a considerable amount of surface detail, the observations were regularly continued. The ease with which micrometric measures can be made on photographs suggested the utilization of the plate material not only for descriptive purposes but also for obtaining accurate areographic longitudes and latitudes of numerous markings and for the determination of some of the constants such as the diameter and flattening of *Mars* and the position of its axis of rotation. A preliminary account of the observations was given in the *Publications of the Astronomical Society of the Pacific* (36, 263, 1924).

1. THE OBSERVATIONS

While most planetary photographs so far have been obtained with an enlarging apparatus in the telescope, the long focal length ($58\frac{1}{2}$ feet, 17.6m) of the Lick refractor made it possible to take the photographs

directly at the focus of the objective. A photographic attachment was accordingly constructed at the workshop of the observatory comprising a double slide plateholder for 4x5 inch plates. With careful rating of the clock no guiding was necessary during the short exposures (1.5 to 20 seconds), but the sliding plateholder was convenient for making a large number of exposures (40–50) in rapid succession on the same plate, shifting the plate slightly between the exposures.

The Lick objective, being designed for visual observations, has a good color correction only for yellow and orange light (minimum focus at 5700 Å.). It is necessary, therefore, to use plates sensitized to yellow light and to insert a color filter a short distance in front of the plate to exclude the light of short wavelengths. As the focal image of *Mars* in the 36-inch refractor is a disk of only 2.2 mm. in diameter or less, a fine grain of the plate is of prime importance, and it is further desirable that the plate should have a high degree of contrast in order to bring out the more delicate shadings. Trying several commercial brands of plates, Seed process plates were found most suitable for this work, and they were sensitized to the yellow and orange by bathing in a solution of Pinaverdol according to the instructions of the Bureau of Standards. It is a remarkable fact that these plates, very slow in ordinary photographic work, gain in speed when sensitized to the longer wavelengths. With a yellow filter they required only about twice the exposure time of the fastest commercial orthochromatic plates, while they are far superior in grain and contrast. The plates were developed for 60 seconds with Rodinal developer 1:5 at a temperature of 70° F.

Two different color filters were used. The Flavazine T filter (No. 16 in the Wratten light filter catalogue

¹ *Publ. A. S. P.* 36, 239, 1924; *L. O. Bulletin* 12, 48, 1925.

of 1922 of the Eastman Kodak Co.) is in the following designated as yellow filter. When its absorption is combined with the color sensitivity of the plates, only wavelengths between about 5300 and 5900 Å. are effective and the photographs are practically monochromatic. Wright's observations with the Crossley reflector showed that photographs of *Mars* taken in infra-red light produced greater contrast between the dark markings and the normal surface and would be more advantageous for the study of surface details. Although a refractor is not suited for infra-red photography, some advantage at least might be gained by using the visual red. A filter of red selenium glass, designated in the following as red filter, combined with the same plates utilizes light of the wavelengths 5900 to 6100 Å. and its use resulted in an appreciable increase of contrast, while the chromatic aberration of the objective is not yet sensible in this region of the spectrum. In the later part of the work most plates contain exposures taken with each of the two filters. The yellow photographs required an exposure of $1\frac{1}{2}$ to 2 seconds, the red ones 15–20 seconds. The focus setting of the telescope was determined visually with an eyepiece for every plate and the different exposures of a plate were distributed among 4–5 focus settings varying within a range of about 2 mm.

A few trials were also made to enlarge the image of *Mars* in the telescope with a negative lens placed about 6 inches before the plate (enlargement $2\frac{1}{2}$ times). Fig. 2 on Plate I reproduces the best of these photographs. As a rule the enlarged images did not show more detail and often less than the direct photographs taken on the same night, while the longer exposure and smaller contrast are decided disadvantages.

When time was available between the sets of photographs, the telescope was used for visual observations resulting in 37 drawings or sketches made with magnifications 270, 350, and 520. Most of these, however, are incomplete and were intended only to supplement the photographs and to record some of the more difficult details. A few complete drawings, prepared partly from photographs, partly from visual sketches made on the same night, are published in the *Publications of the Astronomical Society of the Pacific* (36, 263, 1924) and in *Popular Astronomy* (34, March 1926). As the Lick refractor is not provided with a diaphragm, all observations, visual and photographic, were made with the full aperture of 36 inches.

The quality of the photographs depends in the first place on the steadiness and homogeneity of our atmosphere, the so-called "seeing." On a few nights the seeing was too poor to attempt any photographic or visual observations of *Mars*. Most observations were made with fair (average) seeing, when some unsteadiness in the images was noticeable with magnification

270, on a few nights the seeing could be classified as good, but in no case were the conditions so perfect as to give sharp and steady images with magnification 520. Undoubtedly the great zenith distance of *Mars* (55° – 65°) was to some extent responsible for this difficulty. It should be kept in mind, too, that the use of the telescope had to be divided among several observers and that the instrument was available only for a limited number of observing periods fixed in advance.

While the visual observer has to watch the ever changing, moving, blurring or distorting image for the moments of best definition in order to catch the finer details, the photographic observer has to rely on a great number of exposures for obtaining some photographs of good quality. On the other hand one exposure falling on a moment of best definition furnishes a whole picture; it can be studied afterwards with every care and may yield much more information than the visual observer could have gathered in that short moment. The percentage of good images among the 40–50 exposures taken in rapid succession on the same plate varies greatly with the seeing. Even on nights with only fair or average seeing quite good photographs are sometimes obtained, but they will be rare among a large number of worthless ones. Every plate was therefore first examined with a magnifying glass, and the best images were marked for further study. It is very rare that among the selected images of a plate one stands out as the best showing every detail visible on any of the others. As a rule it is found that one image shows one region to best advantage, while other images give better definition for other markings, and a comparison of three or four selected images will reveal considerably more detail than a reproduction of any single one.

This situation is often overlooked when judging the merits of photography for planetary description. Photographs are most desired for the purpose of direct reproduction and publication, and undoubtedly they have considerable value in this respect. But it must not be thought that reproduction of a few of the best images can exhaust the information contained in a series of photographic observations. In the first place, the finer details are so delicate that it is practically impossible to bring them out fully in reproductions. In the second place, it lies in the nature of telescopic seeing that in general no single image shows all details in best definition and that the results have to be gathered from several photographs by the laborious process of comparison. In the third place, many of the finer details, especially some of the so-called canals, are so faint and elusive that at a superficial examination they are overlooked or attributed to the grain of the plate; only a careful comparison of different images

can convince the observer of their reality and at the same time allow him to eliminate the effects of the plate grain as well as distortions due to the seeing. With the photographic method then the main work lies in the examination and comparison of the original photographs, and the results of this work can be adequately presented only by drawings. It was therefore decided to construct a chart of *Mars* reproducing as faithfully as possible the appearance of the photographs and combining all the details which seem to be real on account of repeated visibility.

During the opposition of 1924 about 1700 photographs of *Mars* were obtained. About 150 or 9% of these were found to be of some value for descriptive purposes, and it is of interest to note that the collection of best images on which our chart is essentially based represents a total exposure time of only 15 minutes. Plates I and II reproduce 24 of the best photographs. In order to minimize the loss of detail in the process of reproduction a rather great enlargement (12 to 18 times) was chosen, and the fainter canal-like markings may be somewhat better visible if the plates are viewed from a distance of 2 to 3 feet, when the grain becomes less disturbing. With each figure the longitude of the central meridian (λ_0) and date of observation are given, while the letters y and r indicate whether the photograph was made with the yellow or red color filter. Those of Fig. 1 and 3 of Plate II were taken on Seed 23 plates used in the earlier experiments; they show plainly the coarser grain. The image of Plate I, fig. 2 was enlarged in the telescope as already mentioned.

2. MEASUREMENT OF THE PLATES

Accurate areographic longitudes and latitudes of numerous markings were needed in order to give the chart correct proportions, and it was thought that such data would also have a value in themselves as a basis for the discussion of seasonal and progressive changes in these markings. For this purpose the original negatives were measured in rectangular coordinates with the double micrometer of the eyepiece of the Repsold measuring engine. The microscope was adjusted for a magnification of 11 diameters which corresponds to about 800 for the combined magnification of telescope and microscope. A pencil sketch of the appearance of the image was first made showing every detail visible. All points sufficiently well defined were then bisected with the vertical and horizontal wire in turn and their identification numbers entered on the sketch. Settings on the edges of the disk were made at the beginning and end of the measures of each image. As a rule four images were selected on every plate.

At first (Winter, 1924-25) 13 plates with well distributed central meridians, all taken near opposi-

tion, were measured as completely as possible. These measures were made through the glass, so that the images appeared as seen in the refractor. On seven plates two images were measured in "direct" position (South polar cap at top), two images in "reversed" position (South polar cap at bottom), while on the remaining six plates all images were measured either in the direct or the reversed position. Reducing the measures to the center of the disk as zero point and expressing them in fractions of the planet's radius, rectangular coordinates were obtained with the y axis parallel, the x axis perpendicular to the polar axis, and the results of the four images of the same plate were combined to a mean. As a first approximation the polar cap was used for the orientation of the images. The longitudes and latitudes were then calculated with the data of the *American Ephemeris* and orientation corrections applied as derived by comparing the overlapping parts of the different images. Most of the better markings had been measured on 3-5 plates, and by combining these results with appropriate weights a preliminary catalogue of areographic longitudes and latitudes was formed.

Full benefit of the accuracy of photographic measures of Martian markings can only be gained, if the constants needed to convert the rectangular coordinates into longitudes and latitudes are sufficiently reliable. This did not seem to be the case and a new determination of these constants from the photographs themselves had to be undertaken. It seemed especially doubtful, whether the settings on the edge of the disk would give the diameter of the solid surface to be used in the reduction. This last question had been raised by Wright's work, which proved that the apparent diameter of the photographic image of *Mars* varies considerably with the color in which the photographs are taken; violet or ultraviolet images have larger diameters than red or infra-red ones. This fact suggested that the visible limb of *Mars* does not always correspond to the solid surface of the planet, but rather to a certain layer of its atmosphere; different colors including more or less of this atmosphere. It seems then likely that the generally adopted value of the diameter of *Mars*, derived from visual measures of the limb, is affected in a similar way, and the question: what is the true diameter of the Martian surface? at once gains a new interest in itself, aside from its importance for the reduction of the measures.

Wright² describes a method for determining this quantity which was employed, with some modifications. On account of the rotation of *Mars* the surface markings appear to be slowly moving across the disk. A series of photographs taken at regular intervals during a period of 6-8 hours in the same night will furnish a

² *Publ. A. S. P.* 36, 252, 1924.

large part of this apparent elliptical path for certain markings. If a well defined point near the equator is selected and measured on all the exposures, the complete ellipse of its path can be constructed with the aid of the times of the exposures and the known period of rotation. The major axis of this ellipse gives the diameter of the solid surface, while its orientation and excentricity determine the position of the axis of rotation. Since the period of rotation of *Mars* is very accurately known, it is not necessary to take all the exposures on the same night. It may even be preferable to combine the best photographs of different nights, as it is difficult to obtain a long series of observations on the same night under uniformly good atmospheric conditions, and the low altitude of *Mars* at the 1924 opposition made a sufficiently long observing period impossible.

As very few changes in the markings were noticed during our brief period of observation, it seemed desirable to utilize all successful photographs for the application of this method, and it was decided to determine at the same time the diameter of *Mars* from settings on the limb so as to obtain a close comparison between the two methods. Forty plates, altogether, were found suitable in quality, and their measurement was undertaken in the summer and autumn of 1925 with the same apparatus as described above. This time, however, the plates were measured film-side up, and great care was taken to prevent any changes in the adjustment of the microscope during the whole period. The position angle of the polar axis to which the plates were oriented was taken from the *American Ephemeris*. Four images were again selected on most plates. The settings on the limb were made at the endpoints of the polar and equatorial diameters; they were repeated for each image in four positions of the plate; for the markings, one image was measured in each of the four positions. In this second series of measures no attempt was made to include all markings visible, as the preliminary measures of 13 plates had already given a sufficient number of reference points for the chart. The measures were confined to three classes of markings:

1. Points on the outline of the polar cap and details in its immediate surroundings.
2. Six to ten well defined points each, near the eastern and western limb of the planet and situated not very far from the equator. Such points of course are of greatest weight in the determination of the diameter and of the position angle of the axis of rotation.
3. Any marking appearing in exceptionally good definition or of unsatisfactory weight in the preliminary catalogue.

Many points on the 13 plates of the first series were thus remeasured, and this made it possible to reduce

the results of the first measures to the more homogeneous system of the second measures by appropriate zero and scale corrections applied to the rectangular coordinates.

3. SCALE VALUE AND ZERO POINT OF POSITION ANGLE

When the *Mars* photographs were taken it was not anticipated that they would be used for such extensive measures, and no provision was made at the time to furnish the scale value and position angle orientation of each plate. These two constants had therefore to be determined from photographs of wide double stars taken during the following summer (1925). For the scale value, which varies very little for focal images of a large refractor and has to be known only with an accuracy of 1 part in 1000, this method seemed perfectly justified. Some doubts were, however, entertained at first, whether the position angle orientation of the plate would remain sufficiently constant. While plate and plateholder could not appreciably change their position relatively to the camera attachment, it was necessary to investigate whether the latter, when screwed in at the end of the telescope, would always stop at the same point. The double star observations of 1925 made in exactly the same way as the *Mars* photographs, however, dispelled all doubts on this point and showed that even over a period of five months no changes in position angle orientation of more than a few tenths of a degree occurred.

The double star photographs were measured interspersed among the *Mars* photographs, each image being measured in the four positions of the plate. The results are tabulated in Table 1. The number of the pair in the first column is that of Burnham's *General Catalogue*, the adopted position (apparent) in the second and third columns was kindly furnished by Dr. Aitken from the best recent measures of double star observers. Only for B.G.C. 7018 these data were calculated from Boss's *Preliminary General Catalogue*, which gives the positions of both stars. For this pair the distance was too large for direct measurement with the micrometer and was subdivided into intervals of 40"-50" by fine lines drawn with a needle on the film. After the date of the photographs (5th col.) and the temperature (6th col.) during the observations follows the number of images measured on the plate and their quality (g=good, f=fair, p=poor). The last three columns contain the correction to the position angle reading of the measuring engine, the value of a screw revolution (mean of both screws, there is no appreciable difference between them), and the weight adopted for each result of the screw value. The latter was chosen according to the separation of the pair, the accuracy of its adopted position, the number and quality of the images measured. For the position angle cor-

TABLE 1
POSITION ANGLE CORRECTION AND SCREW VALUE

B.G.C.	Adopted position 1925 Pos. A.	Distance	p.E.	Date of obs. 1925	Temp. C.	Images measured	Corr. to Pos. A.	Screw value 1 revol.	Weight
6212	271.1	20.26	±.05	July 1	+20.0	3 p	-3.1	5.923	0.5
7018	314.3	230.98	.08	May 22	+11.5	4 f	-3.7	.887	2
8914	103.5	22.10	.05	July 18	+24.7	6 g	-3.0	.896	1
9005	208.0	37.42	.10	Aug. 29	+17.8	5 g	-3.8	.851	0.5
9225	2.8	57.50	.06	July 17	+26.7	6 g	-3.0	.877	1
9374	54.3	34.52	.05	July 17	+26.7	4 p	-3.0	.890	0.5
9707	170.1	35.69	.06	July 25	+18.2	6 g	-4.1	.894	1
11103	349.0	39.22	.09	Oct. 17	+ 8.9	6 g	-3.7	.889	1
Mean:					+18.5		-3.4±0.1	5.888±.004	

rection no weights were employed. The mean values of the constants used in the reduction of the *Mars* photographs are given in the last line together with their probable errors. The weighted mean temperature of the double star observations is +18.5, that of the *Mars* observations +16.5.

4. DETERMINATION OF THE DIAMETER OF MARS FROM MEASURES OF THE LIMB

The settings on the edges of the disk made on 40 plates during the second series of measures furnish the material for a determination of the apparent diameter of *Mars*. One plate (368) was rejected, because, being overexposed, it had been reduced with a solution of ferricyanide of potassium, a treatment which considerably changed the appearance of the edge of the images. On the other hand, a later plate (459), taken when *Mars* was more distant, could be used for the diameter, although not suitable for measuring markings. On this plate only the cusp diameters were measured which, however, are nearly parallel to the polar axis. There were thus 147 images available for each of which the polar and equatorial diameters had been measured in four positions of the plate; 103 of these had been taken with the yellow filter, 44 with the red one; there should further be added the four images of plate 459, mentioned above.

The yellow images have a rather sharp edge at the limb, and to this the micrometer wires were made tangent. The red images show a more gradual fading toward the limb, but still there was some appearance of an edge at which the micrometer settings could be made. Great difficulty was encountered at the terminator where the transition from illumination to darkness is very gradual; in this case the settings had to be made to the extreme limit at which any exposure of the film was noticeable, and they are necessarily more uncertain.

The measured diameters were first corrected for the thickness of the micrometer wire (0".118) and for refraction and expressed as corrections (O—C) to the values

of the *American Ephemeris*. Table 2 presents the results of the diameter measures, giving in the first column the number of the plate, in the second the mean date of the four exposures in Greenwich Mean Time, in the third the mean quality of the images estimated on the following scale: 1.2 for excellent, 1.0 for good, 0.7 for fair, 0.3 for poor; in the fourth column the diameter of *Mars* according to the *American Ephemeris*. The equatorial and polar diameters of the illuminated part of the disk are obtained by subtracting the phase corrections q_e and q_p of col. 5 and 6 from the Ephemeris diameter. The calculation of the phase corrections needs some explanation.

Phase Correction.—As the terminator is an ellipse, while the limb is a circle, the two points T and T' (see Fig. 1) at which the micrometer wire is tangent to the

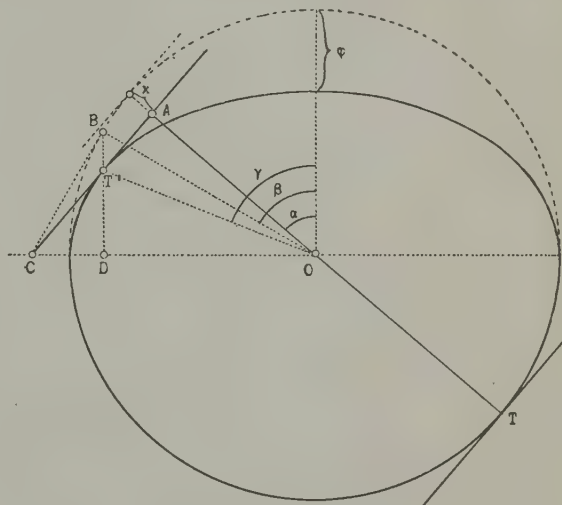


Figure 1.

illuminated disk do not lie on the same diameter. In this figure, x represents the phase correction to be calculated; it can be expressed as a function of ϕ (greatest defect of illumination) and α (angle between micrometer wires and cusp diameter). Using the

TABLE 2
MEASURES OF THE DIAMETER OF MARS

Plate	Date of expos. G.M.T.	Qual. of im.	Ephem. diam. d	Phase Correction		Yellow Filter O-C			Red Filter O-C			Flattening		Yellow-red diam.	Wt.	Plate mean O-C	Wt.	Diam. at unit dist.	Corr. to pos. A. of pol. Axis	Wt.
				φE	φF	Equ.	Pole	No.	Equ.	Pole	No.	Yellow	Red							
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
	1924											%	%	%						
333	Aug. 20. 910	1.0	25.09	.02	.04	+.14	-.14	4				+1.1				+.12	1.0	9.40	-4.5	2.9
339	21. 840	.7	25.10	.01	.04	+.42	+.31	2				+.4				+.49	.35	.54	-1.8	1.3
340	21. 869	.6	25.10	.01	.04	+.15	-.17	2				+1.3				+.11	.3	.40	-.4	1.3
344	22. 754	.6	25.11	.00	.05	+.28	-.10	1	-.07	-.42	1	+1.5	+1.4	+1.3	.8	+.16	.3	.42	-.9	1.8
345	22. 811	1.0	25.11	.00	.05	+.07	-.22	3	-.40	-.57	1	+1.2	+.7	+1.6	1.2	.00	1.0	.36	-.6	4.2
346	23. 798	.6	25.10	.00	.05	+.15	-.06	4				+.8				+.17	.6	.42	+.7	3.3
348	23. 928	.9	25.09	.00	.05	+.07	-.16	4				+.9				+.08	.9	.39	+.6	3.3
350	24. 764	.6	25.08	.00	.05	+.15	-.24	4				+1.6				+.08	.6	.39	+1.3	3.8
351	24. 854	1.1	25.08	.00	.05	+.15	-.15	4				+1.2				+.12	1.1	.41	+0.9	4.3
355	25. 812	.7	25.05	.00	.05	-.15	-.49	4				+1.4				-.20	.7	.29	+1.1	4.5
358	26. 790	1.0	25.02	.01	.05	+.12	-.24	4				+1.4				+.06	1.0	.38	0.0	4.5
361	28. 769	1.0	24.92	.03	.06	+.14	-.15	2	-.02	-.38	2	+1.2	+1.5	+.8	1.5	+.13	1.0	.41	-0.5	4.5
362	28. 851	1.0	24.91	.03	.06	+.10	-.07	2	+.05	-.34	2	+.6	+1.6	+.6	1.5	+.17	1.0	.42	+.2	3.7
363	29. 812	1.2	24.85	.05	.06	+.02	-.18	3	-.05	-.31	1	+.8	+1.0	+.4	1.2	+.08	1.2	.39	+1.5	4.6
368	30. 792	1.0	24.78	.06	.07			4											-1.2	4.0
371	31. 790	1.0	24.70	.08	.07				-.13	-.24	4		+.4			+.16	1.0	.42	-2.2	4.6
372	31. 837	.6	24.70	.08	.07	+.15	+.01	4				+.6				+.21	.6	.44	-.8	4.9
377	Sept. 1. 825	.9	24.61	.11	.07	+.27	-.07	4				+1.4				+.23	.9	.45	-1.3	5.0
378	2. 820	1.2	24.51	.13	.07	+.07	-.03	2	+.01	-.14	2	+.4	+.7	+.3	1.5	+.21	1.2	.44	-.2	4.2
384	4. 798	1.0	24.29	.19	.07	+.06	-.04	2	+.15	-.08	2	+.4	+1.0	-.1	1.5	+.26	1.0	.46	-.6	4.2
385	4. 874	.7	24.29	.19	.07	-.31	-.36	1	-.43	-.59	3	+.2	+.7	+.7	1.1	-.17	.7	.29	-.9	2.6
386	7. 768	1.1	23.91	.28	.08	+.16	.00	4				+.7				+.23	.8	.45	-1.2	3.8
387	7. 793	1.2	23.91	.28	.08				-.16	-.38	4		+.9			+.07	.9	.39	-1.7	3.3
392	8. 805	.9	23.77	.31	.09	+.04	+.07	1	+.05	-.11	3	-.1	+.7	+.5	.8	+.30	.7	.48	-2.1	3.1
400	11. 723	1.1	23.33	.44	.08	+.36	+.15	1	+.27	-.02	3	+.9	+1.2	+.6	.8	+.44	.8	.53	-.8	4.9
401	11. 824	1.0	23.31	.44	.08	+.15	-.15	2	-.02	-.25	2	+1.3	+1.0	+.5	1.0	+.16	.8	.43	-1.4	4.3
402	13. 808	1.1	22.98	.52	.08	-.11	-.20	2	-.30	-.38	2	+.4	+.3	+.8	1.0	+.01	.8	.37	-2.8	4.5
404	15. 742	1.0	22.65	.59	.08	+.08	+.03	3	-.24	-.25	1	+.2	0.0	+1.3	.7	+.20	.8	.44	-5.5	2.6
409	21. 763	1.0	21.54	.84	.08	+.11	-.09	2	-.32	-.47	2	+.9	+.7	+1.8	.9	+.05	.8	.38	-1.8	2.8
410	21. 853	.9	21.52	.84	.08	+.03	.00	2	-.38	-.41	2	+.1	+.1	+1.9	.9	+.08	.7	.39	-1.8	2.0
412	22. 675	.5	21.36	.88	.08	+.06	+.02	3	-.21	-.12	1	+.2	-.4	+.9	.6	+.21	.4	.45	-1.8	2.0
413	22. 771	.5	21.35	.88	.08	+.46	+.22	2	+.32	+.07	2	+1.1	+1.2	+.7	.9	+.50	.4	.58	-1.5	2.1
425	26. 706	1.0	20.58	1.01	.08	-.26	-.13	2	-.44	-.37	2	-.6	-.3	+1.2	.5	+.05	.5	.38	-3.2	1.7
427	26. 830	.8	20.55	1.03	.08	+.14	+.26	4				-.6				+.47	.4	.57	-4.3	1.9
428	29. 701	.8	19.98	1.11	.07	-.32	-.20	2	-.58	-.47	2	-.7	-.6	+1.3	.5	-.05	.4	.34	-1.0	2.2
438	Oct. 8. 705	.5	18.23	1.34	.07	+.07	-.03	1				+.5				+.15	.1	.44	-1.8	0.3
439	8. 732	.8	18.22	1.34	.07	+.08	-.09	4				+.9				+.09	.4	.41	-2.6	1.2
440	11. 716	1.0	17.66	1.39	.07	-.17	-.20	4				+.2				-.03	.5	.34	-5.0	1.6
441	12. 647	.8	17.49	1.42	.07	+.08	-.05	4				+.7				+.12	.4	.43	-2.1	1.6
443	12. 763	1.0	17.46	1.42	.07	-.15	-.15	4				+.1				+.02	.5	.37	-1.3	1.5
459	22. 659	.6	15.72	1.51	.07		+.05	4								+.20	.3	.48		

radius of the planet as unit and letting $OT'=r$, we find from the triangle $OT'A$:

$$(1) \quad x = 1 - r \cos(\gamma - \alpha) \\ = 1 - r \cos \gamma \cos \alpha - r \sin \gamma \sin \alpha;$$

from the triangles ODT' and ODB :

$$(2) \quad r \cos \gamma = (1 - \varphi) \cos \beta, \quad r \sin \gamma = \sin \beta;$$

and from the triangles CBD and $CT'D$:

$$(3) \quad (1 - \varphi) \tan \beta = \tan \alpha.$$

r and γ can be eliminated from (1) by means of equations (2):

$$x = 1 - (1 - \varphi) \cos \beta \cos \alpha - \sin \beta \sin \alpha$$

Further, eliminating β by (3) and developing according to powers of φ , we have:

$$(4) \quad x = \varphi \cos^2 \alpha - \frac{\varphi^2}{8} \sin^2 2\alpha - \frac{\varphi^3}{8} \sin^2 2\alpha \cos^2 \alpha \dots\dots\dots,$$

a formula which seems better adapted to the use of

the Ephemeris data than Bessel's treatment of the problem. In the notation of the *American Ephemeris*:

$$\varphi = 2(1-k) = \frac{2q}{\text{diameter}}.$$

For polar diameter: $a = P - Q$

For equatorial diameter: $a = 90^\circ + P - Q$.

Near opposition it is sufficient to use the first term of formula (4), for a few of the later plates with large phase it was necessary to include the second term. The third term never amounts to more than 0.002 and can nearly always be neglected.

Correction for image intensity.—Because of differences in exposure time, variations in the transparency of the air and in the sensitivity of the plates the images of *Mars* are not all of the same intensity, and it must be expected that the results of the individual diameter measures (not given in Table 2) depend somewhat on this factor. To investigate this question the observations were divided into three periods and for each period the yellow and red images, as well as the equatorial and polar diameters, were treated separately. In every subgroup the diameter residuals were then arranged according to the intensity of the image measured in the microphotometer and represented by a linear formula

$$O - C = a + b(J - J_0),$$

where J is the intensity of the image approximately expressed in magnitudes and J_0 an arbitrary constant close to its mean value. The coefficients b determined by least squares solutions for each subgroup are given in Table 3 together with their weight and the number of images (in brackets). In the last period the equatorial diameters were not used on account of the large phase.

which only in two cases reaches 0".1. The means of the corrected residuals taken for each plate are found in Table 2, col. 7-8 for the yellow images, and col. 10-11 for the red images, with the number of images on which each mean is based in col. 9 and 12. The fact, that in our case a threefold overexposure produces an error of only ".06 in the diameter measures, illustrates clearly the advantages of the photographic method for measuring planetary diameters.

Flattening of Mars and phase effect.—We designate as true flattening F that seen at a great distance by an observer situated in the plane of the planet's equator and measure it, as usual, by the fraction

$$F = \frac{\text{equat. diam.} - \text{polar diam.}}{\text{equatorial diameter}}.$$

The apparent flattening f seen from the Earth is related to it by the series of formula (4) of which we need to retain only the first term:

$$f = F \cos^2 D$$

where D is the areocentric declination of the Earth referred to the Martian equator as given in the *American Ephemeris*. On the other hand, the apparent flattening is derived from the measured polar and equatorial diameters d_p and d_e by

$$f = \frac{d_e - d_p}{d_e} \text{ or } f = \frac{R_e - R_p}{d}$$

if R_e and R_p represent the residuals of Table 2, col. 7-8 and 10-11 and d the Ephemeris diameter of col. 4. The results for f obtained by this formula are found in col. 13 and 14 of Table 2. Taking means for the three observing periods, as in col. 4, 5, 6 of Table 4, it becomes evident that the observed flattening decreases with increasing phase defect; for the sake of comparison col. 2 and 3 of Table 4 give the mean phase cor-

TABLE 3
DEPENDENCE OF RESIDUALS ON IMAGE INTENSITY

Period							
Aug. 20-Sept. 4	4	Equat. diam.	$b = +".03$	Wt. 3.9	} (56)	$b = +".04$	Wt. 1.5
		Polar diam.	$+.08$			$-.10$	
Sept. 7-Sept. 22	22	Equat. diam.	$.00$	.9	} (22)	$+.20$	1.1
		Polar diam.	$-.13$			$-.04$	
Sept. 26-Oct. 22	22	Polar diam.	$+.10$	10.4	(29)	$+.16$	1.1
		Mean:	$b = +".068$			$b = +".043$	
				Wt. 20.0			

The small weights of the solutions in some of the groups are mostly due to small variations in the intensity of the images. Although the dependence is not very pronounced, both yellow and red images show independently that as a whole the diameters are measured slightly larger on the stronger images. All images were then reduced to equal intensity by the small correction:

$$-".062 (J - 4.75)$$

rections for each period expressed in fractions of the radius: φ_e in the direction of the equator and φ_p in the direction of the pole. The gradual fading at the terminator must have introduced a systematic error in the micrometer settings. It seems most likely that this error is proportional to the phase defect and the observed values of the flattening f' are then represented by the formula:

$$f' = F \cos^2 D - \sigma(\varphi_e - \varphi_p).$$

TABLE 4
EFFECT OF PHASE ON OBSERVED FLATTENING

Period	Mean phase cor.		Observed flattening f'		Mean	Formula
	φ_E	φ_P	Yellow im.	Red images		
Aug. 20-Sept. 4	.002	.002	+.0103 (56)	+.0091 (18)	+.0100 (74)	+.0096
Sept. 7-Sept. 22	.026	.004	+.0056 (22)	+.0070 (22)	+.0063 (44)	+.0066
Sept. 26-Oct. 22	.067	.004	+.0013 (25)	-.0045 (4)	+.0004 (29)	+.0009

The true flattening F and the coefficient of the additional phase correction σ were determined by a least squares solution from the mean f' of each plate (combining the data of the red and yellow images) and using weights proportional to the square of the apparent diameter. The results of this solution are:

$$F = +.0104 \pm .0006$$

$$\sigma = +.134 \pm .021$$

On the photographic images the phase defect appears about 13% too large, and the micrometer settings must have been made 0.1 or more inside of the terminator in the later observations although they were made to the extreme edge visible. On photographs with exposures suited for studying the markings the true terminator does not seem to be visible at all, but the images stop short at a region where the Sun is still 1.5 above the horizon. It is interesting to note that this situation is opposite to that prevailing in visual observations. Lowell³ finds that visual micrometer measures on the terminator give too small a phase; they must include the so-called twilight arc.

The polar flattening of *Mars* seems well determined, having a probable error of only 6%. After applying the additional phase corrections the flattening can be derived separately for the yellow and red images:

$$\text{Yellow images } F = +.0106 \pm .0007$$

$$\text{Red images } F = +.0098 \pm .0011$$

The agreement is very good, but the discussion of this result will be taken up later.

Difference in diameter of yellow and red images.—For all plates on which exposures with both color filters had been made the differences between the diameter measures of the yellow and the red images were formed. Taking means for the equatorial and polar diameters for each plate these differences are given in percentages of the diameter in col. 15 of Table 2. Out of the 19 figures 18 are positive, demonstrating in an impressive way that yellow photographs of *Mars* appear larger in diameter than red ones. The weights (col. 16) of the individual results depend, of course, on the numbers n_1 and n_2 of the two kinds of images according to the formula $\frac{n_1 n_2}{n_1 + n_2}$; they are further proportional to the square of the apparent diameter, and finally the equa-

torial diameters were given half weight in the second period and were not used at all in the last period on account of the uncertainty introduced by the phase. The weighted mean of the systematic difference between yellow and red images is

$$\text{Yellow} - \text{red} = 0.82\% \pm 0.08\%$$

This result is based only on a part of the plates and another determination was made (Table 5) by calculating the mean residuals of the yellow and red images for each observing period (2d and 3d col.) after correcting the individual data for phase error and flattening. The differences between the mean residuals divided by the mean diameter (col. 4) furnish the results of the 5th column.

TABLE 5
SYSTEMATIC DIFFERENCE OF YELLOW AND RED IMAGES

Period	Mean residuals		Mean diam.	Yellow-red diam.	Wt.
	Yellow im.	Red im.			
Aug. 20-Sept. 4	+0.11 (56)	-0.09 (18)	24.7	+0.81%	13
Sept. 7-Sept. 22	+0.22 (22)	0.00 (22)	22.7	+0.97	7
Sept. 26-Oct. 22	+0.13 (29)	-0.21 (4)	20.3	+1.68	1
Mean.....				+0.90%	

While the first method is free from small systematic errors peculiar to each plate, the second utilizes the available material more completely. Since both methods agree closely, their mean, 0.86%, has been adopted. *Mars* photographs taken with the red filter then have an apparent diameter 0.86% smaller than those taken with the yellow filter, and our observations confirm Wright's discovery of such an effect. Wright⁴ finds a difference of 3% in diameter between the violet ($\lambda = 4400$ A.) and infra-red ($\lambda = 7600$ A.) images. In our case the mean wavelengths of the two colors differ much less, being about 5600 A. for the yellow and 6000 A. for the red. Relating the diameter difference to the wave-length difference we have:

	Wave-length interv.	Diff. of diam.	Change per 1000 A.
Wright.....	4400-7600 A.	3%	0.9%
Trumpler.....	5600-6000	0.86%	2.1%

³ *Ann. Lowell Obs.* 1, 59.

⁴ *L. O. Bull.* 12, 53, 1925.

The drop in the apparent diameter evidently is not uniform for the whole color range. Wright's measures, combined with our results, indicate that between the ultraviolet, violet and yellow the *Mars* images undergo little alteration in diameter. Between the yellow and red or infra-red a rapid shrinking of the image takes place, which seems to mark the transition from the disk of *Mars* seen with its surrounding atmosphere (or at least a part of it) to the disk representing the solid surface alone. The evidence supporting the view that the infra-red photographs present the size of the solid surface will be given later (page 33).

Results for the diameter of Mars.—We proceed now to make all our diameter measures homogeneous, reducing them to the system of equatorial diameters of yellow images by the following corrections:

Additional phase correction:	Equat. diam.	+0.134 q_B
	Polar diam.	+0.134 q_P
Correction for app. flattening:	Polar diam.	+0.0104 $d \cos^2 D$
Correction for color:	Red images	+0.0086 d

Column 17 of Table 2 contains the plate means of the corrected residuals. As the micrometer settings on the terminator increase in uncertainty with the phase the equatorial diameters received full weight for the first period (Aug. 20–Sept. 4), half weight for the second period (Sept. 7–Sept. 22), and were rejected in the last period, where they serve only the purpose of studying the phase effect. The weights (col. 18) of these plate means are the product of the number of observations used and the quality of the images (col. 3). Unit weight is thus attributed to the result of a plate of “good” quality on which both equatorial and polar diameters of four images had been measured in four positions of the plate. Col. 19 gives the diameters at unit distance corresponding to the residuals of col. 17.

The correction a to the Ephemeris value (9".36) of the diameter of *Mars* at unit distance was first determined from the residuals of col. 17 by a least squares solution with equations of condition of the form

$$\frac{a}{\Delta} = O - C \quad (\Delta = \text{distance of } \textit{Mars} \text{ in astr. units})$$

with the result $a = +0".053$, or,

$$\text{Equatorial diameter of } \textit{Mars}: 9".413 \pm ".009$$

(First solution).

It has, however, been a general experience that micrometric measures of planetary diameters, especially those made visually, are affected by considerable systematic errors, mostly due to irradiation and diffraction as well as to a personal equation in the conception of the edge. The effect of such errors is essentially independent of the size of the disk or the distance of *Mars* and can be determined together with the diameter, if the measured

disks vary sufficiently in size (Method of Hartwig and Campbell). A second solution was accordingly made by introducing a constant term b into the equations of condition

$$\frac{a}{\Delta} + b = O - C$$

with the result $a = +0".01 \pm ".06$ and $b = +0".10 \pm ".16$, or,

$$\text{Equatorial diameter of } \textit{Mars}: 9".37 \pm ".06$$

(Second solution).

Both solutions represent the observations about equally well, leading to a probable error of $\pm ".078$ for unit weight.

In the second solution, unfortunately, the two unknowns are not well separated and the probable error of the result is therefore large. All we can learn from it is, that the systematic measuring error is small, of the order of 0".1, in the sense that the images were measured too large. Another means for estimating the possible size of some of these systematic errors is furnished by the comparison of strong and weak exposures. When correcting our measures for differences in the intensity of the photographic images a median intensity ($J=4.75$) was somewhat arbitrarily chosen as standard. If a greater or smaller intensity had been selected as standard all the corrected diameter measures would have been increased or decreased by the same amount, but in no case could this systematic difference have exceeded 0".1. The constant b would have been zero for the weakest measurable images. It is improbable, therefore that our measures are affected by a systematic error of more than 0".1, which enters into the final result only by one-third of its amount, and it seems justifiable to adopt the first solution leading to the following results:

Yellow images:	Equatorial diameter	9".41
	Polar diameter	9.32
Red images:	Equatorial diameter	9.33
	Polar diameter	9.24
Polar flattening of <i>Mars</i>	$0.0104 \pm .0006 = 1/96$	

Red images are 0.86 percent smaller than yellow images.

The polar diameter is that seen by an observer situated in the plane of the equator of *Mars* at a distance of 1 astronomical unit. The probable error of the diameter results is about $\pm ".02$ including accidental and systematic sources of error.

The polar diameter of the yellow images should be quite comparable with the results of visual observers and agrees indeed very closely with the best of these:

Hartwig ⁵ (Heliometer)	9".352
Campbell ⁶ (Micrometer)	9.25

⁵ *Publ. d. Astron. Ges.* **15**, 77, 1879.

⁶ *Astron. Jour.* **15**, 145, 1895.

5. DIAMETER, FLATTENING AND POSITION OF POLAR AXIS FROM MEASURES OF SURFACE MARKINGS

The measures of the markings made on 151 photographic images of 40 plates were described on page 22. Each plate furnishes a list of rectangular coordinates referred to the center of the disk, oriented to the pole and equator of *Mars* according to the data of the *American Ephemeris* and expressed in fractions of the equatorial radius of the planet (with the approximate diameter value of 9".248).

On page 21 a method was outlined in its main ideas by which it is possible to derive the diameter of the solid surface from such measures, as well as the position of the axis of rotation. Instead of establishing the apparent elliptical path of a marking across the disk directly from the observations, it is more convenient to calculate this path with approximate values of the areographic position of the marking and of the planetary constants, to compare the calculated rectangular coordinates with the observed ones and to determine from the resulting residuals differential corrections for the desired quantities. For this purpose we have to set up the formulae by which the rectangular coordinates are related to the areographic longitude λ and latitude β . This is easily done if we assume first that the surface of the planet is a perfect sphere and define the position of any point on it by three auxiliary rectangular coordinates ξ, η, ζ (fig. 2); the ξ axis being

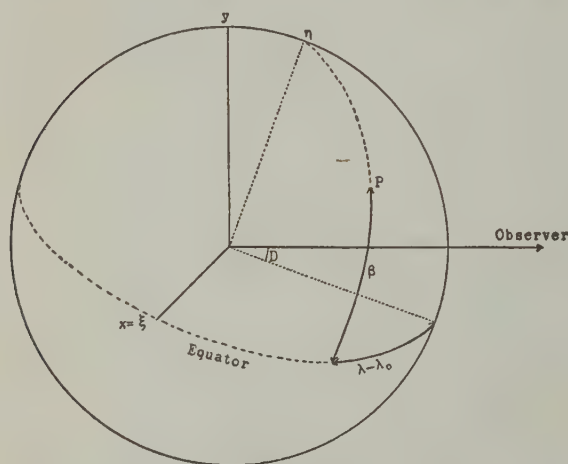


Figure 2.

perpendicular to the plane of the central meridian, the η axis falling into the polar axis of the planet (positive toward the North pole), the ζ axis in the intersection of the central meridian with the equator. We choose the equatorial radius of *Mars* as unit and designate with λ_0 the longitude of the central meridian; ξ, η, ζ are then found by the equations:

$$\begin{aligned} \xi &= \cos \beta \sin(\lambda - \lambda_0) \\ (5) \quad \eta &= \sin \beta \\ \zeta &= \cos \beta \cos(\lambda - \lambda_0) \end{aligned}$$

From these auxiliary coordinates the measured ones x, y are obtained by simply rotating our coordinate system around the ξ axis by the angle D which is the declination of the Earth referred to the center and equator of *Mars* and is given by the *American Ephemeris*. The transformation of coordinates representing such a rotation is:

$$\begin{aligned} x &= \xi \\ y &= \eta \cos D - \zeta \sin D \end{aligned}$$

and eliminating ξ, η, ζ by means of (5) we have

$$\begin{aligned} (6) \quad x &= \cos \beta \sin(\lambda - \lambda_0) \\ y &= \sin \beta \cos D - \cos \beta \sin D \cos(\lambda - \lambda_0) \end{aligned}$$

If, however, *Mars* is assumed to be an ellipsoid of rotation, flattened at the poles by an amount F as defined on page 25, these equations have to be slightly altered. Fig. 3 presents a cut through the meridian of the point P in consideration. Besides the surface of the planet the concentric sphere tangent at the equator is also drawn. Projecting the point P of the ellipsoid to the sphere at P' by a line parallel to the polar axis, we designate with β' the latitude of the projected point and find for the coordinates η and ρ of P (remembering that $OP' = 1$)

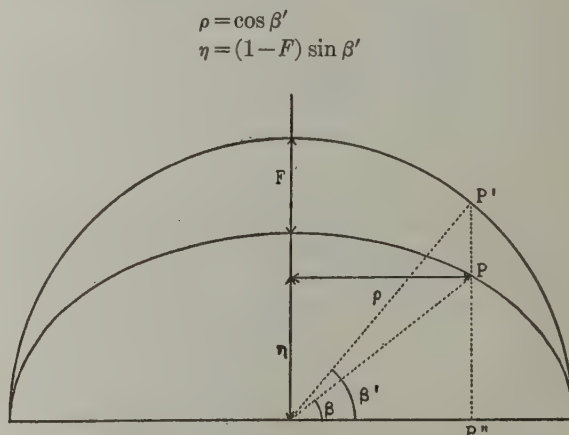


Figure 3.

It is evident then, that by replacing in (5) and (6) $\cos \beta$ by $\cos \beta'$ and $\sin \beta$ by $(1 - F) \sin \beta'$ we obtain the corresponding formulae for the flattened ellipsoid:

$$\begin{aligned} (6') \quad x &= \cos \beta' \sin(\lambda - \lambda_0) \\ y &= (1 - F) \sin \beta' \cos D - \cos \beta' \sin D \cos(\lambda - \lambda_0) \end{aligned}$$

But the true planetocentric latitude is β and not β' .

The relation between these two quantities is derived from the triangles $OP'P''$ and OPP'' (fig. 3):

$$\frac{tg\beta}{tg\beta'} = \frac{PP''}{P'P''} = 1 - F$$

$$tg\beta = (1 - F)tg\beta'$$

According to Lagrange this can be developed into a power series for the small quantity F :

$$(7) \quad \beta = \beta' - \frac{F \sin 2\beta'}{2 \sin 1^\circ} + \dots$$

where β and β' are expressed in degrees. Higher order terms may be neglected, as for a flattening of $F=0.01$, $\beta-\beta'$ amounts at the extreme to 0.3 . It is convenient to determine β' for each marking and to apply the small corrections (7) only in the final catalogue.

For a selected marking the equations (6') represent its apparent path across the disk as a function of the variable parameter λ_0 . Since the period of rotation of *Mars* is very accurately known, λ_0 , the longitude of the central meridian, can be interpolated from the *American Ephemeris* for the time of each exposure on which our marking was measured, and we may assume these data to be free from error, even when the exposures are taken on different days. With approximate values of F and D and of the longitude and latitude of our marking its rectangular coordinates for the date of each plate are calculated by (6'). The differences between these calculated coordinates x_0 , y_0 and the observed ones x_0 , y_0 must then be due to the following causes, aside from the accidental measuring errors:

- (a) Errors in the assumed values of λ and β'
- (b) Errors in the constants F and D
- (c) Errors in the orientation of the plate and in the scale of the x_0 , y_0 ; the latter are introduced by expressing the coordinates as fractions of an incorrect value of the planet's radius.

The six errors mentioned, or the corresponding corrections, can be determined from the residuals by a least squares solution for which we have now to derive the equations of condition. The first four corrections affect the calculated coordinates, and we have to differentiate the equations (6'):

$$dx_0 = \cos \beta' \cos (\lambda - \lambda_0) d\lambda - \sin \beta' \sin (\lambda - \lambda_0) d\beta'$$

$$dy_0 = \cos \beta' \sin D \sin (\lambda - \lambda_0) d\lambda + \{ \cos \beta' \cos D + \sin \beta' \sin D \cos (\lambda - \lambda_0) \} d\beta' - \sin \beta' \cos D dF - \{ \sin \beta' \sin D + \cos \beta' \cos D \cos (\lambda - \lambda_0) \} dD$$

Since F is a small quantity, terms containing the products $F d\beta'$ and $F dD$ can be neglected. The corrections of the observed coordinates for orientation and scale are:

$$dx_0 = -x_0 dR - y_0 dP$$

$$dy_0 = -y_0 dR + x_0 dP$$

where dR is the fraction of the radius by which its assumed value should be increased, while dP is the correction in arc measure to the position angle of the polar axis as given in the *Ephemeris*. Since

$$x_0 + dx_0 = x_0 + dx_0$$

$$y_0 + dy_0 = y_0 + dy_0$$

our equations of condition take the form:

$$(8) \quad \begin{aligned} x_0 - x_0 &= x_0 dR + y_0 dP + \cos \beta' \cos (\lambda - \lambda_0) d\lambda \\ &\quad - \sin \beta' \sin (\lambda - \lambda_0) d\beta' \\ y_0 - y_0 &= y_0 dR - x_0 dP + \cos \beta' \sin D \sin (\lambda - \lambda_0) d\lambda \\ &\quad + \{ \cos \beta' \cos D + \sin \beta' \sin D \cos (\lambda - \lambda_0) \} d\beta' \\ &\quad - \sin \beta' \cos D dF - \{ \sin \beta' \sin D \\ &\quad + \cos \beta' \cos D \cos (\lambda - \lambda_0) \} dD \end{aligned}$$

As long as the n measures of a single point are considered, they furnish $2n$ such equations for the determination of 6 unknowns; but in no case can all these be obtained with satisfactory weight from one point alone. Let us take for example a point at the equator of *Mars* ($\beta'=0$) and assume that three observations of it are available, made at such times that $\lambda - \lambda_0$ has the values -60° , 0° , $+60^\circ$. Although we have six equations with six unknowns, dF remains indeterminate, because its coefficients in the equations are all zero; for dR , dP , $d\lambda$ the solution is quite satisfactory, while $d\beta'$ and dD have small weight on account of not being well separated. On the other hand the measures of a marking near the pole furnish $\cos \beta' d\lambda$, $d\beta'$, dP and the difference $dR - dF$ with good accuracy, but they contribute little toward the determination of dR and dD . It is clear then that the flattening dF must be found by combining equatorial points with others near the pole, and a satisfactory accuracy for dD has to rest on a large number of observations. For all markings it is important that the observations cover a wide range of areocentric hour angles. Since most configurations appear somewhat distorted when seen near the limb, great care has to be exercised in the correct identification of the selected points, and it is desirable to use numerous markings of different character in order to eliminate as far as possible systematic errors due to this source.

Our measures seem to satisfy these conditions and 192 points between latitudes -79° and $+50^\circ$ were selected for the solution with a total of 1409 observations (one observation designates the mean result of the different images, generally four, of the same plate). Only points with too few or poorly distributed measures were omitted. The problem before us is to determine λ and β' for 192 points as well as the four constants dR , dP , dD , dF , a total of 388 unknowns, from 2818 equations of condition. Several simplifications, however, seem possible without much loss in accuracy.

In the first place the determination of dP was separated from the other unknowns. It has been mentioned before that the zero point of position angle was taken over from the double star observations of the following year, and it seemed desirable to determine dP for each plate so as to test the constancy of the zero point by the agreement of the different plates. Only the y measures of two groups of 6–10 equatorial points were used for this purpose, one group being situated near the eastern limb, the other near the western limb. The second equation of (8) shows that for such points the coefficient of dP is large and has opposite sign for the two groups. The preliminary catalogue (mentioned on page 21) furnished approximate areographic positions for the points needed for the calculation of y_0 . Subtracting the mean residual $y_0 - y_c$ of the western group from the corresponding mean residual of the eastern group and dividing by the mean x difference of the two groups dP is obtained, neglecting the other differential corrections. Although this method is not quite rigorous, it can be shown that the omissions are of little consequence and the results for dP , expressed in degrees, are given with their weights in the last two columns of Table 2. The weighted mean is

$$dP = -1^{\circ}0 \pm .2.$$

The probable error also takes account of the uncertainty in the zero point.

Correcting the residuals for this value of dP we eliminate one unknown from our equations of condition. With close approximations for the λ 's obtained from our preliminary catalogue the terms with $\cos \beta' d\lambda$ can be neglected in the determination of the three constants dR , dD , dF . Not only are the products $\cos \beta' d\lambda$ small, but they have the character of accidental errors and are independent of the other unknowns whenever the observations of a point are symmetrically arranged with respect to the central meridian, a condition which is well fulfilled for most of our points. The equations of condition (8) can now be simplified by leaving out the terms with dP and $d\lambda$ and we can further replace on the right hand side x_0 and y_0 by x_c and y_c or by their expressions in λ and β' according to (6').

$$\begin{aligned} x_0 - x_c &= -\sin \beta' \sin(\lambda - \lambda_0) d\beta' + \cos \beta' \sin(\lambda - \lambda_0) dR \\ (9) \quad y_0 - y_c &= \{ \cos \beta' \cos D + \sin \beta' \sin D \cos(\lambda - \lambda_0) \} d\beta' \\ &\quad - \cos \beta' \sin D \cos(\lambda - \lambda_0) dR - \{ \sin \beta' \sin D \\ &\quad + \cos \beta' \cos D \cos(\lambda - \lambda_0) \} dD - \sin \beta' \cos D d\Phi \end{aligned}$$

where dF was replaced by the new unknown

$$d\Phi = dF - dR.$$

While it would be possible to set up the 195 normal equations of which all but three contain only four unknowns, much computing work can be saved by the

following consideration. The solution would be very simple if we could neglect the $d\beta$'s in a similar way as the $d\lambda$'s. Unfortunately this is not possible because the $d\beta$'s are closely related to the constants dD and $d\Phi$. They are not like accidental errors but rather like systematic errors showing a certain run with β' and this run depends on the corrections dD and $d\Phi$. In view of the symmetrical arrangement of our observations, the $d\beta$'s will, however, be purely accidental for all points which have the same latitude, and for our solution it is sufficient to retain the mean systematic corrections as a function of the latitude, neglecting the individual corrections of each point. Our 192 points were accordingly divided into 16 latitude intervals in general 10° wide, except near the South pole where they are narrower. For all points of the same latitude interval the correction $d\beta'$ was assumed to be the same. There were thus 19 normal equations to be set up from the 2818 observations. The calculation of the coefficients was greatly facilitated by the fact, that for the different observations of the same point only $\lambda - \lambda_0$ varies, and further by substituting the mean β' of the latitude interval for the individual values of each point and the constant mean value $-16^\circ 25'$ for D , which varies little during the period of observation. The weight of each observation was adopted proportional to the number of images on which it rests (weight 1 for 4 images). As the different corrections $d\beta'$ are independent of each other, the solution of the normal equations is relatively simple and led to the following results:

	$dR = -.0078 \pm .0015$	$R = 9^\circ 176 \pm .014$
(1st	$dD = +.0019 \pm .0033$	$dD = +0^\circ 11 \pm .18$
solution)	$d\Phi = +.0142 \pm .0033$	$dF = d\Phi + dR = +.0064 \pm .0036$
		$F = +.0163 \pm .0036$

The value of the flattening of *Mars* is considerably larger than that derived from the settings on the limbs. This suggests that in the latter case the polar diameter was measured too large on account of the great intensity of the polar cap on most of the photographs. But the irradiation of the polar cap will also affect the determination of the center of the disk by a similar systematic error. If this interpretation is correct, all the measured y coordinates need a constant negative correction, and a second solution was made including as additional unknown such a correction c . The only change in the equations of condition is that the term $-c$ has to be added to the second equation of (9), and the procedure of the solution was quite similar:

	$dR = -.0076 \pm .0015$	$R = 9^\circ 178 \pm .014$
(2d	$dD = +.0058 \pm .0041$	$dD = +0^\circ 33 \pm .24$
solution)	$d\Phi = +.0090 \pm .0046$	$F = +.0113 \pm .0048$
	$c = -.0070 \pm .0043$	

Our supposition concerning the effect of the irradiation of the polar cap on the determination of the

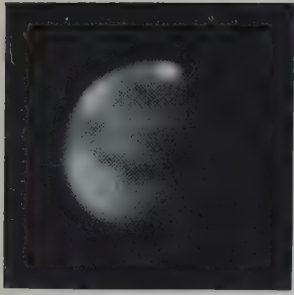


Fig. 1. $\lambda_0 = 5^\circ$, Oct. 11, *y*.

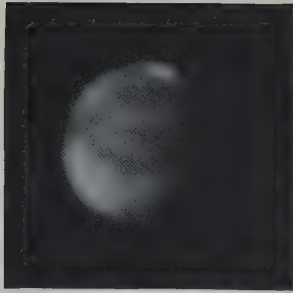


Fig. 2. $\lambda_0 = 12^\circ$, Oct. 12, *y*.

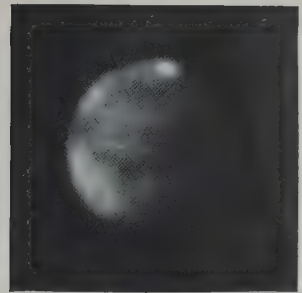


Fig. 3. $\lambda_0 = 12^\circ$, Oct. 12, *y*.

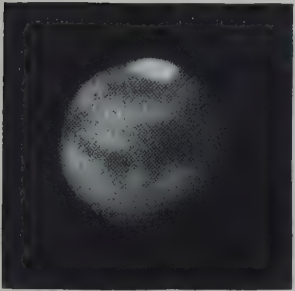


Fig. 4. $\lambda_0 = 13^\circ$, Sept. 4, *y*.

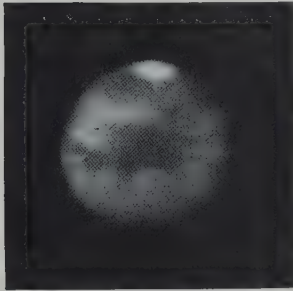


Fig. 5. $\lambda_0 = 37^\circ$, Sept. 2, *r*.

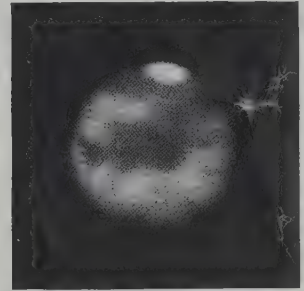


Fig. 6. $\lambda_0 = 44^\circ$, Aug. 31, *r*.

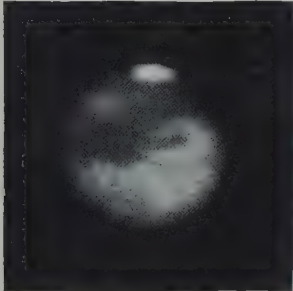


Fig. 7. $\lambda_0 = 64^\circ$, Aug. 28, *r*.

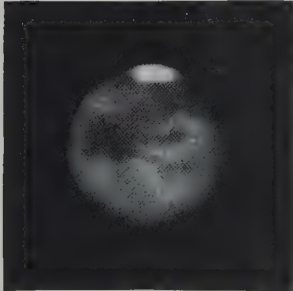


Fig. 8. $\lambda_0 = 70^\circ$, Aug. 29, *y*.

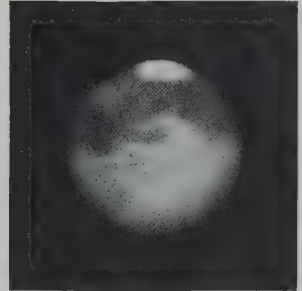


Fig. 9. $\lambda_0 = 89^\circ$, Aug. 26, *y*.

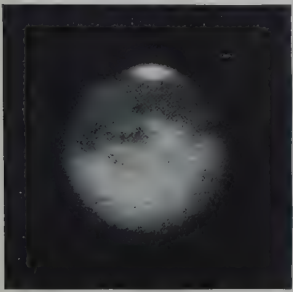


Fig. 10. $\lambda_0 = 92^\circ$, Aug. 28, *r*.

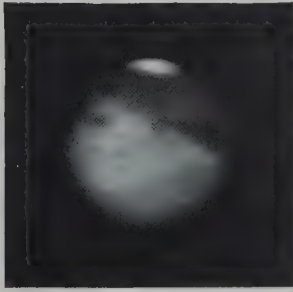


Fig. 11. $\lambda_0 = 128^\circ$, Aug. 24, *y*.

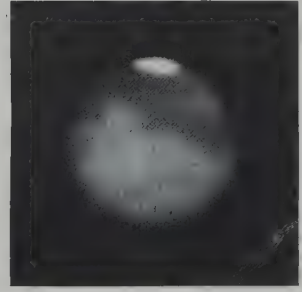


Fig. 12. $\lambda_0 = 130^\circ$, Aug. 22, *y*.



Fig. 1. $\lambda_0 = 150^\circ$, Aug. 21, *y*.

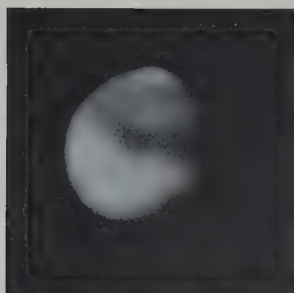


Fig. 2. $\lambda_0 = 184^\circ$, Sept. 26, *y*.

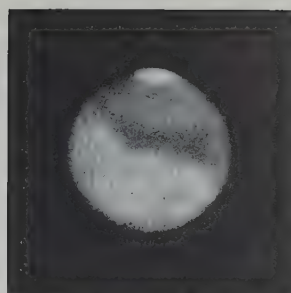


Fig. 3. $\lambda_0 = 184^\circ$, Aug. 20, *y*.

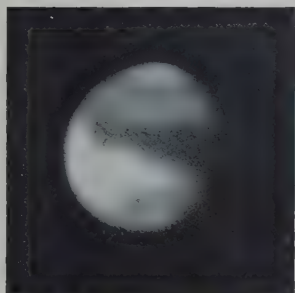


Fig. 4. $\lambda_0 = 207^\circ$, Sept. 21, *r*.

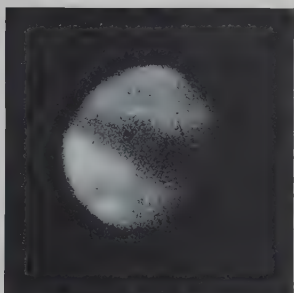


Fig. 5. $\lambda_0 = 238^\circ$, Sept. 21, *r*.

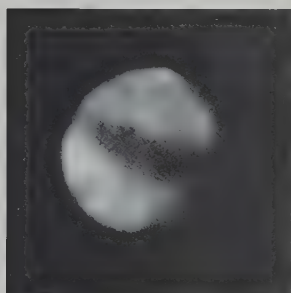


Fig. 6. $\lambda_0 = 252^\circ$, Sept. 15, *r*.

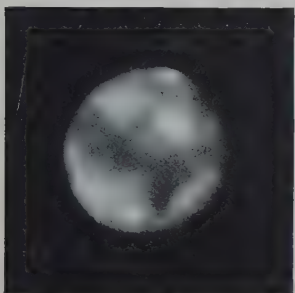


Fig. 7. $\lambda_0 = 283^\circ$, Sept. 11, *r*.

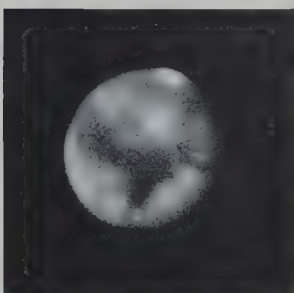


Fig. 8. $\lambda_0 = 294^\circ$, Sept. 13, *y*.

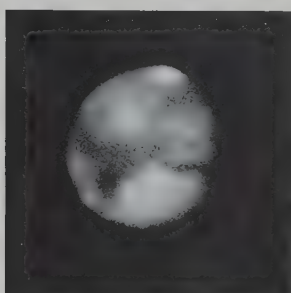


Fig. 9. $\lambda_0 = 318^\circ$, Sept. 11, *r*.

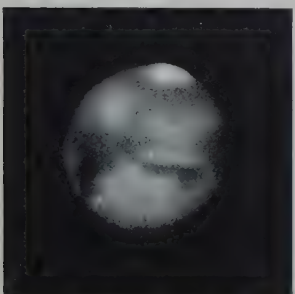


Fig. 10. $\lambda_0 = 335^\circ$, Sept. 7, *y*.

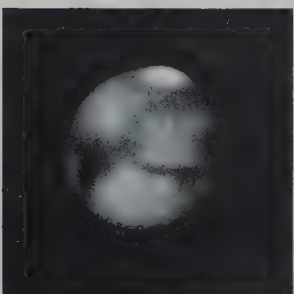


Fig. 11. $\lambda_0 = 335^\circ$, Sept. 7, *y*.

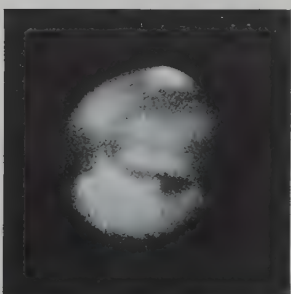


Fig. 12. $\lambda_0 = 343^\circ$, Sept. 7, *r*.

center of the disk is indeed confirmed by the negative value of c and the results of the second solution were adopted as final, although the introduction of a new unknown has resulted in a reduction of the weight of dD and $d\Phi$.

6. THE POLAR FLATTENING OF MARS

We have now determined the polar flattening of *Mars* by two methods entirely different in character but with results in close agreement:

From measures of equat. and polar diam.	$F = .0104 \pm .0006$
From measures of markings:	$F = .0113 \pm .0048$
Mean:	$F = .0108 = 1/93$

As our figures are contradicted by theoretical considerations, a careful discussion of the possible sources of systematic errors seems desirable. It might be thought that imperfect guiding would slightly elongate the photographic images in the direction of the daily motion which for our observations differs only about 10° from the Martian equator. This would, of course, affect the measures of the limb as well as those of the markings. This source of error, however, must be ruled out. In the first place, it is unlikely that short exposures of $1\frac{1}{2}$ seconds like those of the yellow images should be systematically affected by guiding. In the second place, such an effect should increase with the exposure time, and the red images with 18 seconds exposure should give a larger flattening, while in fact their result is even a trifle smaller (page 26).

The direct diameter measures made by settings on the limb or the terminator could be influenced by variations in the illumination of the different parts of the edge. For the terminator such an effect was indeed found, discussed and corrected for. Any effect of the phase on the determination of the flattening is completely eliminated for the period from Aug. 28 to Sept. 1, when the line connecting the cusps forms an angle of 45° with the polar axis; the phase defect of the polar diameter is then the same as that of the equatorial diameter. The six plates of this period taken separately give $F = .0103$ a result practically identical with that of the whole series.

There remains the irradiation of the polar cap which could only have operated in the sense of producing too small a flattening. On seven plates taken in September the polar cap is very faint and pale. These plates alone when corrected for phase give $F = .0101$, while for the 24 earlier plates on which the polar cap is brilliant we find $F = .0107$; the difference is again inappreciable.

Still more convincing is the comparison with the results of other observers. It is true that visual micrometric measures have yielded widely divergent data for the flattening of *Mars* ranging from 0 to .06. But if

we confine ourselves to the best and most complete series of recent times, the results are more consistent.

1. Hartwig⁷ discussed the measures of the diameter of *Mars* made with the heliometer or double image micrometer at Königsberg, Leiden, Oxford, Berlin, Paris, Strassburg, and gives for the flattening:

$$F = 1/96 = .0104 \quad \text{Wt. 2.}$$

2. From a special series of observations with the Breslau heliometer made in 1879 for the purpose of studying the flattening Hartwig⁸ finds:

$$F = 1/116 = .0086 \pm .0022 \quad \text{Wt. 1.}$$

3. Among the measures with the filar micrometer we may mention those of Young⁹ made with the $9\frac{1}{2}$ -inch refractor at Princeton in 1879:

$$F = 1/219 = .0046 \pm .0018 \quad \text{Wt. } \frac{1}{2}.$$

4. Barnard's¹⁰ measures with the micrometer of the 36-inch refractor of the Lick Observatory in 1894 give:

$$F = 1/109 = .0091 \quad \text{Wt. 1.}$$

5. Numerous micrometric measures of the equatorial and polar diameters of *Mars* were made at the Lowell Observatory in the years 1894 and 1896. Using only the observations near opposition the following values of the flattening are obtained:

1894	$F = .0061$	(Oct. meas.) ¹¹
1896	.0248	(Oct.-Nov. meas., first reduction) ¹²
Mean	$F = .0154$	Wt. 1.

The small result by Young rather falls out of line, but it must be mentioned that these measures are strongly affected by irradiation and give a value of the diameter of *Mars* $1''.46$ too large; it is on this account that they were given half weight. Taking means for each method of observation with the weights indicated we have:

Heliometer	$F = .0098$
Filar micrometer	.0107
Photogr. meas. of diameter	.0104
Photogr. meas. of markings	.0113

and we may adopt their mean as the best available

$$\text{observational result for the polar flattening of Mars} \\ F = .0105 = 1/95.$$

The good agreement of four different methods leaves no doubt that the disk of *Mars* appears in the telescope

⁷ *Publ. d. Astron. Ges.* **15**, 78, 1879.

⁸ *A. N.* **150**, 313, 1899.

⁹ *Amer. Jour. of Science*, 3d series, **19**, 206, 1880.

¹⁰ *A. J.* **17**, 145.

¹¹ *Ann. Lowell Obs.* **1**, 61-62.

¹² *Ann. Lowell Obs.* **2**, 246.

flattened by about 1 percent and that this phenomenon is not due to purely instrumental or personal errors.

There are two theoretical considerations bearing on the polar flattening of *Mars*. In the first place, such a flattening will cause secular perturbations in the motions of the satellites. H. Struve¹³ calculated by this method the position of the polar axis and the amount of flattening, for which he obtained:

$$F = 1/190 = .0053$$

a value much smaller than the observed one. Struve's result, however, can hardly be considered as being established beyond any doubt or question. The measures of the satellites, which are very difficult, may be subject to considerable systematic errors introduced principally by the uncertainties in the center of *Mars* to which they are referred. The observations are furthermore not continuous but confined to short periods around the favorable oppositions and separated by several years. This is well illustrated by the fact that for both satellites the eccentricities derived show a wide range. The value of the flattening is principally based on the motion of the perimartium of *Phobos* and on the changes in the orbit plane of *Deimos*. While the latter have not even completed a period, the different determinations of the perimartium of *Phobos* taken alone, could equally well be represented by a shorter period (0.90 years), corresponding to a larger flattening. It should also be mentioned that the perturbations of the satellites on each other were neglected and that the position of the plane of the equator derived by this method is not in good agreement with the results of direct observation.

Clairaut's Theorem states the condition for liquid equilibrium on the surface of the planet, which is:

$$F \leq \frac{5}{4} \varphi$$

where φ , the ratio of the centrifugal force to the gravitational attraction at the equator, can be calculated from the mass, period of rotation and the equatorial radius of the solid surface. Adopting for the latter the result (4".59) of our measures of markings and for the mass of *Mars* Struve's value 1:3 090 000, we obtain;

$$\varphi = .00434 = 1/231$$

and Clairaut's Theorem would require:

$$F \leq .00542 \quad (1/185)$$

where the sign of equality applies to the case of uniform density in the interior of the planet, while greater density at the center leads to a smaller value of F .

¹³ *A. N.* 138, 217, 1895; *Mem. de l'Acad. Imp. des Sciences de St. Petersbourg* 8^e series, 8, No. 3, 1898; *Sitzungsber. Berlin Akad.* 1911, p. 1056.

Our observational results for the flattening are incompatible with Clairaut's Theorem and there are only two assumptions, both equally improbable, which could explain such a discrepancy; either that the density of *Mars* is greater near the surface than at the center, or that the surface of *Mars* is not in liquid equilibrium. Although *Mars* does not seem to have any large water surfaces, there can be no doubt that its atmosphere must be in liquid equilibrium. It will presently be shown that the visual or photographic measures of the limb refer to a certain layer of the atmosphere and the flattening derived from them should, therefore, conform to Clairaut's Theorem.

Unless we are willing to admit an abnormal density distribution in the interior of *Mars*, the only way out of the dilemma seems at present to lie in a suggestion made by Hartwig, W. H. Pickering and others that the observed values of the flattening are influenced by peculiarities in the Martian atmosphere. This atmosphere must then be visible up to a higher level (52 miles) at the equator than at the poles (42 miles), and consequently different amounts of it are included in the measures of equatorial diameters than in those of polar diameters. Such an effect might easily be produced by the circulation in the Martian atmosphere. As the limbs of the red images refer to a lower atmospheric level, about $\frac{2}{3}$ in elevation compared with the corresponding level of the yellow images, they should give a flattening intermediate between that of the yellow images and the theoretical value (ca. .005) of the solid surface. In fact the result of the red images taken alone (page 26) is a trifle smaller, although not as much as we should expect, but the difference lies within the limits of its probable error. On the other hand our suggestion does not explain the value of the flattening derived from the measures of the markings. Considering, however, its rather large probable error, it is not impossible that the agreement with the results of the limb measures is purely accidental. This question could probably be cleared up in a more definite way by measuring the flattening on infra-red images, the limbs of which do not seem to include any of the atmosphere, and by further determinations of the flattening from measures of markings.

7. THE DIAMETER OF MARS

From the rotational movement of the markings we have obtained (2d solution, page 30) the following results for the

Diameter of the solid surface of Mars at unit distance:

Equatorial diameter	9".178 ± ".015 = 6652 ± 11 km. (4133 ± 7 miles)
Polar diameter	9".075 ± ".047 = 6577 ± 34 km. (4087 ± 21 miles)

The probable errors include the uncertainty of the scale value and for the polar diameter that of the flattening. This is the first attempt to determine the diameter of the solid surface and, as we expected, this constant is considerably smaller (ca. 2%) than the generally adopted value of the diameter of *Mars* which is based on measures of the limb. We thus confirm Wright's conclusion that the apparent disk of *Mars* as seen or photographed in ultraviolet, violet or yellow light includes part of the planet's atmosphere, and that the smaller diameters of red or infra-red photographs are due to partial or complete elimination of the scattered light of the Martian atmosphere.

It is surprising to find that Wright's diameter measures¹⁴ of the infra-red images are still smaller (8".86) than our value for the solid surface. On closer examination it is evident that Wright's measures are systematically smaller than ours even for yellow images taken with the same filter. A correction of +0".5 applied to Wright's measures will bring his result for the yellow images (9".17) into agreement with ours for the same color (9".36 mean of polar and equat. diameter). If the measures of infra-red images are similarly corrected, their mean result is 9".06 and comes close to the corresponding value 9".13 which we find to be valid for the solid surface. It appears then that the infra-red photographs represent approximately the disk of the solid surface, although for short exposures the gradual fading of light at the limb may even cause the measures to give slightly smaller results.

Comparing now the diameter of the solid surface with our measures of the limb we obtain some information on the height of the Martian atmosphere at the equator as far as it is visible on the photographs of the two colors.

Height of visible atmosphere at equator:

Yellow images	84 km. = 52 miles
Red images	55 km. = 34 miles

Even the figure of the yellow images is rather a lower limit for the extent of the atmosphere, and its highest parts are probably too tenuous to be visible at the limb by scattered light. In the twilight arc our own atmosphere becomes visible by scattered sunlight up to a height of 68 km. (42 miles).¹⁵ We have then here a direct proof that the Martian atmosphere is quite extensive; unless its scattering properties are larger, we should conclude that it reaches a height similar to that of the Earth's atmosphere. On account of the smaller gravitational attraction its density and pressure at the surface must nevertheless be much lower.

8. POSITION OF THE AXIS OF ROTATION

With respect to the position of the polar axis of *Mars* we found (page 30) the following corrections to the data of the *American Ephemeris*:

$$dP = -1^{\circ}0 \pm ^{\circ}2 \quad dD = +^{\circ}33 \pm ^{\circ}24$$

The first refers to the position angle, the second to the inclination with the line of sight. Applying these corrections we find for the equinox and equator of 1925.0:

Position of the North pole of Mars:

$$R.A. = 315^{\circ}9 \pm ^{\circ}3 \quad Decl. = +54^{\circ}7 \pm ^{\circ}2$$

The method of deriving the position of the polar axis from the rotational movement of equatorial markings does not seem to have been employed before. It has the advantage of giving a complete determination of both coordinates from the observations of a single opposition. Our result is in good agreement with that of Lowell and Slipher¹⁶ obtained from observations of the polar caps made in the years 1901-1911; which, reduced to 1925, is:

$$R.A. = 316^{\circ}0 \quad Decl. + 54^{\circ}2$$

According to both determinations the Ephemeris position needs only a small correction in the R.A. of the North pole.

W. H. Pickering,¹⁷ on the other hand, finds a quite divergent result; reduced to 1925 it is:

$$R.A. = 315^{\circ}2 \quad Decl. = +51^{\circ}8$$

His method of comparing the latitudes of certain markings derived at three different oppositions from drawings is open to two serious objections. Latitudes taken from drawings based on eye estimates are likely to be affected by systematic errors of a personal character. It is well known that estimates of the center of a round disk, for example, may be consistently too high or too low by a few percent even for an experienced observer. A change in the personal equation of Prof. Pickering between 1914 and 1922 would be sufficient explanation of the discrepancy. The other assumption on which Pickering's method rests is that most of his markings have not undergone any change in latitude during a period of 8 years and at entirely different seasons. But our measures when compared with older observations show, on the contrary, that considerable systematic changes must have taken place during the last ten or twenty years especially in the latitude of the northern boundary lines of some of the southern maria on which many of Pickering's points are situated. These have then all been affected in the

¹⁴ *L. O. Bull.* 12, 54, 1925.

¹⁵ R. Wolf, *Handbuch der Astronomie* 1, 475.

¹⁶ *Lowell Obs. Bull.* 2, 28, 1912.

¹⁷ *Report on Mars* No. 33; *Pop. Astr.* 34, 1926; *Harv. Ann.* 82, 106.

same sense (advance toward the North). Pickering's result therefore hardly deserves any confidence as long as his observations do not cover a full cycle.

In his study of the perturbations of the satellites of *Mars* by the flattening of the planet H. Struve also derives the position of the equator of *Mars*, leading to

$$\text{R.A.} = 312^{\circ}6 \quad \text{Decl.} = +52^{\circ}8$$

for the position of the North pole (1925), a result that could not be reconciled with our numerous measures of markings.

9. AREOGRAPHIC LONGITUDES AND LATITUDES OF 228 SURFACE MARKINGS

For the final calculation of the areographic longitudes and latitudes of the measured markings the corrected constants of the second solution were employed, namely:

$$\begin{aligned} \text{Equatorial diameter of } Mars & 9''.178 \\ \text{Flattening} & .0113 \\ \text{Position of North pole R.A.} & = 315^{\circ}9 \quad \text{Decl.} = +54^{\circ}7 \\ \text{Zero correction to } y \text{ measures} & - .007 \end{aligned}$$

and on these values our catalogue is based. Designating with x'_0 and y'_0 the rectangular coordinates calculated with the improved constants, the equations (8) go over into:

$$(10) \quad \begin{aligned} x_0 - x'_0 &= \cos(\lambda - \lambda_0) dL - \sin \beta' \sin(\lambda - \lambda_0) d\beta' \\ y_0 - y'_0 &= \sin D \sin(\lambda - \lambda_0) dL + \{ \cos \beta' \cos D \\ &\quad + \sin \beta' \sin D \cos(\lambda - \lambda_0) \} d\beta' \end{aligned}$$

where $dL = \cos \beta' d\lambda$. For every point we have such a system of equations of condition which were solved for dL and $d\beta'$ by least squares solutions with the one simplification that the y coordinates were not used for the determination of dL , as in the second equation of (10) the coefficient of dL is always small. The areographic longitude (Table 6, col. 2) is obtained by applying the correction $d\lambda = dL \sec \beta'$ to the approximate value of λ used in the calculation of x_0 and y_0 , while the true areocentric latitude of the flattened ellipsoid (Table 6, col. 4) is found by adding the two corrections $d\beta'$ and $\beta - \beta' = -0.32 \sin 2\beta'$ (equation (7), page 29) to the approximate value of β' . According to this method the longitudes and latitudes were also derived for a number of points not used in the determination of the constants. Table 6 gives the results for a selection of the best measured or most important points; the others have been used only for the construction of the chart and can be read off from this about as accurately as they are determined.

To insure correct identification, all points of Table 6 were entered on two skeleton charts (Plate III, fig. 2, and Plate IV, fig. 2) together with their number (Table

TABLE 6
AREOGRAPHIC LONGITUDES AND LATITUDES OF 228 MARKINGS

No.	Longitude	Wt.	Latitude	Wt.	No. plates	Description*
1	1.3	6.1	+ 6.0	8.4	12	N.W. point <i>Sabaeus Sin.</i>
2	2.4	5.0	-26.3	6.6	8	
3	6.6	2.3	+21.4	1.8	5	
4	9.1	4.0	-42.2	4.8	7	
5	9.5	5.2	-19.0	8.2	10	
6	9.5	8.8	- 9.4	13.2	15	S.W. limit <i>Sabaeus Sin.</i>
7	10.6	3.2	+35.2	2.7	7	<i>Siloe F.</i>
8	11.5	2.4	+14.5	2.7	5	
9	12.5	0.6	-68.3	1.3	4	
10	12.7	8.1	- 2.8	12.2	14	W. limit <i>Sabaeus S.</i>
11	13.4	2.2	+18.5	1.7	6	
12	15.1	3.1	-61.9	2.7	7	
13	18.0	5.6	+11.1	6.7	11	<i>Ozia Pal.</i>
14	18.2	2.6	-26.3	4.1	5	
15	21.2	2.5	-47.1	3.4	6	
16	21.4	9.5	+ 5.6	12.0	14	Point of <i>Margaritifer S.</i>
17	21.9	2.4	+49.7	0.5	4	
18	26.4	3.3	-11.3	4.9	5	
19	26.8	1.7	+25.5	1.3	4	E. part <i>Niliacus L.</i>
20	28.1	2.6	-27.4	3.2	4	
21	29.1	2.9	-55.0	3.5	7	
22	36.1	4.7	- 3.3	6.4	9	<i>Aromatum Prom.</i>
23	36.5	4.9	+30.2	3.1	9	<i>Niliacus L.</i>
24	39.9	4.4	-27.0	6.6	8	<i>Protei Lac.</i>
25	40.2	3.2	-40.5	4.0	8	
26	41.5	3.3	-19.2	4.5	8	
27	41.9	2.6	-62.5	3.6	7	
28	43.6	2.5	+48.7	0.7	4	
29	44.4	1.6	- 3.2	2.4	3	N. limit <i>Aurorae S.</i>
30	46.1	3.7	+ 3.7	4.2	8	
31	46.6	3.0	+14.1	2.9	10	Inters. <i>Iamuna, Hydraotes</i>
32	47.0	4.4	-12.5	8.1	10	<i>Auri Lac.</i>
33	52.0	2.9	- 9.7	4.1	8	Light spot, <i>Aurorae S.</i>
34	52.2	5.0	-42.7	6.9	10	<i>Aridus Lac.</i>
35	54.4	6.4	-25.8	11.9	15	
36	54.9	2.5	+34.3	1.5	6	<i>Craneum</i>
37	58.1	7.2	- 3.6	10.0	12	N.W. point, <i>Aurorae S.</i>
38	62.0	6.3	-11.8	10.5	13	W. limit, <i>Aurorae S.</i>
39	63.7	6.2	- 0.4	7.9	12	<i>Juventae F.</i>
40	64.4	3.2	-25.3	5.2	8	<i>Nectaris F.</i>
41	64.4	2.2	-62.2	1.7	4	
42	66.9	3.7	-18.7	5.5	8	
43	68.9	2.5	+11.3	3.2	7	<i>Lunae Lac.</i>
44	69.0	4.6	-67.8	5.2	13	
45	70.8	4.8	-35.5	7.1	10	
46	73.3	8.5	- 9.1	12.9	15	<i>Maesia Silva</i>
47	74.3	2.4	+21.3	1.9	4	<i>Labeatis Lac.</i>
48	75.9	3.2	-23.7	4.4	7	
49	76.4	2.0	-34.7	2.8	3	N.W. limit, <i>Bosporus</i>
50	76.8	2.8	+ 0.5	3.7	7	<i>Messeis F.</i>
51	84.4	2.9	+ 8.7	3.1	7	
52	84.6	4.4	-45.0	5.9	9	<i>Ambrosiae Lac.</i>
53	84.6	2.8	- 5.1	3.8	6	E. part, <i>Tithonius Lac.</i>

* The terms east and west are used like on terrestrial charts: east in the sense of decreasing longitude, west in the direction of increasing longitude.

TABLE 6—(Continued)

No.	Longi- tude	Wt.	Lati- tude	Wt.	No. plates	Description*
54	85.8	1.3	-26.6	2.5	4	Center, <i>Solis L.</i>
55	86.7	5.3	-31.4	7.9	10	S. point, <i>Solis L.</i>
56	89.4	6.4	-20.8	9.7	11	N. point, <i>Solis L.</i>
57	91.9	1.9	+32.8	1.3	5	
58	92.2	1.8	-11.6	2.5	4	
59	92.9	4.4	-40.2	6.7	9	S. limit of <i>Thaumasia</i>
60	93.9	1.9	+45.9	0.7	4	
61	94.0	6.3	- 7.1	9.4	12	<i>Tithonius Lac.</i>
62	97.5	2.6	+21.5	2.4	8	<i>Ascræus L.</i>
63	98.3	7.3	-27.9	10.8	12	W. point, <i>Solis L.</i>
64	99.9	2.4	-58.6	2.0	5	
65	104.7	1.7	-20.1	2.4	5	
66	104.9	5.0	-68.2	4.1	9	
67	105.2	6.4	-35.2	11.1	13	N.E. point, <i>Aonius S.</i>
68	107.0	3.1	-47.2	3.5	6	
69	109.0	6.7	-14.0	11.6	15	<i>Phoenixis L.</i>
70	109.2	5.0	-42.0	6.9	10	<i>Bathys L.</i>
71	109.4	1.5	- 2.3	2.1	6	<i>Arsine</i>
72	109.7	3.9	+11.8	4.0	8	<i>Ceraunius</i>
73	112.9	2.5	+28.2	1.5	5	
74	114.6	4.0	-21.1	5.2	8	
75	119.4	0.9	+ 2.2	1.1	3	<i>Biblis F.</i>
76	120.9	1.4	-55.1	1.0	3	
77	122.1	6.2	-31.0	10.5	12	
78	124.1	6.4	-25.2	11.9	13	N.E. point, <i>Sirenum M.</i>
79	124.6	3.2	-49.6	3.5	7	
80	125.8	3.5	- 6.5	4.3	8	Inters. <i>Sirenius, Eumenides</i>
81	126.3	3.2	+14.8	2.6	5	<i>Bandusiae F.</i>
82	128.5	2.5	+30.0	1.6	6	<i>Cyane F.</i>
83	130.3	2.2	-67.2	2.2	6	
84	131.0	6.7	-31.5	9.4	12	
85	134.2	5.7	-23.2	8.9	12	
86	135.9	2.6	-54.6	2.2	4	
87	136.5	2.7	+20.7	2.0	4	<i>Nix Olympica</i>
88	138.6	4.0	- 7.3	6.0	10	<i>Nodus Gordii</i>
89	139.1	2.7	+ 5.0	3.1	6	
90	139.6	5.8	-27.1	8.5	10	S. limit of <i>Memnonia</i>
91	142.4	2.4	-40.0	2.5	3	N. end of <i>Thermodon</i>
92	143.3	5.9	-33.6	9.9	12	
93	145.6	1.5	+27.5	1.0	4	
94	145.7	3.0	-51.5	3.6	6	<i>Palinuri Fret.</i>
95	147.1	1.6	+16.8	1.5	5	<i>Memnonius L.</i>
96	150.6	2.2	+ 1.1	2.7	5	<i>Maricae L.</i>
97	152.0	1.4	+43.5	0.6	3	<i>Propropontis</i>
98	152.6	3.4	-10.5	4.7	9	
99	154.2	6.9	-21.0	11.0	13	<i>Gorgonis L.</i>
100	158.1	2.4	-60.6	2.8	8	
101	160.5	2.9	-70.8	2.2	6	
102	162.0	5.8	- 3.4	7.8	9	
103	163.0	3.0	-53.8	3.3	8	
104	164.5	2.5	+ 5.2	3.7	7	<i>Ammonium</i>
105	164.6	1.5	-79.8	1.7	3	
106	164.9	2.6	-45.5	3.4	5	
107	171.8	2.1	- 0.6	3.0	6	

* The terms east and west are used like on terrestrial charts: east in the sense of decreasing longitude, west in the direction of increasing longitude.

TABLE 6—(Continued)

No.	Longi- tude	Wt.	Lati- tude	Wt.	No. plates	Description*
108	172.0	4.8	-34.4	7.7	10	
109	172.3	6.8	-10.9	11.5	12	<i>Titanum Sin.</i>
110	174.3	4.9	-19.1	8.7	9	<i>Titanis Lac.</i>
111	176.6	1.1	-57.5	1.6	4	
112	178.7	1.9	-48.3	2.2	4	
113	178.7	3.8	-64.4	3.7	9	<i>Ulyxis Fret.</i>
114	179.3	1.1	+46.1	0.4	2	<i>Propontis</i>
115	183.6	1.5	-38.9	2.5	6	
116	183.6	4.5	- 0.7	6.7	10	<i>Aquæ Apollinares</i>
117	187.1	8.9	-13.4	6.2	8	S. limit, <i>Zephyria</i>
118	188.9	1.4	-52.0	2.1	5	
119	189.5	1.2	-24.4	2.7	4	
120	191.6	1.5	+ 8.7	1.7	5	
121	196.7	4.2	+17.0	4.4	8	S.E. point, <i>Trivium Charontis</i>
122	201.9	4.3	-60.0	5.0	9	
123	203.1	6.5	- 7.8	8.7	10	<i>Laestrygonis Sin.</i>
124	203.2	1.2	+35.6	0.6	2	<i>Hecates L.</i>
125	203.4	2.2	+ 1.0	2.9	8	
126	204.7	4.4	-30.8	6.8	9	<i>Scamandri Lac.</i>
127	205.6	3.7	-74.7	2.6	6	
128	206.8	3.5	-15.0	5.9	8	<i>Laestrygonis Lac.</i>
129	208.4	2.1	+21.7	1.6	3	S.E. limit, <i>Elysium</i>
130	208.5	2.6	-51.3	3.7	5	<i>Mare Chronium</i>
131	216.7	2.9	+41.2	1.3	4	
132	221.8	5.0	+10.2	6.0	8	<i>Lucrinus L.</i>
133	221.8	3.4	-22.7	5.0	6	
134	224.3	3.3	-15.7	4.5	6	<i>Cyclopum L.</i>
135	225.1	3.5	- 6.3	5.2	7	
136	232.4	2.3	-75.5	2.5	5	
137	232.7	4.2	-31.0	7.8	10	<i>Cynia L.</i>
138	232.9	3.4	-51.0	5.2	8	<i>Tiphys Fret.</i>
139	233.3	2.5	+23.1	2.8	5	E. part, <i>Hephaestus</i>
140	233.4	3.4	+ 1.6	4.8	7	
141	237.5	4.8	- 9.9	7.8	8	<i>Cimmerius L.</i>
142	238.2	1.7	+17.6	1.6	3	W. part, <i>Hephaestus</i>
143	239.2	2.4	+ 7.4	3.0	4	
144	250.5	3.0	-20.5	5.2	6	<i>Hesperidum L.</i>
145	251.8	2.6	-31.2	4.4	5	N. limit, <i>Ausonia austr.</i>
146	252.8	2.2	+ 3.5	2.9	4	<i>Minor L.</i>
147	253.4	2.6	-55.1	2.7	5	
148	255.7	1.7	+26.6	1.5	3	<i>Lethes L.</i>
149	256.2	2.2	+41.0	1.1	3	<i>Aquæ Calidae</i>
150	260.0	2.7	- 2.7	4.0	5	N.E. point, <i>Syrtis Minor</i>
151	260.0	2.6	-40.0	4.1	5	E. point, <i>Hadriacum M.</i>
152	260.2	3.4	-63.7	3.2	6	
153	262.7	1.5	+14.0	1.4	2	<i>Tritonis L.</i>
154	263.4	1.9	-11.5	3.7	6	
155	267.0	2.9	-29.8	4.4	6	
156	267.3	2.7	-51.1	3.1	5	
157	267.7	3.4	-41.5	7.0	8	
158	272.7	2.4	- 7.7	2.9	3	
159	273.0	2.5	+ 0.2	3.0	4	
160	276.7	0.6	+47.2	0.2	2	<i>Isidis L.</i>

* The terms east and west are used like on terrestrial charts: east in the sense of decreasing longitude, west in the direction of increasing longitude.

TABLE 6—(Continued)

No.	Longi- tude	Wt.	Lati- tude	Wt.	No. plates	Description*
161	278.0	3.3	-76.3	2.5	5	
162	278.0	1.5	+11.9	1.4	3	<i>Moeris L.</i>
163	280.3	2.6	-54.3	3.7	6	
164	280.7	3.9	- 0.8	6.9	8	S.W. point, <i>Libya</i>
165	280.8	2.4	+ 5.9	3.1	5	E. limit of <i>Syrtis Maior</i>
166	281.0	2.6	+33.1	1.3	3	<i>Nilus L.</i>
167	281.4	3.3	-33.0	5.8	8	
168	282.9	2.0	+23.9	1.9	3	N.E. point, <i>Syrtis Maior</i>
169	282.9	2.2	-13.7	3.9	4	<i>Lemurum L.</i>
170	286.8	2.1	-35.3	2.6	3	N.E. limit, <i>Hellas</i>
171	287.8	2.8	-62.7	3.1	6	
172	289.3	2.5	-43.7	3.4	7	
173	292.2	3.7	+ 3.1	4.9	6	<i>Maior Lac.</i>
174	294.0	3.0	-49.0	2.9	6	
175	294.9	2.2	+22.7	1.8	3	N.W. point, <i>Syrtis Maior</i>
176	297.7	3.6	-18.5	5.7	6	
177	298.0	2.4	-29.5	2.9	3	N. limit, <i>Hellas</i>
178	299.2	2.8	+43.4	1.3	4	<i>Coloe Palus</i>
179	299.6	2.4	+21.5	2.4	4	
180	301.2	3.8	+ 4.3	4.4	5	S.E. limit, <i>Aeria</i>
181	301.2	2.5	-43.6	2.7	5	Inters. <i>Alpheus, Peneus</i>
182	302.2	4.2	-68.8	4.4	8	
183	302.7	3.1	-10.9	3.9	4	
184	303.9	3.6	-58.8	4.0	8	
185	306.9	2.0	-51.3	1.8	4	S. limit, <i>Hellas</i>
186	307.0	2.6	- 4.2	3.6	4	<i>Hipponis L.</i>
187	307.6	3.8	+30.2	2.3	7	<i>Hipponitis Palus</i>
188	307.6	2.8	-20.4	4.0	4	
189	310.4	2.7	-31.0	2.7	3	N.E. point, <i>Yaonis L.</i>
190	311.6	3.1	- 0.1	4.6	6	N.W. point, <i>Deltoton S.</i>
191	311.8	4.6	-40.2	5.1	7	S.E. point, <i>Yaonis L.</i>
192	313.2	2.8	-10.8	3.9	4	<i>Hammonis Cornu</i>
193	314.6	3.7	-21.9	6.2	7	
194	316.4	1.9	+31.7	1.1	4	
195	317.0	2.1	-64.9	1.9	4	
196	317.1	3.1	-35.2	4.0	6	<i>Yaonis L.</i>
197	318.0	1.7	-79.0	1.8	4	
198	318.2	3.8	-14.4	6.2	7	<i>Hammonis Lac.</i>
199	321.2	2.2	-48.7	1.9	3	
200	321.4	2.9	-33.3	3.4	5	N.W. point, <i>Yaonis L.</i>
201	321.5	5.9	-73.5	7.3	13	<i>Novissima Thyle</i>
202	322.4	2.7	-52.6	2.3	5	
203	323.4	3.4	- 9.9	4.6	6	
204	324.4	3.7	-24.9	5.4	8	
205	325.7	1.2	+41.8	0.5	4	
206	326.8	2.3	-15.6	3.5	4	E. limit, <i>Deucalionis Reg.</i>
207	327.6	5.0	-46.3	6.2	8	
208	328.7	4.0	-19.3	6.0	7	
209	329.1	3.6	-61.1	2.8	6	
210	332.5	3.7	-25.3	5.4	7	<i>Erythraeus Lac.</i>
211	335.8	4.0	- 1.8	6.8	8	<i>Portus Sigeus</i>
212	338.6	3.7	+42.4	1.7	7	<i>Ismenius L.</i>
213	339.6	2.9	-35.4	4.0	7	
214	340.7	4.0	-11.4	7.3	8	S. limit, <i>Sabaeus Sin.</i>
215	341.5	4.0	-55.5	4.2	6	

* The terms east and west are used like on terrestrial charts: east in the sense of decreasing longitude, west in the direction of increasing longitude.

TABLE 6—(Continued)

No.	Longi- tude	Wt.	Lati- tude	Wt.	No. plates	Description*
216	343.7	1.4	-46.1	1.4	3	
217	345.0	3.7	-68.0	3.1	6	
218	346.0	5.5	-19.9	8.5	11	<i>Pandorae Fretum</i>
219	346.9	2.4	-31.6	2.6	5	
220	349.7	6.3	- 1.8	9.4	12	<i>Edom Prom.</i>
221	349.7	1.0	-27.8	2.5	5	
222	352.8	6.7	+ 5.3	8.9	12	N.E. point, <i>Sabaeus Sin.</i>
223	355.2	3.7	-48.2	5.3	8	
224	355.3	2.6	-36.8	3.2	6	
225	356.8	3.1	-10.6	4.0	4	S. limit, <i>Sabaeus S.</i>
226	358.7	4.0	+ 1.4	5.0	7	<i>Fastigium Aryn.</i>
227	358.8	2.7	+38.4	1.3	5	<i>Eulaei Lac.</i>
228	359.1	1.4	-60.4	2.3	6	

* The terms east and west are used like on terrestrial charts: east in the sense of decreasing longitude, west in the direction of increasing longitude.

6, col. 1). If a point is marked by an open circle, the measures refer to the center of a dark spot (oasis) or of a condensation in the dark areas, or they represent the intersection of two or several dark lines (canals), and in a few cases the centers of light spots or areas; in short, the open circles always refer to markings of a more or less symmetrical character. Measures of the boundary between dark and light areas, or the points of projections (caret, sinus, promontorium) are indicated by a cross or star. The weights of cols. 3 and 5 of Table 6 are those of the least squares solution; they represent the accuracy of the result in great circle measure. Col. 6 gives the number of plates on which the point was measured and the last column the description of the marking in the usual nomenclature wherever the identification seemed beyond doubt. The names were mostly taken from the chart at the end of Flammarion's book *La planète Mars*, vol. 2, or from the chart at the end of *Lowell Observatory Annals*, vol. 2.

The system of areographic longitudes, with respect to its zero point, is based on the longitudes of central meridian given in the *American Ephemeris* for 1924. If the *Fastigium Aryn* were adopted as zero point, a correction of +1°3' would have to be applied to the longitudes of Table 6. As this correction is small and may partly be due to errors or changes in this single point it seemed preferable to retain the system of the *Ephemeris*.

The probable error of unit weight was calculated from the residuals of the individual x and y measures; 32 markings with a total of 246 measures in each coordinate were for this purpose selected at random, being the first eight of every quadrant of longitude (excluding, however, those with less than 5 measures). These points should represent a fair average and give

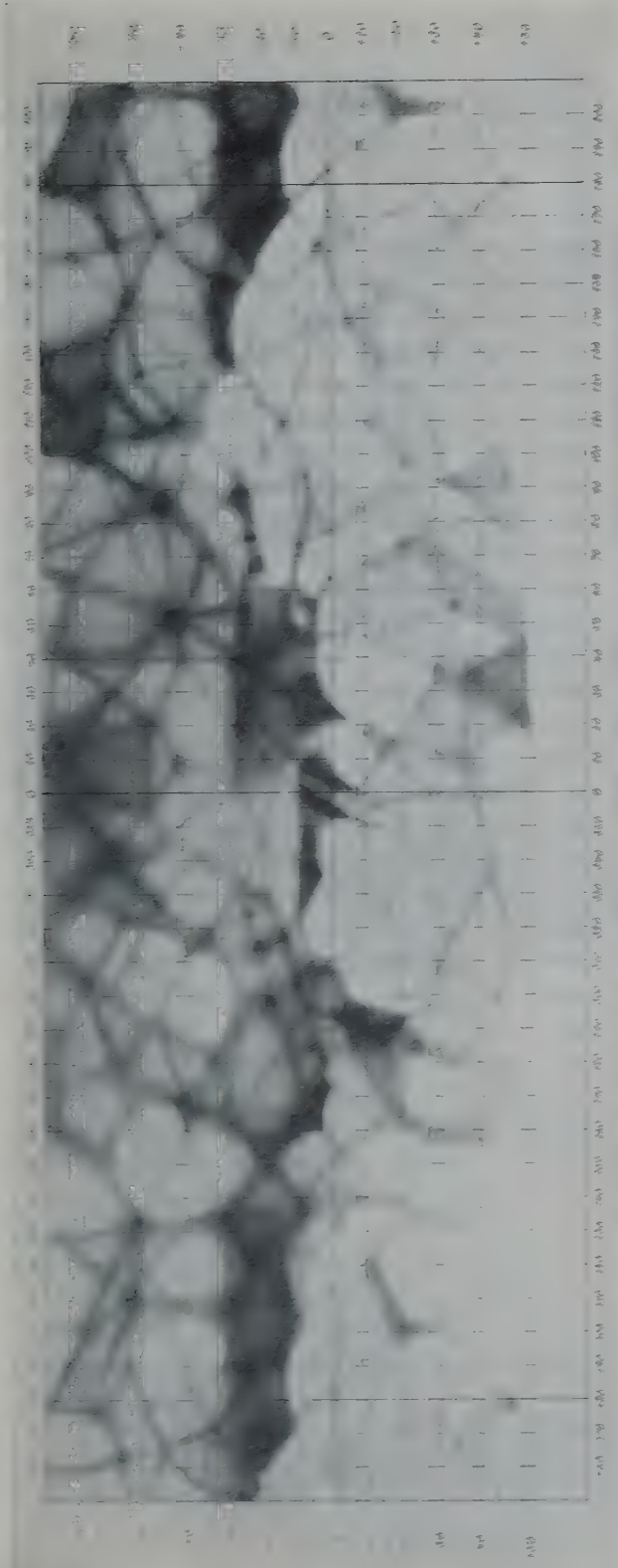


FIG. 1

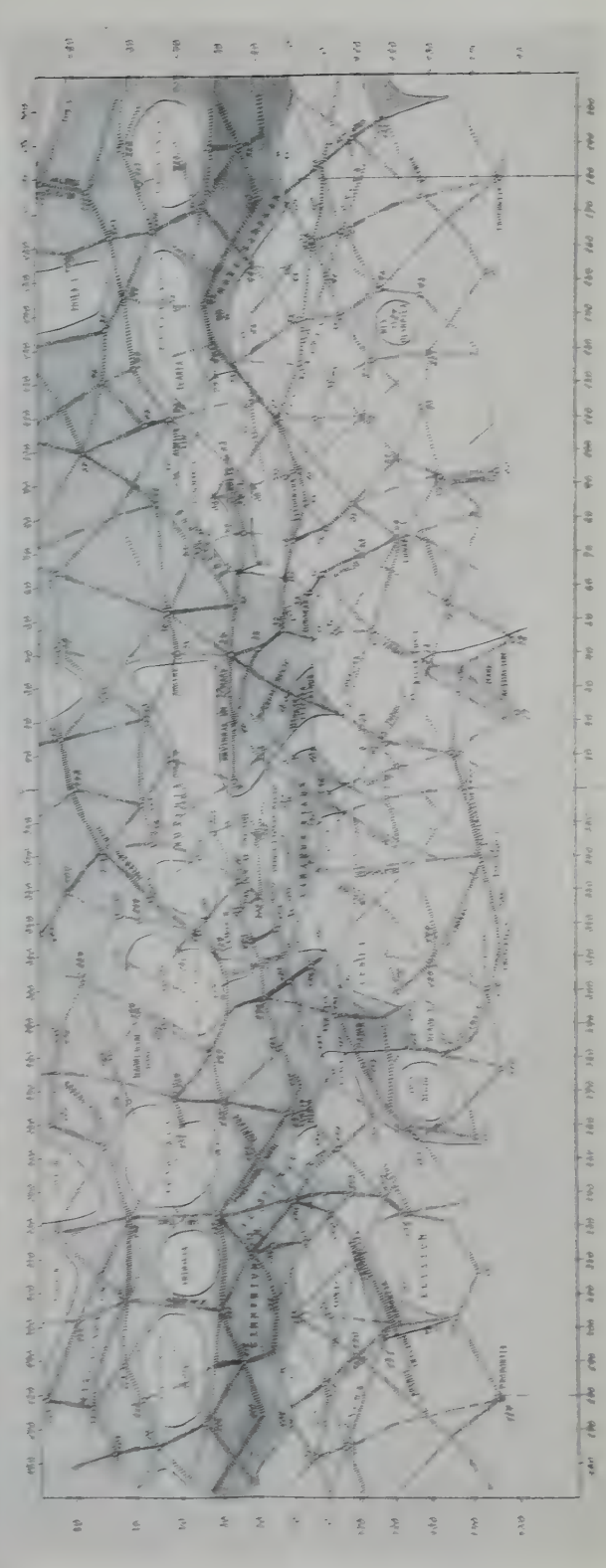


FIG. 2

for the probable error of unit weight ϵ expressed in fractions of the planet's radius:

$$\begin{aligned} x \text{ coordinate } \epsilon &= \pm .0206 \\ y \text{ coordinate } &.0193 \\ \text{Mean } \epsilon &= \pm .020 \end{aligned}$$

The accuracy is practically the same in both coordinates; the four measures of one plate furnish the position of a marking with a probable error of 1 per cent of the diameter, or $\pm .24$. This probable error seems rather large compared with the accuracy of photographic star positions. It should not be forgotten, however, that the surface markings of *Mars* are of quite different character from star images; they are mostly faint, difficult to identify and to measure, and many of them are unsymmetrical. The short exposures of *Mars* are also more subject to atmospheric distortions of the image, while in the longer exposures of star images these distortions nearly balance each other in all directions.

Expressed in degrees of great circle at the center of the disk the probable error of unit weight is

$$\epsilon = \pm 1.1$$

and the probable errors of the catalogue positions are calculated from the weights p (Table 6, cols. 3 and 5) by the formulae:

$$\begin{aligned} \text{probable error of longitude} &= \pm \frac{1.1}{\sqrt{p}} \sec \beta \\ \text{probable error of latitude} &= \pm \frac{1.1}{\sqrt{p}} \end{aligned}$$

The average weight of the catalogue positions is 3.5 in longitude, 4.4 in latitude, and the corresponding probable errors are 0.6 and 0.5 . A few of the best observed markings have probable errors as small as 0.3 which is equivalent to 11 miles on the surface of *Mars*. For points north of latitude $+30^\circ$ the latitudes are very uncertain because of the proximity of these markings to the limb. Points near the South pole are comparatively better determined, as they are visible on many plates and as their latitudes are also obtained from the x measures near greatest eastern and western elongation.

The advantage of the photographic method for measuring the positions of Martian surface details does not lie so much in the increased accuracy of an individual measure as in the ease and rapidity with which numerous measures are accomplished and in the possibility of eliminating systematic errors by making the measures in different positions of the image. For symmetrical markings like the many dark spots and canal intersections it may confidently be expected that our measures are free from systematic errors. It is, however, doubtful whether the same could be said for

points on the boundaries of dark and light areas. These boundaries appear generally as somewhat gradual transitions and the limit must be set arbitrarily (the greatest gradient of intensity was chosen as a rule). The fact, that the measures of the limb of the planet, made in a similar way, depend little on the exposure time or contrast and show no evidence of sensible systematic errors seems to justify the anticipation, that the same is true for the boundary lines, wherever a sharp boundary really exists.

10. THE CHART OF MARS

From the observational material obtained at the 1924 opposition a chart of the surface of *Mars* was prepared by pencil drawing in two parts. The first (Plate III, fig. 1) presents the equatorial regions from latitude -65° to $+50^\circ$ on the Mercator projection, the second (Plate IV, fig. 1) the Southern hemisphere between latitude -30° and the South pole in stereographic projection.

A preliminary draft of both charts was based on the measures and sketches of 13 plates made during the winter 1924-25. During the second series of measures (autumn, 1925) all images measured for diameter (Table 2) were examined on the original negatives with the microscope and compared with the preliminary charts, making notes about any corrections or additions required. A similar comparison was made with all the sketches obtained by visual observation. The definitive charts as reproduced were then completely redrawn. All the points of Table 6 as well as numerous others less frequently measured were first entered according to their longitudes and latitudes on the reseau of the charts. The markings were then executed with the help of the preliminary charts and the notes concerning their improvement. Greatest effort was made to have the chart resemble the photographs as closely as possible in general appearance. The relative intensities of the shadings were estimated on enlarged positives, making a compromise whenever they seemed to vary with the position on the disk. As some slight changes have undoubtedly taken place on *Mars* during the period of observation, preference was always given to the observations nearest to opposition (August 20-September 21). When the chart was completed, it was again compared with all drawings and photographs until the observer was satisfied of its correctness.

By careful comparison of about 150 of the best photographs it was possible to eliminate the effects of the plate grain as well as lack of definition or atmospheric distortion in some parts of the images. No detail was entered on the chart unless visible on several photographs or confirmed by visual observations.

While the chart is essentially a combination of all the photographs, it was, of course, necessary to enhance the finer details, especially the spots and lines of the network, in order to make them easily visible.

Practically every detail observed visually can be found on some of the photographs, but the latter may show only a trace of something without revealing its character or appearance and the visual observations were a great help in the representation of the fine structure, such as sharpness, width, regularity, etc., of boundary lines or canals. On the other hand many markings are quite satisfactorily shown by the photographs which were not noticed in the visual observations. This is especially true of the details in the dark areas and in the region around the South pole. While the eye is perhaps more sensitive for picking up dark lines and spots on the normal yellow-orange colored surface, the photographs as a whole give better resolution in the dark areas and in close proximity to the limb.

Our chart of *Mars* should, of course, not be considered as a permanent map of the planet; it is intended only as a representation of the surface for the period of observation. If it has been constructed with such care and precision, this was principally for the purpose of subsequently studying the changes depending on the seasons and other causes. The favorable weather conditions on Mt. Hamilton and the use of the photographic method made it possible to accumulate sufficient observations for a detailed chart in the short time of one month. The Martian season to which our results apply is that immediately preceding the winter solstice of the Northern Hemisphere corresponding in our terrestrial calendar to the period from November 26 to December 16 (Pickering's Martian Dates November 22–December 2). The polar cap, on account of its rapid variations, is not entered on the chart, but is represented by separate drawings for each day of observation (Plates IV and V).

In regard to general features our chart is in good agreement with those of other observers; there are, however, notable differences in many details, most of which are undoubtedly peculiar to the 1924 opposition. Much discord still prevails concerning the character and interpretation of the surface markings of *Mars* and it seems inevitable to take up the discussion of these problems on the basis of the visual and photographic observations with the 36-inch Lick refractor, especially as many years have elapsed since this excellent instrument was used for the study of *Mars*. It must be emphasized, that before the 1924 opposition the writer had never made any systematic observations of *Mars* and was not familiar with the Martian topography. The present description was therefore made as of something new or unknown and could not have been influ-

enced by any knowledge of the appearance of *Mars* in smaller telescopes.

11. THE DARK AREAS OF MARS

The largest part of the surface of *Mars* (especially the Northern hemisphere) is of fairly uniform brightness and of a characteristic yellow-orange color: we may call it the normal surface. It corresponds to the white background of our chart. Among the different classes of markings detaching themselves from this background we take up first the extended dark areas (generally designated by the term "mare") which with irregular and partly connected outlines cover a large part of the Southern hemisphere. In the 36-inch refractor they appear of a distinctly blue-green tinge. This color estimate is perhaps enhanced by the contrast with the normal surface, but it is independently supported by a comparison of the photographs taken with the red and yellow filters. If both are obtained on the same plate and if the exposure times are so chosen that they have equal intensity for the normal surface, then the dark areas are darker on the red images than on the yellow ones. Remembering that the yellow photographs are made with wave-lengths 5300–5900 Å. (green to orange) while the red ones utilize only a narrow interval (5900–6100) in the orange, our observation shows that the dark areas reflect relatively more light of the shorter wave-lengths or more green light. Of course it is desirable that such color determinations be made on a numerical photometric basis, but even the qualitative comparison of the two filters is quite unmistakable.

None of the larger dark areas was found to be of uniform shading. On close examination the photographs (especially those of the red filter) reveal a great amount of structure in them, consisting principally of dark spots and bands and lighter areas or passages; many of these were confirmed by visual observation. This circumstance excludes the interpretation of the dark areas as regions covered by deep water and furnishes evidence for the nonexistence of larger seas or oceans on *Mars*.

Some of the boundaries of the dark areas appear quite sharp, especially the northern limits of *Erythraeum M.*, *Sirenum M.*, *Cimmerium M.*, and the western limit of *Syrtis Major*, while *Sabaeus Sin.* presents a sharply bounded area all around. In many places, however, the dark areas fade out in gradual transition to lighter areas without any marked limits. The depth of shading also differs very widely; the northern part of *Syrtis Major*, *Tyrrhenum M.* with *Syrtis Minor*, *Sirenum M.*, and *Sabaeus Sin.* were estimated to be the darkest. Between these and the normal surface practically every intermediate step is represented. Large

canal for all these various markings has perhaps led to such a misconception; it is partly for this reason that the writer prefers to speak of a network, partly because the structure of the dark areas is not "canal-like" at all.

Not only do the lines of the network differ widely in visibility, some being of the most easily visible, others among the most difficult details, but similar differences exist in regard to width, definition, uniformity, etc. That the canals always are somewhat diffuse on the photographs is undoubtedly due to the grain of the plate, but even in visual observations under the most favorable atmospheric conditions the canals did not as a rule present the appearance of fine, sharp, uniform, straight lines as they are described by Schiaparelli and Lowell, and in many cases the lines have a well marked width. *Nectar* (40-63), *Cerberus*, *Styx*, *Thoth*, with widths of 250 to 400 miles (400-650 km.), might at the 1924 opposition be called broad bands and classified with the dark areas. Others like *Arazes* (69-78), *Coprates* (38-46), *Eumenides*, etc., fitting better the description of canals, have still a noticeable width of 60-130 miles (100-200 km.). Even the fainter lines for which the widths could not be estimated with certainty made the impression of diffuse, hazy shadings. A few like *Ganges* appeared rather as the edge of slightly shaded areas. When comparing many photographs it is occasionally noticed that a certain part of a canal is better visible on most of them than the rest, thus indicating that the same canal changes in intensity along its course, and the observer does not feel certain that there are not cases where they break off entirely before reaching an endpoint (oasis or dark area).

Strongly curved canals are rare and a certain directness or straightness of line predominates; sharp changes of direction are generally marked by an oasis or lacus. The straightness is, however, more striking in visual observations than on the photographs; there can be little question that the visual observer has the tendency to smooth out minor irregularities and line up the markings. It should be stated that on the photographic negatives, where the canals appear as light lines on a dark background, they are equally well if not better shown than on enlarged positives. On the other hand, dark lines on a light background (corresponding to light lines in the dark areas) are rarely picked up on the negatives except as broad passages. As examples of canals well visible on the reproductions of Plates I and II the following may be cited:

Plate I: Fig. 4: *Gehon*, *Oxus*. Fig. 7: the fairly sharp straight line connecting *Juventae Fons* (39) with *Aurorae Sin.* (37). Fig. 10: *Coprates* (38-46), *Arazes* (69-78), *Eumenides*, the line (47-69). Fig. 12: *Arazes* (69-78), *Eumenides* and *Elison* (78-88) form a well marked triangle of straight lines; (81-102) is broad and strong.

Plate II: Fig. 2: *Tartarus* (102-132), (99-102) and (102-114). Fig. 6: *Cerberus*, *Eunostos* (132-139), *Pactolus* (132-148), *Aethiops* (141-142), *Hyblaeus* (131-139). Fig. 7: *Cerberus* (132-140), *Aethiops* (141-142), *Triton*, *Thoth*. Fig. 10: *Phison*, *Euphrates*, *Astaboras* as well as the canal (190-227) are faintly visible.

The network in the dark areas is best illustrated by Plate I, figs. 5, 8, 10, 11 and Plate II, fig. 7.

Before taking up the interpretation of the network it is necessary to decide whether the canals are real surface markings or whether their observation is due to optical illusions as some astronomers have asserted. The issue is perhaps more clearly defined by the question: Are there features on the surface of *Mars* which are arranged in spots and lines forming a network, although their fine structure may in reality somewhat differ from the smooth lines observed? Or the alternative: Are the canals merely lines which the eye, observing with insufficient resolution and magnification, picks out from a slightly irregular granulation of the surface, while the real markings have no network character at all?

A. Köhl¹⁰ has recently investigated this problem in the light of his theory concerning the contrast effects in telescopic diffraction images. He comes to the conclusion that the surface of *Mars* in reality is probably covered with numerous fine, sharply defined details too small for the resolving power of the telescope. The transitions between areas where these details are of different density produce in the eye of the observer the optical-physiological effect of lines of contrast, most of which lie below the limit of perception. At certain places, however, between just resolvable planetary markings they are reinforced and become visible as "canals." Köhl illustrates this effect by cutting a printed page to circular shape, placing it on a black background and painting on it a few gray spots as well as a larger gray area with pointed projections. If this disk is seen at such a distance that the details of the print disappear, some faint dark canal-like lines are noticed produced by the irregular distribution of the printed letters, running between the gray spots and the projections of the gray areas.

This experiment is of great interest for the correct interpretation of observational data and may have an application to some of the finer and more difficult canals, but it does not fit the case of the Martian network in general. In the first place, it starts from the erroneous conception that the canals are all similar in visibility and character, which we have already dismissed as being far from the truth; and the faint contrast lines of Köhl's figure do not come anywhere near showing the variety of intensities and widths exhibited by the network lines of our chart. In the second place,

¹⁰ *VJS der Astron. Ges.* 59, 196, 1924.

areas of very light shading occur quite outside of the "Mare" as e.g. in the region of the *Ganges* and S.W. of *Nix Olympica* (87).*

When our chart is compared with those of other observers made at former oppositions a considerable difference in the latitude of the sharp northern boundary of *Sirenum M.* and *Cimmerium M.* is immediately noticed. On our chart these dark areas extend 10°–15° farther north and appear correspondingly wider. A real change in this boundary line must have taken place as so large differences could not be accounted for by errors of observation; this is in agreement with W. H. Pickering's¹⁸ statement concerning the changing latitude of *Titanum Sin.* It appears then that the dark areas are not quite permanent in outline and extent besides showing some well known variations in shading.

It is hard to find any hypothesis that can be reconciled with the observational data except the assumption that the dark areas are regions covered with vegetation. Not only is this in harmony with the observed color, but various forms of plant life and varying density of growth can account for the different degrees of shading. Climatic conditions and available supply of moisture, partly depending on the seasons, partly progressing over longer periods, will greatly influence the development of vegetation and may thus lie at the origin of the changes observed in the dark areas.

12. THE NETWORK (CANALS AND OASES)

The results of our observations as embodied in the chart indicate the existence of a complicated network of dark lines and spots covering the whole surface of the planet, the dark areas as well as the normal surface. Visual observers have paid most attention to the structure of this network in the latter regions, where it is most striking. There the lines appear rather narrow, so that the term "canal" has been applied to them while the spots, relatively small and well defined, have received the designation of "oasis" or "lacus." A few observers like Lowell and Douglass and others already have noted numerous canals and oases in the dark areas, and our chart shows the network equally well pronounced in these; with the difference, however, that here the lines often become broad diffuse bands and the dark spots appear as condensations of shading without any sharp edge, so that we might more appropriately speak of a "framework." Despite these differences there is good evidence that the framework of the dark areas is of the same character as the network of canals and oases on the normal surface. First, we find in both the same general design as to size and

arrangement of the meshes; the lines running most frequently between dark spots or radiating from them. Secondly, the whole network is one continuous system; it does not stop at the boundaries of dark areas, and many lines can be traced on a course leading through light and dark regions indiscriminately. Thirdly, there are instances where a network line may be narrow like a canal at one time and widen to a broad band at another time (*Nectar*, *Cerberus*, *Thoth*, etc.). Most convincing is perhaps the case of *Pandorae Fretum* which in 1924 appeared as a light area with a network of faint canal-like lines as shown in the chart. At the 1926 opposition this whole region presented the aspect of a dark area and these same lines then took the character of the more broad and diffuse bands typical of the dark areas.

The network of our chart coincides to a great extent with the canals and oases drawn by other observers. These markings having been the subject of a widespread controversy, a closer description of their appearance may be of interest. Before the 1924 opposition the observer had never seen any of the canals and entertained a rather critical point of view concerning their reality. But even on the first observing night it did not seem possible to make a faithful drawing of what was seen in the 36-inch telescope without entering on it some of the more prominent network lines which like *Araxes* (69–78), *Eosphoros* (54–69), *Eumenides* were among the most easily visible markings. With increasing observing practice more of the canals were recorded, and at the same time a careful examination and comparison of the photographs plainly revealed parts of this network. On a good night the more prominent canals were steadily visible while many others appeared only at intervals during the moments of best definition. The principal gain in observing practice was not in seeing more of the lines, but in remembering and recording on the drawings what was seen. Some of the denser parts of the network, like the regions between *Nodus Godii* (88) and *Titanum Sin.* (109) or that south of *Trivium Charontis* (121), or that of *Tithonius Lac.* made, most of the time, the impression of a slightly shaded area. But when, for a moment, the definition improved they suddenly resolved into a maze of well defined lines, too complicated, however, to be retained by the memory from such short flashes, even if they repeated themselves. The observer has thus never been able to satisfy himself of having correctly represented the region of *Nodus Gordii* (88) e.g. as seen at the few glimpses under the most favorable atmospheric conditions.

The question of the canals of *Mars* is often treated as if all these objects were of the same character and appearance; yet nothing is farther from the results of our observations. The use of the same technical term

* The numbers in brackets refer to those of the skeleton charts (Plate III, fig. 2 and Plate IV, fig. 2).

¹⁸ Report on Mars No. 5, *Pop. Astr.* 22, 303, 1914.

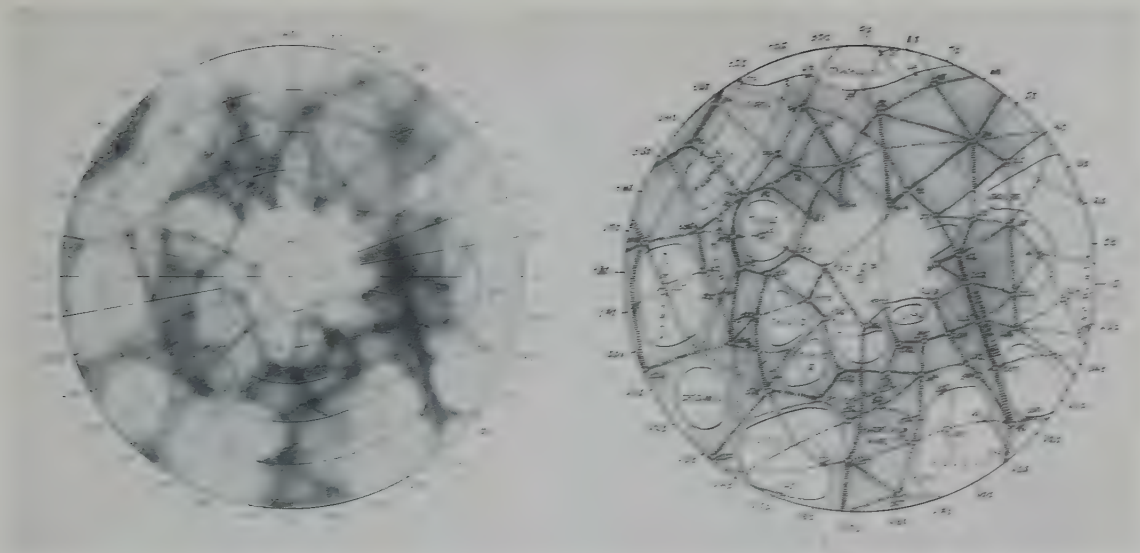


FIG. 1

FIG. 2

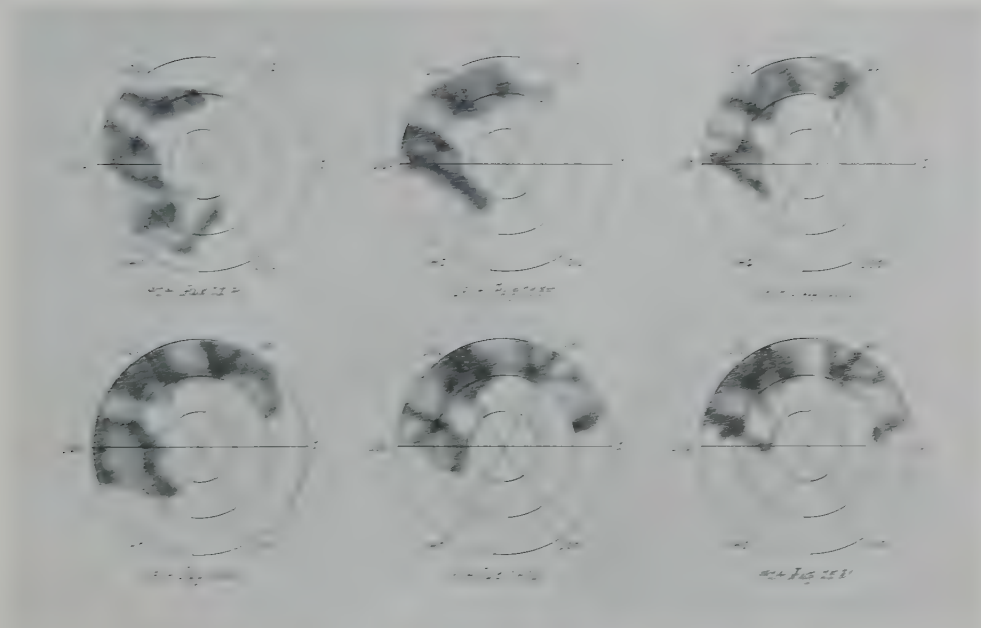


FIG. 3. Diagrams of the same type as those in Fig. 1 and 2. The same method of construction as in Fig. 1 and 2.

the visibility of the contrast lines (Kühl's fig. 2 and 3) is far inferior to that of the stronger canals on our photographs. In the third place, the observations do not corroborate the idea that the canals become noticeable thanks to dark markings (spots or projections) at their ends, and our photographs show many canals interrupted by the edge of the disk (see *Eumenides* on Plate I, figs. 9 and 10) or ending in spots which are no stronger than they themselves. Finally, Kühl overlooks the fact that the network also covers the dark areas, and the broad bands in these can certainly not be treated as contrast lines; yet we have given strong reasons above, why the framework of the dark areas must be of similar topographic character to the canal network of the light areas.

Nobody examining our photographs can doubt the reality of markings like *Nectar* (40-54), *Cerberus*, *Thoth*, *Nasamon* (160-168); if seen at the 1924 opposition alone they would probably not be classified among the canals. But at other oppositions these same objects appeared as faint lines like most of the other canals. Should we then admit the reality of such a marking at one opposition when it is strong and reject the same marking as an illusion at another opposition when it is faint? Even if we were willing to drop the fainter canals as optical illusions it would be difficult to draw the line between real and illusory canals since the network contains lines of every degree of visibility from the strongest to the faintest ones. All evidence furnished by direct observation or deduction, it seems to the writer, supports the claim that most parts of the Martian network must be real surface features, although it is possible that a few of the faintest lines may be illusions due to contrast effects.

Some of the students of *Mars*, notably Lowell, have made the assertion that the canal network of *Mars* is of such striking geometrical regularity that it cannot be explained as a natural phenomenon, that it must be artificial, the work of intelligent beings. Our representation of the network can hardly support such a conclusion; it presents considerable irregularity and does not make the impression of artificiality. In some places the lines are closely crowded, intersecting without apparent system, in other parts the meshes are wide. Occasionally several lines cross each other without passing through the same point. Even the lines themselves are diffuse and not always uniform. The observer is under the impression that the network is drawn with rather too great regularity on the chart; many of the smaller irregularities could not be ascertained with sufficient certainty; it is one thing to notice slight irregularities in a line and another thing to represent these correctly in a drawing. The writer is therefore inclined to interpret the network as natural features of the topography of *Mars*.

It was not possible to obtain any estimate of the color of the canals from visual observations. A comparison of the yellow and red photographs taken on the same plate, however, shows that the canals behave like the dark areas, i.e. they are relatively darker on the red photographs, and we conclude from this fact that they are probably of the same blue-green tinge as the dark areas. The hypothesis that the latter are regions covered with vegetation then requires that the network is also being made visible by vegetation while the framework in the dark areas would indicate places where the vegetation is densest. This view is supported by the temporary broadening of certain canals like *Nectar*, *Cerberus*, *Thoth* into bands resembling the dark areas and further by the evident relationship of the network to the outline of the dark areas. On our chart the network appears like the skeleton of the dark areas, the latter forming projections (sinus, caret) along the lines, while in the open spaces of the meshes the light areas often advance into the dark ones (promontories).

There remains now the question, why in the light regions of the normal surface vegetation should be confined to the spots and narrow lanes of the network and why in the dark areas it should be most abundant there. The view that the network follows natural geological formations of the crust of the planet is perhaps the most obvious, and one would in the first place think of depressions of the surface in which moisture accumulates and where the temperature is higher, both factors favoring growth of vegetation. There is, however, a possibility that other geological features such as faults or volcanic cracks are responsible for the canals. It cannot be denied that the system of valleys on the Earth is very different from the Martian network, but there is no reason why the two must be similar. A comparison of the Moon and the Earth gives a vivid illustration of how different the surface formations of planetary bodies may be. In regard to size, mass, force of gravity at the surface, scarcity of water, density of atmosphere, *Mars* takes a position somewhat intermediate between the Earth and the Moon, and it would therefore be hard to predict what its surface structure should be.

Unfortunately, our knowledge about elevations and depressions on *Mars* is very limited. Observations of the terminator are difficult on account of the twilight effect and because they can only be made when *Mars* is at greater distances. One striking projection at the terminator was observed visually (1924, November 15, 6^h40^m to 8^h30^m Pac. S. T.) over the region of *Argyre*; being of large dimensions and long visibility it undoubtedly represented a cloud phenomenon as such have been observed before. Of more interest would be the smaller irregularities of the terminator for which

all our photographs were carefully scanned. With the gradual fading of illumination the terminator appears faint and diffuse on the photographic images and is not susceptible of accurate measures. Only eye estimates of the deviations from its normal elliptical shape could be made and led to the following notes:

Plate	Date	Projections	Defects
425	Sept. 26. 706	<i>Zephyria, Electris,</i> Region E. of <i>Trivium Charontis</i>	<i>Chronium M., Cimmerium M.</i> (slight)
427	26. 830	<i>Ausonia, Aethiopis</i> (plain)	<i>Cimmerium M.,</i> <i>Tyrrhenum M.</i> (slight)
428	29. 701	<i>Phaetontis, Electris</i> (slight)	
438	Oct. 8. 705		<i>Lunae Lac.</i> (47).
439	8. 732	<i>Thaumasia</i> (plain), also <i>Ophir I.</i> betw. <i>Lunae L.</i> and <i>Tithon Lac.</i>	
440	11. 716	<i>Protei Reg.</i>	<i>Aridus Lac.</i> (34)
441	12. 647	<i>Argyre</i> (plain)	<i>Erythraeum M.</i>
443	12. 763	<i>Thaumasia, Thyle I.</i>	

It will be noticed that the protrusions all refer to light regions, the defects to dark ones which is indeed in harmony with our assumption. The evidence is, however, not as good as it might appear, since the gradual fading of illumination and the photographic irradiation must produce a similar effect. The latter can be tested at the limb where it should be increased by the greater contrast. The original negatives* on the contrary show the limb to be much smoother than the terminator, so that at least a part of the observed irregularities may be real, indicating higher levels for the light regions, lower levels for the dark areas and the network. This also agrees with Lowell's observations.

Another source of information concerning elevations on *Mars* is furnished by the melting of the polar cap. It will be shown later that this melting process seems to progress more rapidly along the lines and near the spots of the network. A higher temperature is thus suggested for the network, and this most likely has its cause in lower levels.

No duplication or gemination of canals was observed either visually or photographically. Of course there are a few cases where lines run parallel or nearly so for some distance; but they are neither close enough nor sufficiently frequent to attract special attention. It must, however, be stated that the period of observation falls into the Martian season which according to Schiaparelli is unfavorable to duplication, and the present observations cannot furnish any valid proof against its occurrence.

* On the illustrations of Plates I and II the irregularities of the limb are much enhanced by the process of reproduction, while the real terminator is hardly visible.

13. WHITE SPOTS AND AREAS

Besides the South polar cap a number of spots and areas were observed as being lighter in shade than the normal surface and presenting a decidedly more whitish appearance. The most remarkable of these objects was a circular spot at the place where Schiaparelli in 1879 detected *Nix Olympica* and for which we have therefore adopted the same name. The first photographs favorable for this region were taken on August 21, and on some of these a small area is noticeable, surrounded by dark network markings, but hardly any brighter than the normal surface.* On August 22 (see Plate I, fig. 12) *Nix Olympica* stands out as a uniform white spot of 10° (360 miles, 580 km.) diameter but much less brilliant than the polar cap. Its appearance remained the same on photographs of August 23 and 24 (see Plate I, fig. 11) and a few of the best images still show a trace of it visible on August 28 close to the limb. The same spot was also noted on three sketches made by visual observation on August 23, 24, 25 with a diameter of about 6°; but the visual estimate is less reliable than the photographic measures. When the region came to view again in September no white spot could be detected either visually or photographically. *Nix Olympica* then appears to have persisted for a period of at least six days with a sharp outline, constant size and identical location (the photographic measures of its center on four days shows an extreme range of only 2°). No appreciable diurnal variation was evident; the spot was well visible not only in the forenoon hemisphere but as late as 2 hours past the central meridian. Mr. Wright having kindly allowed me to examine his plates, it is of interest to state that the white spot does not show on the ultraviolet images but is well marked on the infra-red ones. This circumstance as well as the invariability in position and outline leaves little doubt that *Nix Olympica* in 1924 was a surface phenomenon. One might think of a frost or snow deposit if the brilliancy were not rather low for this.

The other whitish markings were observed closely nestling toward the northern boundary of the southern maria. Two elliptical white areas to the east and west of *Titanum Sin.* (109) were seen during most of the observing period (August 20-24 and September 21-26). The first of these lying in *Memnonia* stretched along the north-eastern boundary of *Sirenum M.* from *Titanum Sin.* about 10° to the south-east. A trace of it is visible on Plate I, fig. 11. The second, situated in *Zephyria* close to the northern limit of *Cimmerium M.* was larger and more pronounced; it is shown in Plate II, figs. 1, 2, 3. Varying somewhat in size it extended at its maximum from *Titanum Sin.* (109) to *Laestri-*

* The reproduction of Plate II, fig. 1 does not give good definition in this part of the disk.

gonum Sin. (123) with a width of nearly 10° . A smaller white spot (ca. 7° diameter) was occasionally noticed right north of *Titanum Sin.* (109). A similar marking (10° – 12° diameter) seemed to overlie *Edom Prom.* (220) on October 8 and November 15.

Wright²⁰ was the first to draw attention to the existence of "impermanant white configurations" exhibited by the violet and ultraviolet photographs, which he interpreted as "clouds of fine haze." He found these quite persistent along the borders of *Sirenum M.* and *Cimmerium M.*, and they are undoubtedly identical with the white areas observed in the same position visually and on yellow photographs. Wright sought to distinguish between objects of this kind and the extremely bright temporary patches on the disk of *Mars* sometimes seen with the telescope. As a result of our combined observations it appears then that we have to distinguish two classes of bright markings. The first, like *Nix Olympica*, are not visible on violet or ultraviolet photographs while fairly strong on yellow or red images; they must be on the surface of *Mars* or immediately above it. To this class the great white spot observed by Slipher²¹ in 1922 seems to belong. In the second class we have the objects described by Wright which are more striking on violet or ultra-violet photographs, but also noticeable by visual observations; they are probably phenomena of higher atmospheric levels.

A rather exceptional formation for *Mars* is the small light spot in *Aurorae Sin.* (33) which was well observed visually on September 1 and 2 with magnification 520 and drawn as of 3° or 4° diameter; it is also shown on many of the photographs (Plate I, fig. 8), but did not distinguish itself by any peculiar color. Attention may here be drawn further to the brightness of *Ophir I.* during the period August 26–September 2. Southwest of *Fons Juventae* (39) and west of the *Ganges* it detaches itself as a sharply defined light band (see Plate I, figs. 7 and 8) enhanced somewhat by the contrast with the slightly shaded areas on both sides.

14. THE SOUTH POLAR CAP

The increase in contrast necessary for the reproduction of the photographs in Plates I and II has suppressed most of the details in the polar cap. The original negatives, however, do not suffer from this defect and are very well suited for a study of the seasonal changes in the South polar regions. From each of the forty plates a sketch of the polar cap was made with the aid of the microscope and a number of points were measured along its border and in its surroundings. Computing the areographic longitudes and latitudes and plotting the measured points on a chart

reseau in stereographic projection, the 26 polar cap charts of Plates IV and V were prepared by pencil drawing. The latitude circles are given for intervals of 10° and the charts cover the area from -60° latitude to the pole. Combining the results of the different plates taken on the same night, only one chart was made for each day of observation.

The eccentric position of the South polar cap is easily noticed on the drawings. The many irregularities of the outline render a determination of its center rather uncertain.

Period	Center of South polar cap	
Aug. 20–Sept. 12	$\lambda = 55^\circ$	$\beta = -83^\circ$
Sept. 22–Oct. 12	45	–87

In looking over the photographs of Plates I and II the poor visibility of the polar cap on figs. 2, 4, 5, 6, 7 of Plate II is surprising, while on later photographs (Plate I, figs. 1, 2, 3) it appears again of normal strength but smaller. It might be suggested that a temporary obscuration of the polar cap by atmospheric cloudiness had taken place between September 11 and September 26. On closer investigation, however, this suggestion must be dropped. Of the two series of photographs taken on September 11, the earlier one (Plate II, fig. 11, $\lambda_0 = 283^\circ$) shows the polar cap considerably weakened, the later one (Plate II, fig. 9, $\lambda_0 = 318^\circ$) again at normal strength. On September 26 the reverse is the case: polar cap normal on earlier plate ($\lambda_0 = 141^\circ$), faint on later plate ($\lambda_0 = 184^\circ$). This makes it evident that the visibility of the polar cap depends on the longitude of the central meridian and not on the date of the observation. For central meridians between 160° and 280° the South polar cap on account of its eccentric position and elongated outline must appear smaller and situated close to the limb, and in these longitudes it is moreover bordered by light areas. The oblique position of the surface to the Sun's rays and to the direction of view of the observer combine with the increased absorption in the Martian atmosphere so close to the limb in robbing the polar cap of its normal brilliancy. Only on the best photographs taken in these longitudes is the real polar cap clearly shown, detached from the surrounding light areas, as a small spot close to the limb; in visual observations it was always distinctly visible.

When the polar cap is turned toward the observer, there is hardly any noticeable difference between the yellow and red photographs with respect to the relative intensity of the polar cap, compared to the normal surface; perhaps the cap might be estimated just a trifle stronger on the yellow images. For central meridians between 160° to 280° , however, where the polar cap is close to the limb and small, it is considerably fainter on the red images. This is probably

²⁰ *L. O. Bull.* **12**, 61, 1925.

²¹ *Publ. A. S. P.* **34**, 215, 1922; *Pop. Astr.* **33**, 593, 1925.

related to the fact that the red photographs of *Mars* show a marked fading in the brightness of the disk as the limb is approached, while the yellow images present a more uniform disk with a sharper edge. As to size and outline, the polar cap appears pretty much the same in both colors.

The 26 charts of Plates IV and V give a good illustration of the way in which the South polar cap is gradually receding. The outline of the cap is very irregular with many projections and indentations; these are most numerous during the period August 20–September 13 and do not seem to be of a temporary nature, but often persist for many days. The projections or lobes as a rule are directed toward light areas. One of the most prominent of these, in longitude 150° (August 20–29), points toward *Thyle I*; others in longitudes 90° and 40° advance toward *Dia* and *Argyre*. The projection of *Novissima Thyle* (longitude 320°) is the entrance of a light passage leading toward *Hellas*. The indentations all occur near dark spots or where network lines enter the cap. One of these, at longitude 70° south of the spot (44) is well visible on Plate I, fig. 8. Another indentation at longitude 170° first (August 20) found south of the point (101) gradually recedes along a dark broad band toward (105). One cannot avoid the impression that the polar cap adapts itself closely to the topographic features of the planet and must be on the surface; our observations thus support the widely accepted hypothesis of a region covered with snow or ice, melting under the influence of the Sun's rays. This is not in contradiction to Wright's work, who finds very strong images of the polar cap in the ultra-violet, which he interprets as representing an atmospheric phenomenon. At the same time he admits the existence of a surface cap showing on infra-red photographs, and it is evidently with the latter that we are concerned in our yellow and red photographs. There does not seem to be anything unreasonable in the assumption that the melting snow covered area is associated with peculiar atmospheric conditions. These also seem to affect the visual observations to some extent, more than the yellow photographs. It may be due to this fact that the different parts of the cap were often of unequal intensity, and that some of the protruding lobes at its border were noted (visually) for exceptional brilliancy.

The observer who compares the apparent size of the polar cap from day to day finds its shrinking to progress very unevenly, as illustrated in Table 7, which gives the diameters of the South polar cap measured on the photographs at right angles to the central meridian. As no suitable photographs were available after October 12 the last five data were taken from later drawings made visually and are based on eye estimates.

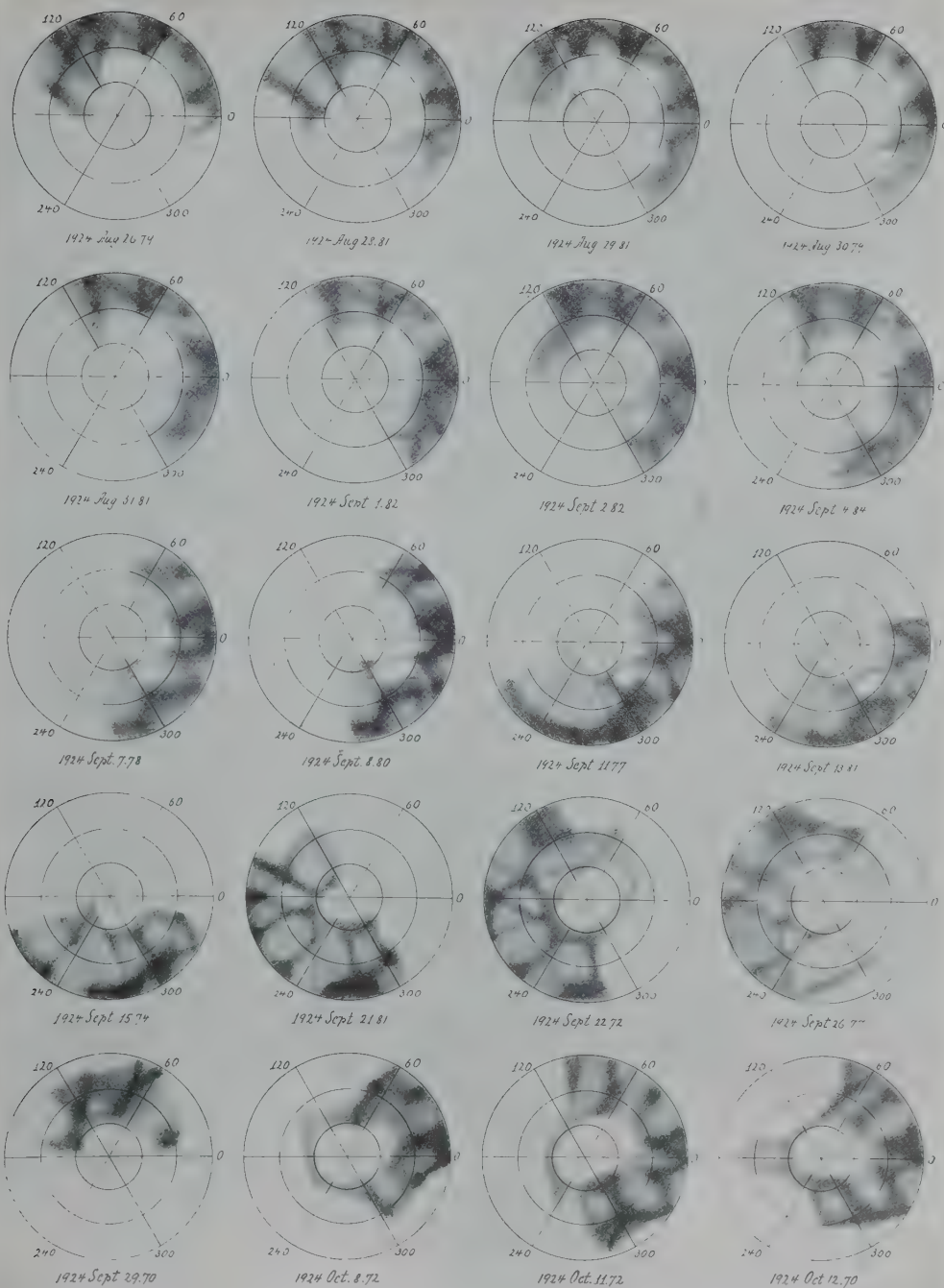
TABLE 7
APPARENT DIAMETERS OF SOUTH POLAR CAP

	Date	No. plates	Central meridian	Diameter
1924	Aug. 20	1	184°	24°
	Aug. 21	2	154	28
	Aug. 22	2	121	28
	Aug. 23	2	140	26
	Aug. 24	2	112	30
	Aug. 25	1	105	29
	Aug. 26	1	88	26
	Aug. 28	2	78	28
	Aug. 29	1	69	27
	Aug. 30	1	54	27
	Aug. 31	2	52	24
	Sept. 1	1	48	27
	Sept. 2	1	37	25
	Sept. 4	2	25	22
	Sept. 7	2	338	21
	Sept. 8	1	337	23
	Sept. 11	2	300	19
	Sept. 13	1	294	17
	Sept. 15	1	253	15
	Sept. 21	2	238	11
	Sept. 22	2	183	9
	Sept. 26	1	140	14
	Sept. 29	1	111	16
	Oct. 8	2	34	15
	Oct. 11	2	348	15
	Oct. 12	1	12	15
	Oct. 18		251	(18)
	Oct. 22		215	(16)
	Oct. 30		140	(9)
	Nov. 1		118	(9)
	Nov. 15		344	(8)

Besides the gradual decrease due to the progress of the season there is a large fluctuation which must be partly attributed to an elongation of the cap, partly to its eccentric location. Without eliminating these two effects no conclusions can be drawn as to the regularity of the melting process; it would be possible to accomplish this by combining simultaneous observations made at observatories of widely differing terrestrial longitudes.

Graff,²² from visual estimates made at Hamburg-Bergedorf and corrected only for elongation of the cap, finds considerable fluctuations in the size which he represents by a curve and considers as real. Our observations, however, do not confirm these fluctuations and deviate mostly in the opposite sense. If our results are combined with those of Graff, the irregularities of the polar cap and the effects of its eccentric position are to some extent eliminated, since observations of the same date refer to central meridians differing on an average by 128° . We then obtain a fairly smooth curve, but the melting is most rapid from August 20 to September 15 and gradually slows up for the later periods.

²² *Astr. Abhandl. d. Hamburger Sternwarte in Bergedorf*, 2, No. 7, 1926.



Drawings of the South Polar Cap and its Surroundings.
The circles represent latitudes -60° , -70° and -80° .

The charts of Plates IV and V also indicate a rather steady progress in the contraction of the polar cap, and only in a few cases are any small advances in the outline of the cap noticeable; e.g. between August 21 and 22 in longitude 90° , between August 28 and 29 in longitude 50° , between September 4 and 8 in longitude 0° , or between September 22 and 26 in longitude 120° . None of these variations is too large, however, to be explained by accidental errors in the measures.

Numerous dark lanes are seen to cross the cap, especially between August 22 and September 7; measures of their crossings and starting points made it possible to fix their location with fair accuracy. They seem to be rather permanent in character and it is probably due to the greater distance of *Mars* that they do not show on the later photographs. Most likely they represent parts of the network piercing through the snow and some of them are actually later found as regular network features. This fact as well as the irregularities of the outline makes it evident that the South polar cap is melting more rapidly along the dark network markings and that these become visible even when the open meshes are still covered with snow. We must conclude, therefore, that the dark spots and lines are warmer than the light areas, and it has been

mentioned before that this furnishes support to the hypothesis that the dark markings are depressions in the surface.

As a good illustration of the melting process attention may be drawn to the gradual changes of *Novissima Thyle* (201) also called *Mitchell's Mountain*. On August 29 this region still appeared as a regular part of the polar cap; between August 30 and September 1 a fine dark lane developed dividing it from the polar cap. For about 10 days *Novissima Thyle* was visible as a brilliant white spot well separated from the polar cap (the reproductions of Plate II, figs. 10 and 12 do not show the separation as clearly as the original negative). On September 15 the spot had lost its brilliancy and appeared thereafter as an ordinary light area.

Most of the suggestions concerning the interpretation of Martian phenomena presented in this paper are, of course, not new. They were merely stated to bring the various observational facts into relationship and to indicate the personal views of the observer or his support of certain hypotheses.

March, 1927.

Issued April 26, 1927.

LICK OBSERVATORY BULLETIN

NUMBER 388

ELEMENTS AND EPHEMERIS OF COMET *d* 1927 (STEARNS)

BY

H. THIELE

Comet *d* 1927 was discovered at Middletown by Stearns on March 10. Its estimated magnitude at discovery was ten. Maxwell describes it as having a faint nucleus but no tail. Maxwell also has reported striking similarities in appearance on April 5 to the neighboring nebulae NGC 5846 and NGC 5850. The chief point of similarity is the faint nucleus which would make observations taken with large telescopes more comparable than a combination of such observations with measures of the integrated appearance in smaller telescopes.

A preliminary parabolic orbit was based on the following observations:

1927, U.T.	α (1927.0)	δ (1927.0)	Observer
Mar. 10.420	15 ^h 16 ^m 06 ^s	-7°21'43"	Stearns, Middletown
Mar. 13.4121	15 15 18.5	-6 24 56	v. Biesbroeck, Yerkes
Mar. 15.3049	15 14 43.2	-5 48 03	v. Biesbroeck, Yerkes

A direct solution by Leuschner's method without differential correction gave:

ELEMENTS

T	1927, Mar. 18.3102 U.T.
ω	10° 16' 15"
Ω	214 33 55
i	87 16 31
$\log q$	0.566338

RESIDUALS

O-C	I	II
$\Delta\alpha$	+9.7	+3.0
$\Delta\delta$	-2.5	+0.3

An observation of March 14.0835 by Vinter Hansen and Möller of Copenhagen was represented as follows:

$$O-C, \Delta\alpha = +3.0, \Delta\delta = +3.3$$

As the first telegraphic position was approximate, no attempt was made to improve this orbit.

The elements and ephemeris were telegraphed to Harvard College Observatory and are published in *H. C. O. Announcement Card No. 24*, March 21, 1927.

By a single differential correction of the geocentric distance and the heliocentric velocities derived from this orbit for March 26.49442, a second orbit has been derived from the following observations:

1927, U.T.	α (1927.0)	δ (1927.0)	Observer
(1) Mar. 13.4121	15 ^h 15 ^m 18 ^s .5	-6°24'56"	v. Biesbroeck, Yerkes
(2) Mar. 26.49442	15 09 40.70	-1 55 16.7	Maxwell, Lick
(3) Apr. 5.48441	15 03 07.97	+1 48 14.9	Maxwell, Lick

ELEMENTS

T	1927, Mar. 20.02161 U.T.
ω	10° 36' 11".5
Ω	214 35 43.2
i	87 32 21.0
e	0.991805
$\log q$	0.563875

As these three observations were taken with large telescopes and as the motion was principally in declination it seemed advisable to derive the elliptic orbit which would represent the observations exactly, rather than to attempt the best parabolic orbit with residual in the first declination.

CONSTANTS FOR THE EQUATOR 1927.0

$$\begin{aligned} x &= r[9.915686] \sin (282^\circ 17' 58.4 + v) \\ y &= r[9.829585] \sin (241^\circ 03' 49.7 + v) \\ z &= r[9.968649] \sin (356^\circ 32' 54.7 + v) \end{aligned}$$

EPHEMERIS

1927, U.T.	$\alpha(1927.0)$	$\delta(1927.0)$	$\log p$	Br.
Apr. 13.0	14 ^h 57 ^m 8 ^s	+ 4° 41' 0	0.442	1.21
17.0	53 38	6 12.5	0.440	
21.0	49 57	7 42.9	0.440	
25.0	46 10	9 11.4	0.440	
29.0	42 17	10 37.2	0.441	
May 3.0	38 22	11 59.9	0.444	
7.0	34 28	13 18.6	0.447	1.17
11.0	30 36	14 33.1	0.451	
15.0	26 51	15 42.9	0.456	
19.0	23 14	16 47.8	0.461	
23.0	19 47	17 47.7	0.468	
27.0	16 33	18 42.5	0.475	
31.0	14 13 31	+19 32.3	0.482	0.98

The unit of brightness is that of March 13.

The outstanding features of the orbit are: the perpendicularity to the ecliptic, the large perihelion distance of 3.7, and the proximity of the perihelion to the ecliptic.

BERKELEY ASTRONOMICAL DEPARTMENT,

April 16, 1927.

PRELIMINARY ELEMENTS OF OBJECT COMAS SOLÀ (1927 AA)

BY

ROBERT S. RICHARDSON, P. A. McNALLY, S. J., and ELIZABETH E. STERNBERG

Object Comas Solà (1927 AA) was discovered by Comas Solà at Barcelona January 10th. The magnitude for January 22d was 12.8. The retrograde motion of the body was very rapid. The discoverer suggested that the object was probably an asteroid.

On receipt, late in March, of *Bulletin de l'Observatoire de Nice, No. 25*, containing the following three photographic positions by Comas Solà:

1927, U.T.	$\alpha(1927.0)$	$\delta(1927.0)$
(1) Jan. 22.87569	8 ^h 00 ^m 06 ^s .6	-0° 55' 56"
(2) Jan. 23.91528	7 58 34.9	-1 08 54
(3) Jan. 24.97986	7 57 02.7	-1 21 52

an attempt was made to derive a general orbit by direct solution. By inspection of the critical determinant:

$$\alpha'^3 \tan \delta - \alpha'' (\tan \delta)' + \alpha' (\tan \delta)'',$$

it was readily recognized that the precision of its numerical value depended exclusively on the accuracy of the accelerations and that the solution would become more or less indeterminate if the positions were not accurate to the nearest second of arc. The general equation resulted in the improbable geocentric distance of about five astronomical units at the middle date. As the geocentric motion indicated a large eccentricity of the orbit, and as the indeterminateness is reduced by a conditioned solution, a parabola was derived as a first approximation with the following residuals:

O-C	I	II
$\Delta\alpha$	+21".2	+25".0
$\Delta\delta$	+ 7.0	+ 7.4

Without computing the corresponding elements differential correction leading to a general orbit was undertaken to remove these residuals, with the unfavorable result:

O-C	I	II
$\Delta\alpha$	+7".8	+6".8
$\Delta\delta$	+0.6	+0.4

As the parabolic residuals should readily have disappeared by differential correction, an inconsistency in the observations was suspected and no further attempt was made to improve the orbit, particularly since in the meantime a longer arc had become available. The inconsistency of the observations is established by the orbit given below. The elements resemble those based on the longer arc and in case of need would have well served for ephemeris purposes. It is interesting to note that it was possible to derive an approximate orbit under the existing unfavorable circumstances.

BERKELEY ASTRONOMICAL DEPARTMENT,

April 16, 1927.

ELEMENTS AND EPHEMERIS OF OBJECT COMAS SOLÀ (1927 AA)

BY

P. A. McNALLY, S. J., and ELIZABETH E. STERNBERG

In view of the uncertainty of the foregoing preliminary orbit derived with Mr. R. S. Richardson from an arc of two days, no attempt was made to improve the same by differential correction on the basis of a longer arc, but an independent direct solution was based on the following photographic positions published by Comas Solà in the *Bulletin de l'Observatoire de Nice*, Nos. 24, 25, 26.

1927, U.T.	$\alpha(1927.0)$	$\delta(1927.0)$
(1) Jan. 12.0451	8 ^h 19 ^m 22 ^s .4	+2° 21' 24"
(2) Jan. 23.91528	7 58 34.9	-1 08 54
(3) Feb. 5.88750	7 41 50.4	-3 12 08

Owing to the unfavorable position of the body in its orbit, near opposition, the direct solution gave the following large residuals:

O-C	I	II
$\Delta\alpha$	+8' 34".6	-8' 46".5
$\Delta\delta$	+1 8.6	+0 25.0

These residuals indicate that a preliminary correction of the heliocentric velocities, without changing the geocentric distances, would lead to a representation of the observations by a single complete differential correction. At the suggestion of Dr. H. Thiele the constants for the equator were computed from the first and third geocentric distances of the direct solution and the corresponding observed positions. With these constants for the equator, improved heliocentric velocities were computed according to Leuschner's formula (*Lick Observatory Publications, Vol. VII, page 329*). For the correction of the time intervals for aberration the geocentric distance resulting from the representation of the middle place was used together with geocentric distances for the first and third places derived in the same ratios that resulted from the original direct solution. With these improved data the residuals of the first and third places became:

O-C	I	II
$\Delta\alpha$	-33".8	+20".5
$\Delta\delta$	+22.0	-35.3

A single differential correction produced the following results with residuals within the errors of observation:

ELEMENTS

T	1926, Sept. 20.1930 U.T.
ω	147° 12' 11".7
Ω	279 24 20.0
i	25 19 34.6
$\log q$	0.233242
a	2.375806
e	0.279838
P	3.66198 yrs.
μ	968".928

CONSTANTS FOR THE EQUATOR 1927.0

$$\begin{aligned} x &= r[9.957409] \sin (157^\circ 35' 17".6 + v) \\ y &= r[9.957007] \sin (55 00 26.2 + v) \\ z &= r[9.776774] \sin (106 10 44.6 + v) \end{aligned}$$

RESIDUALS

O-C	I	II
$\cos \delta \Delta\alpha$	+0".2	-0".5
$\Delta\delta$	-1.0	-0.7

EPHEMERIS

1927, U.T.	$\alpha(1927.0)$	$\delta(1927.0)$	$\log \Delta$	Mg.
Apr. 10.0	7 ^h 47 ^m 42 ^s	-4° 08' 0	0.2382	14.33
14.0	51 54	07.7		
18.0	7 56 21	08.5	0.2646	
22.0	8 01 03	10.4		
26.0	05 59	13.6	0.2897	
Apr. 30.0	11 07	18.2		
May 4.0	16 26	24.2	0.3136	
8.0	21 55	31.6		14.86
12.0	27 33	40.5	0.3360	
16.0	33 19	-4 50.9		
20.0	39 12	-5 02.8	0.3573	
24.0	45 11	16.1		
May 28.0	51 16	31.0	0.3772	
June 1.0	8 57 26	-5 47.2		
June 5.0	9 03 42	-6 04.9	0.3961	15.30

The magnitudes are based on that of January 23.

Representation of the observations used for the short arc preliminary orbit is as follows:

O-C	I	II	III
$\Delta\alpha$	+10".9	0".0	-1".4
$\Delta\delta$	-1.9	0.0	-8.2

These residuals clearly indicate that with the prevailing uniform motion reliable accelerations could not be derived for the preliminary orbit with reference to the equator.

BERKELEY ASTRONOMICAL DEPARTMENT,

April 16, 1927.

NOTE ON THE ORBIT OF THE SPECTROSCOPIC BINARY 112 HERCULIS

BY

W. F. MEYER

After publishing the results on the orbit of the spectroscopic binary 112 *Herculis* in *Lick Observatory Bulletin No. 381*, a letter was received from Dr. Frank Schlesinger, drawing my attention to the large probable errors of the elements. Upon his suggestion I recomputed the coefficients of the normal equations and in so doing discovered errors in three of the coefficients. The resulting least squares solution did not seriously affect the corrections to the elements nor to the residuals, but the probable errors were materially reduced.

The new least squares solution gave the following corrections to the preliminary elements:

$$\begin{aligned}\delta T &= +0.183 \text{ days} \\ \delta e &= +0.016 \\ \delta \omega &= +10^{\circ}53 \\ \delta K &= -0.315 \text{ km.} \\ \delta \gamma &= +0.03 \text{ km.}\end{aligned}$$

FINAL ELEMENTS

$$\begin{aligned}P &= 6.3624 \text{ days} \\ T &= \text{J.D. } 2424589.683 \pm 0.166 \\ e &= 0.116 \quad \pm 0.017 \\ \omega &= 195^{\circ}53 \quad \pm 5^{\circ}81 \\ K &= 17.68 \text{ km.} \quad \pm 0.28 \text{ km.} \\ \gamma &= -19.63 \text{ km.} \\ a \sin i &= 1,537,000 \text{ km.}\end{aligned}$$

The probable error of a Lick observation is 1.05 km. The probable error of a normal place of weight unity is 0.70 km. The sum of the squares of the weighted residuals was reduced from 14.6165 to 13.069. The maximum difference between the residuals of a normal place from the elements and the condition equations was 0.08 km.

BERKELEY ASTRONOMICAL DEPARTMENT,

April 21, 1927.

Issued April 26, 1927.

LICK OBSERVATORY BULLETIN

NUMBER 389

PHOTOGRAPHS OF MARS AND OF JUPITER TAKEN BY LIGHT OF
DIFFERENT COLORS DURING 1926

BY

WILLIAM H. WRIGHT

This paper is a preliminary report on photographs of *Mars* and of *Jupiter* taken by light of different colors during the opposition of 1926. The observations are in continuation of some made of *Mars* in 1924 which are described in two earlier papers.¹ A general account of the purpose of the work will be found in these, so that it will suffice here to describe only the modifications in apparatus and observing procedure that were made for the present observations.

The most important of the changes is the addition of a number of colors to those used in 1924. Four were then used: ultra-violet, violet, orange, and infra-red; and of these the first and third were employed only occasionally, which left but two in systematic use. During 1926 six colors were regularly employed, so that the present series is, in that respect, more complete. The approximate wave-lengths of the colors, and the plates and filters by which they were isolated, are given in Table 1.

ally in the comparative study of the diameters of images rendered in the several colors, to have the plates as uniform in quality as possible, and to this end Dr. Mees had a special emulsion prepared, parts of which were sensitized separately, in the process of manufacture, with erythrosine (for green and orange), pinacyanol (for red) and kryptocyanine (for infra-red). Two separate treatments with erythrosine were adopted, one of which gives the maximum in sensitivity in the orange which is usually provided by this dye. The other gives a very remarkable extension from the ordinary region of sensitivity in the blue, which falls gradually to zero in the red. The emulsion prepared by the second of these processes was used exclusively in taking the "green photographs," and very largely in the orange. Ordinary erythrosine plates are perhaps better for the latter color, but it was more convenient, in observing, to use the others. The wave-lengths of effective sensitivity given in

TABLE 1

Wave-length (approximate)	Plate	Filter
3700	Eastman Special Process No. 4407 A	Chance ultra-violet glass
4400	Eastman Special Process No. 4407 A	Chance clear glass
5300	Eastman Process Green No. 4407 D	Chance green glass
5700	Eastman Process Green No. 4407 D	Chance orange glass
6500	Eastman Process Pinacyanol No. 4407 B	Selenium glass
7600	Eastman Process Kryptocyanine No. 4407 C	Selenium glass

The plates used were especially manufactured by the Eastman Research Laboratory under the direction of Dr. C. E. K. Mees, whose interest and cooperation in the prosecution of this work is acknowledged with the greatest pleasure. It seemed desirable, especially

column 1 of Table 1 are only approximate. The range in wave-length is quite wide for each color, except in the infra-red, where kryptocyanine provides a narrow, isolated maximum.

The filters used in 1926 were of colored glass, surfaced to a degree of excellence which is specified by opticians as grade A. The glass was supplied by

¹ *Lick Obs. Bull.*, 12, 48, 1925; *Publ. Ast. Soc. Pac.*, 36, 239, 1924.

Messrs. Chance Bros., of Birmingham, England, and the surfaces were worked by Hilger. The filters employed in 1924 were of the gelatine-in-glass kind popularly used in general photography. There were two reasons for making this change. One is explained on page 49 of the *Lick Observatory Bulletin* already cited; the other was that better color stability was hoped for in glass than is obtainable in gelatine, a matter of much importance where observations taken years apart are to be compared. In 1924 the filters were placed between the telescopic focus and the enlarging lens. In the present observations they were moved forward to a position within the telescopic focus—in the converging pencil of rays coming from the mirror. Ultra-violet photographs were made in the same manner as the others. It is believed that the absorption of ultra-violet light (of the wave-length used) by the small glass enlarging lens is not a serious matter.

In essentially all of the present observations standard intensity areas were photographed on the plates by light of the color used in making the picture. There are six of these areas, graded in intensity steps of 1:2, the total range provided being thus 1:64. A positive check on the contrast factor of each negative is therefore supplied, and means are at hand for the measurement of relative intensities of different parts of the planet's surface.

Except with respect to the changes indicated above, the observing technique was the same as that employed in the observations of 1924.

PHOTOGRAPHS OF MARS

The general purpose of the work in 1926 may conveniently be indicated in categorical form as follows:

I. The general development of the methods used in 1924, and further exploration of the field opened through their application. Under this head may be considered the employment of additional colors, and other matters referred to in the foregoing paragraphs.

II. The systematic observation of special phenomena observed at the earlier opposition.

These phenomena include:

- a) The dependence of diameter on color. The measurements here contemplated were of two kinds:
 - (1) The direct measurement of diameter on photographs taken in different colors, on plates as nearly uniform in quality as possible; and
 - (2) The determination of diameter by timed measurements of markings, to be reduced in conjunction with the

known period of rotation of the planet. This method was proposed in the earlier papers as an independent means of determining the diameter of the solid surface of *Mars*.

- b) The further interpretation of the impermanent configurations or markings found on the violet and ultra-violet photographs of *Mars*. For a description of these, reference may be had to the *Lick Observatory Bulletin* already cited. The restricted interpretation adopted in that paper is that the several types of configuration there described are cloud formations which differ in certain essential respects from terrestrial clouds.
- c) The investigation of the changes in form of the above described objects by cinematography.

With respect to II (a), the dependence of diameter on color, it may be recalled that the observations of 1924 showed that *Mars* is photographed larger by violet and ultra-violet than by infra-red light. The explanation advanced was that the difference represented the thickness of the atmospheric shell, which was photographically recorded by short-waved light, but not by long-waved. The measurements necessary for the continuation of this branch of the investigation will take a great deal of time, and they have not yet been begun. The material available for the inquiry seems to be of reasonably satisfactory quality. The "seeing" for the opposition was, on the whole, very disappointing, especially near midnights, when the planet was in the best position for observation; and there was a great deal of cloudy weather. On the other hand the plates used were of uniform quality, as has been mentioned, and, profiting by the experience had in 1924, a "softer" developer was used, which very much reduces limb fading in the infra-red images, and makes them more suitable for measurement.

While there is nothing to report in relation to this important division of the work, it may not be out of place to refer here to two recently published observations of diameter. In a recent paper² Professor Frank E. Ross has given measurements of relative diameters of photographs of *Mars* secured in several colors, and has found inequalities similar to those referred to above. The other observation is by Professor R. J. Trumpler,³ who in his discussion of photographs of *Mars* taken in 1924, has determined the diameter of the planet's solid surface by the method mentioned above of timed measurements of the surface markings. He finds the diameter, as thus determined, to be appreciably smaller than that directly measured by the usual method, on photographs taken by yellow light.

² *Astrophys. Jour.*, **64**, 243, 1926.

³ *Lick Obs. Bull.*, **13**, 32, 1927.

The differences found by Ross and by Trumpler are of the same order of magnitude as those reported by me, and while the three observers will probably agree that it will be unprofitable to attempt a numerical comparison of the several results until the material for both oppositions shall have been fully reduced and discussed, there appears to be no reason to question that there is an excess in the diameter of *Mars* as photographed by short-waved light over the actual surface diameter. My own disposition is now, as formerly, to interpret this difference in terms of the atmospheric over-layer.

Item II (b) will be passed for the moment because it is treated in a later part of the paper.

Item II (c) relates to an attempt to secure "motion pictures" of the variable markings shown on the violet and ultra-violet photographs. These objects, severally referred to in the account of the 1924 observations as limb light, mottlings, blotches, and streaks, according to their appearance on the photographs, are subject to pronounced change during intervals of, let us say, an hour or more. It was believed that by taking photographs close together in point of time (three minutes was the interval adopted) the behavior of these peculiar things, as they formed, changed or dissolved in their daily journey across the illuminated disk of the planet, could be advantageously studied by the use of a cinema projector. A portrayal secured by this means of the brilliant clouds of mid October, especially on October 18 when they formed, or burst through the underlying envelope of haze shortly after noon, could not well have been other than fascinating and instructive. There are no technical difficulties in the taking of the photographs, and the period of approximately eight hours during which *Mars* was accessible for observation during this opposition lent some hope that such an undertaking might be successful. Three favorable factors are obviously required for the success of a night's run of these observations: the occurrence on *Mars* of fairly conspicuous objects to be studied, uniformly good seeing, and finally, uniformly transparent atmosphere during the night. As supplementary to the violet photographs, others (showing the surface markings) were taken in the infra-red, the purpose being to throw both sets on the screen, properly synchronized, so that the exact areographic position of every cloud or atmospheric peculiarity would be seen. Because of the approximate equality in length of the Martian and the terrestrial day the photography had to be repeated at intervals of about ten days in order to cover the planet. Furthermore the photographs had to be made as close to opposition as possible.

Many photographs were made with this end in view, I cannot say at present with what success. Suit-

able objects were, fortunately, common enough, but the seeing was seldom good, and clouds, which are to be expected in this locality in the fall, proved very annoying. Frequently, after two or three hours of painstaking work on the part of the observer, they covered the sky, perhaps to vanish in an hour, having spoiled the run of the evening's observations. Whatever the outcome of the cinema project, the photographs will retain their value as "stills." It is hoped that this method will prove useful in the study of some of the phenomena of *Jupiter* at the coming opposition.

While a complete report upon the observations can obviously not be made at the present time it will perhaps not be inappropriate to reproduce a few representative photographs. These will serve to show the general character of the material available for study and may, in addition, prove suggestive of some of the physical conditions existing on the surface and in the atmosphere of *Mars*.

Remarks on the illustrations relating to Mars (Plates VI, VII, and VIII). The methods used in making the prints which were provided the engraver for reproduction in half-tone are described under a separate head, since they relate to photographs of *Jupiter* as well as to those of *Mars*. It is quite essential to an understanding of the illustrations that these methods be understood.

The photographs in the upper two rows of Plate VI were taken during the evening of October 15-16, Mount Hamilton Civil reckoning. Those in the upper row are reproduced approximately in normal contrast, as photographed, except that the red image, No. 5, has, in the process of reproduction, been made a little strong. A fair idea of the natural gradation in contrast is had in the sequence, figures 4, 11, 3, 2, 6, 1. Due to the planet's rotation the photographs give somewhat different presentations (see column 7 of Table 2). In the second row, with the exception of figure 10, the same subjects are shown with some modification in the character of the reproduction. To be specific, figures 7, 8, and 9 are substantially repetitions in exaggerated contrast of the images directly above them, and are introduced for the purpose of bringing out more clearly the cloud on the left side of the planet. In 8 and 9 the picture has been completely sacrificed to this end. The cloud is not visible in red and infra-red pictures. No. 10 has been introduced principally to perfect symmetry in the arrangement of the cuts, and No. 11 as an illustration of the composition of several photographs, three in this instance (compare with No. 5, above). Figure 12, is a composite photograph in which three originals are combined in double contrast. This subject is, furthermore, diffusely printed in order to subdue photographic grain. As stated elsewhere a

number of different methods of printing are here being tried experimentally for the purpose of testing their respective merits for reproduction in half tone. The group of illustrations shows the progressive fading of the features which characterize the violet and ultra-violet images, on the one hand, and the development of the known features of the surface, on the other, as the wave-length of the light used in observation is increased. The phenomenon was fully described in the 1924 observations and need not be dwelt upon here. A curious feature of most of the violet and ultra-violet images, which was not then referred to but which persists in the photographs for both oppositions, is a wedge-like dark area extending inward from the terminator in the north temperate zone, which for these presentations is the winter hemisphere. The pairs: figures 13-14, 16-17, and the group of four at the top of Plate VII, provide records on other days of the cloud group shown in figures 1, 7, 6, and 12. These clouds were first photographed on the evening of October 13-14 (figures 13-14) and appear to have changed little in general character during the following two days. No photographs were taken on October 16-17, but a number were secured on the following night which are of exceptional interest because on that occasion the clouds formed shortly after noon. This was the fifth day after the clouds were first photographed, and in that time the lag in the daily rotation of *Mars* relative to that of the Earth brought the planet under observation several hours earlier by Martian time, and thus afforded the opportunity of observing, during the morning hours, the area in which these clouds occurred. The upper row of Plate VII consists of reproductions in exaggerated contrast of four photographs taken on October 17-18. In the second row the same photographs are diffusely printed. The first three do not record the cloud group in question. There are lighter cloud areas on the disk, but nothing in the position of the objects especially under consideration. The group is however very conspicuous on the fourth photograph. It is necessary to bear in mind, in comparing the pictures of this group, that the planet rotated approximately 60° between the first and last. The presentation of the planet in figure 4 is substantially the same as in figures 7, 13, and 16 of the preceding plate, as may be seen by consulting the central meridian longitudes in the last column of Table 2. In the paper already cited, Professor Ross has published some excellent photographs of this cloud group, taken on October 16. So far as one can judge from his illustrations, the cloud was much stronger after meridian passage than before, and it therefore seems not unlikely that it had just formed. Afternoon clouds are common enough on the Earth, especially in equatorial regions, and, while one hesitates to press the analogy too far, the

explanation in the more familiar case is so simple that it may stand in some relation to that in the other.

The cloudy region just described is without doubt related to the persistent equatorial cloudy area noted during the 1924 opposition as lying north of *Mare Sirenum* and *Mare Cimmerium*.⁴ A photograph of the region is given in Plate II, figure 6, of the earlier Bulletin. It is reproduced here in exaggerated contrast (Plate VI, figure 15) for comparison with the clouds of 1926. The areographic longitude of the center of the disk is 196° , as against approximately 130° for the photographs we have been discussing (or 160° for figures 6 and 12 of Plate VI), so that the clouds of 1926 lie farther to the west (in the direction of diminishing areographic longitude) though there is a considerable overlap. While the 1926 clouds were the stronger, the two cloud groups strikingly resembled one another in general appearance, and this taken in conjunction with the proximity of their locations can be taken as evidence that they are closely related in origin and development. However, the one of September 21-22, 1924, extended clear across the illuminated disk, and could therefore not have been an affair of the afternoons only.

The three pairs of images, Plate VII, figures 9 to 14, show cloud forms over various parts of the planet. Figures 5 and 9 present almost the same face, 5 showing it turned 8° farther toward the left. Figure 9 shows one vertical and two horizontal streaks on the left half of the image. The vertical one is perhaps that seen in figure 5, which can be traced, with some alterations in appearance due principally to perspective, in figures 6 and 7, as it is carried over by the planet's rotation. The crossed cloud streaks of figure 9 are of interest by reason of their complexity. Crossed cirrus clouds are familiar enough on the Earth where they are separate objects at different atmospheric levels. I am not sure that the relation between the Martian objects and terrestrial clouds is sufficiently close to make this comparison a significant one.

The foregoing comments refer to the objects, interpreted as clouds, which are conspicuous in the violet photographs and fade to invisibility in the red. Figure 15 shows a temporary object of a very different kind. The presentation shows the *Syrtis Major*. By turning the plate so that the rows are vertical (the ordinary reading position of the paper) it will be seen that this cut provides a fair representation of a deer's head. The purpose in suggesting the resemblance is not altogether frivolous, for the anatomy of the suggested animal provides a system of reference in the discussion

⁴ In one of the references to this cloud it was mistakenly referred to as being close to the equator and south of these maria (*Lick Obs. Bull.*, 12, 61, ¶5). The maria in question are in the southern hemisphere, so the error in direction is obvious, especially as a photograph of the object was given.

of special features of this part of the Martian surface which is at once unambiguous and free from the rather forbidding nomenclature of areography. The marking to which it is especially desired to call attention is the white spot directly in front of the deer's throat. This is a temporary feature observed only on the evening of November 1-2. My attention was drawn to it by Dr. Trumpler who was observing *Mars* with the 36-inch refractor, and whom I visited on the way to my instrument. In the telescope it was the brightest part of the planet, at least equaling the polar cap, but it was decidedly yellow—as nearly as one could tell of the same color as the brighter areas which give the characteristic color to *Mars*. It is unquestionably the same sort of object as has repeatedly been seen and described by observers with the telescope. The three photographs, 15, 16, and 17, were taken consecutively. No. 17 is by orange light. The quality of the picture is poor, but the spot is shown near the limb, and is thereby proved to have been on the visible disk when the ultra-violet photograph No. 16 was taken. That photograph shows not a trace of the spot. This observation duplicates in all essential respects one by E. C. Slipher of the great temporary marking of July 9, 1922. In that instance the marking photographed well in orange light, but was lost in the blue image. The mobility and change in appearance of Slipher's object leaves little doubt that his interpretation of it as a cloud of some kind is correct. After the color reaction of the object of last November had been detected, Trumpler, from a comparison of photographs taken by both of us in 1924, found a number of similar instances, and has reported them in his recent paper.⁵ *Mars* therefore seems to present to us clouds, or impermanent markings, of two classes:

- I. Those which are visible by light of short wave-length, and become gradually invisible as the wave-length of the observing light is increased, and
- II. Those which are visible by light of long wave-length and fade into invisibility when the wave-length of the viewing light is decreased.

The observational basis for Class II is somewhat fragmentary, it must be admitted, but it seems fairly secure. There are three cases supported by photographic evidence (the only cases in which such evidence appears to be available), and there is the corroborative fact that observers consistently report the temporary areas of great brightness as being yellowish in color. Objects of the kind designated above as Class I, when they have been bright enough to be seen in the telescope, are described as white, or greenish white.

Clouds of Class I, which will be referred to as "blue" clouds, are shown in substantially all of the violet and ultra-violet photographs accompanying this paper. The object at the "deer's throat" in figure 15 of Plate VII had the color reaction of a yellow cloud, and probably was one.

Returning to a consideration of the illustrations, Plate VII, figure 18 is a photograph, of unsatisfactory quality, taken on the night of November 2-3. It is reproduced merely for the purpose of showing that the bright spot of the day before had disappeared. Figure 19 is the violet counterpart of 18, while 20 is essentially a reproduction of 19 in heightened contrast. Attention is especially called to these two photographs because there is a certain similarity between 18 and 19, which when first noticed led to the belief that the latter recorded very dimly the image of the *Syrtis Major*. The exaggerated contrast provided in figure 20 makes it clear that the resemblance is due in part to a large cloud⁶ which outlines approximately the deer's forehead and forward antler, and there is little resemblance between figures 18 and 20 in the general region of the south pole. The dark area extending from the middle of 20, in a direction upward to the left, probably corresponds with the somewhat similar one in 18. In describing the 1924 observations the statement was made that surface detail had not been photographed by violet, that is to say, by unscreened light. Commenting on this, Professor Pickering recalls⁷ that the *Syrtis Major* and some of the other markings are recorded on a few of the photographs taken by him in 1888 and 1890. Professor Ross in his recent paper also reports that surface features are dimly seen on unscreened photographs. I believe that traces of these markings are to be found on unscreened pictures of the present series taken between October 23 and November 9, 1926, though interference by "blue" clouds makes the observation difficult in any one instance. This interval included the date of opposition, November 4, and the conditions for seeing the surface were then probably more favorable than at any other time. Judging from their descriptions of their plates the observers mentioned appear to have less difficulty than I in seeing surface detail on unscreened photographs, and this is probably to be attributed very largely to the fact that my photographs were made almost exclusively on plates quite insensitive to the blue-green region of the spectrum, namely: "Eastman Lantern Slide" and "Eastman Process" plates. Possibly cloud formations have, on occasion, provided the basis for resemblance to surface features, as in figure 20. However that may

⁶ Figures 11 and 19-20 are in longitude nearly 180° apart, so that the cloud on the left of figure 11 has approximately the same areographic position as the one appearing at the lower right side in 19-20.

⁷ *Pop. Astron.*, 33, 437, 1925.

⁵ *Lick Obs. Bull.*, 13, 43, 1927.

be, the question of visibility or invisibility of surface markings on ordinary "violet" or "blue-violet" photographs can hardly be one of great consequence, because they can be plainly photographed by green light (Plate VI, figure 2), and very probably have not been photographed by ultra-violet light. Blue-violet would appear to be the limiting color by which they can be observed.

Plate VIII is devoted to a number of miscellaneous illustrations. The photographs in the upper row are a few shown in Plate VI, further enlarged as an experiment in scale of reproduction, to determine whether the increase in size would be of advantage. Figures 1 of Plate VIII and 12 of Plate VI are identical, except that the latter is diffusely printed. Figure 2 of Plate VIII matches 11 of Plate VI, and 3 of Plate VIII, 16 of Plate VI, except that VIII 3 is diffusely printed. The large pictures do not appear to possess especial merit. Figures 4, 5, and 6 depict the *Syrtis Major* at three separate oppositions. No. 4 was taken by Barnard at the Yerkes Observatory with the use of a yellow, or orange, color screen,⁸ and is here reproduced through the courtesy of Director Frost of the Yerkes Observatory. While figure 4 was made by light of a different color than that used for the other two, the very great differences shown undoubtedly represent actual changes of a kind that have generally been recognized as seasonal. Referring to the stag-head configuration in figure 5, the back part of the neck is much fuller here than in figure 4, and the "beard" (if one may take so great a liberty with the facts of natural history) is better developed. In these respects figure 6 shows an intermediate condition. However, in 6 one antler, strong in the other two photographs, is extremely faint. The bright spot at the throat in figure 5 is a temporary feature referred to elsewhere. Except with respect to the latter feature these seem all to be recognized seasonal changes, and the photographs are given merely as immediate illustrations of the well-known fact, to which there will later be occasion to refer, that changes do take place on the planet's surface.

Interpretations. In the two earlier papers to which reference has been made there was proposed a general explanation of the outstanding phenomena exhibited by such photographs as these, based on the assumption that *Mars* possesses an atmosphere of considerable absorbing and light scattering power.⁹ There is no occasion to repeat the arguments there presented except in regard to an aspect of the interpretation on which light has been shed by the detection of the two classes of clouds, or impermanent markings, described above. The matter I have in mind has to do with the sugges-

tion that the non-appearance of the well known surface markings, in photographs taken by violet and ultra-violet light, is due to the chance matching in reflectivity in those colors, of the substances which compose the Martian surface. This hypothesis was considered in both of my early papers, and perhaps I may be permitted to write in a quotation from one of them:¹⁰

"The violet pictures from their very uniformity offer a startling contrast to the lively presentation made by those taken in the infra-red. In seeking for an explanation of this great difference, two possible hypotheses naturally occur to one. The first is that the several materials of which the surface of *Mars* is composed, while they have very different reflectivities for infra-red light, may have equal reflectivities for light of the violet end of the spectrum. If that were true the violet images would, of course, present no contrasts whatever. The second is that the atmosphere, the existence of which we have inferred from the edge effect of the infra-red photographs, may be obliterating the details. With respect to the first of the proposed explanations we recall that differences in relative reflectivity of substances, corresponding to the several parts of the spectrum, are common enough, and furnish, in fact, the basis of color in ordinary objects. It would, therefore, be physically possible to have a number of materials that would stand out in contrast when viewed by light of one wave-length and could not be distinguished by that of another. Arrangements of the sort can be made artificially, but they are not to be expected in nature. We may photograph a landscape by red and by violet light and the relative values will, in general, be different in the two pictures, but there will be contrast in both pictures though it may be differently distributed. The chance that for a single region of the spectrum the intensities of all the elements of the picture would match exactly seems remote.

"Nevertheless it has been urged in personal conversation that the possibility of the surface of *Mars* being composed of only two kinds of material should not be overlooked, and that if it were, the probability of a chance matching by violet light would not be negligible. * * * * There is nothing in the observations to exclude the possibility implied in the above suggestion. On the other hand I can think of no good reason for assuming that there are exposed on the surface of *Mars* only two materials and that they match in ultra-violet light. Photographs of *Jupiter*, *Saturn*, the Moon, and the Earth's surface show contrast and detail quite as well by violet as by infra-red light of the wave-length used in these observations. On the whole the first of the alternative explanations advanced to

⁸ *M. N. R. A. S.*, 71, 471, 1911.

⁹ See especially *Lick Obs. Bull.*, 12, pp. 57 to 60.

¹⁰ *Loc. cit.*, p. 57.

account for the quality of the violet images hardly seems a probable one."

With respect to the suggestion that there are only two different kinds of substance on the Martian surface, let us consider the bright spot of November 2 (Plate VII, figure 15) and assume that *Mars* has an atmosphere of high transparency to light of all colors, a premise that has been entertained by those who question the adequacy of the theory of atmospheric obliteration as an explanation of the flatness of the short-wave photographs. The spot may have been a surface marking or a cloud. It has been suggested that such objects are dust clouds, and that they derive their color from the surface material from which they originate. This does not appear to be a tenable assumption. Dust which is sufficiently fine to float in the atmosphere is visible by scattered light,¹¹ and that fact determines its color, which will have a considerable bluish content. The Martian object was yellow, and could not be photographed by light of short wavelengths. It could not, therefore, have been a cloud of fine dust. If the dust is coarse enough to show its color, and is kept afloat by whatever process one wills, the particles will be large enough to intercept light, since color is a residual phenomenon of absorption. A cloud of such small bodies constitutes a fairly effective light trap, and if sufficiently dense or extensive to cast a black shadow, may readily be shown to reflect only half of the light which would be sent back by a solid surface of the same material, unless the cloud be, in the strictest mathematical sense, at opposition,¹² when the reflectivity will approach that of the solid surface. In no case, however, could reflectivity of the kind we are now considering equal that of the surface. This object was, however, *brighter* than the brightest part of the surface. It could not therefore, have been a dust cloud composed of particles of the surface material sufficiently large to confer their natural color. We have considered two cases, the one of dust sufficiently fine to scatter light, the other of dust sufficiently coarse to show its natural color, and find that neither of these will fit the observed facts. There remains the possibility of dust of intermediate size. While this is a condition not so easy to discuss in a qualitative way, I think we may, without serious danger of error, consider the light returned by such material as made up of two parts: that which is reflected or diffused

as colored light, and that which is returned as scattered light. The same considerations refer to the diffused light as before; it cannot equal in strength the light from the surface. If the brightness of the cloud is greater than that of the surface, as it is observed to be, the extra light necessary to make it so must be the contribution of the returning beam which is due to scattering. Now there is an excellent reason for believing this not to have been the case with the object observed. The component of the returned light which has its origin in scattering would necessarily be relatively strong in the spectral regions of short wavelength, and could be regarded as negligible in the infra-red. If it were sufficiently strong to add materially to the visibility of the cloud by yellow light, and make the cloud appear brighter in that color than the surrounding surface, we should expect a considerable increase in the blue content of the cloud light, while in the infra-red, assuming negligible scattering, the cloud should appear darker than the surface. As a matter of fact the object could not be seen on ultra-violet photographs, and was bright and more conspicuous on the infra-red than on the yellow (or orange) photographs. If the above analysis is correct, the object could not under the assumption of a homogeneously transparent atmosphere, have been a dust cloud.

Let us now assume that the spot was actually on the surface. In that case it could hardly have been composed of the permanent surface material for two very obvious reasons: it did not stay there, and it was brighter than any other part of the surface. It seems difficult to escape the conclusion that in addition to the two hypothetical materials on *Mars*, the dark and the light, we had for a while at least a third, and this had the same peculiarity which characterizes the other two: it was easily photographed by long-waved light and faded out in light of short wave-lengths. But is there any valid ground for limiting our assumption to three substances on the Martian surface? Consideration of changes, probably seasonal, such as are reported by essentially all observers, and are illustrated in figures 4, 5, and 6 of Plate VIII, certainly suggests a wealth of variety in the material exposed to our view. With the spectroscope proclaiming the distribution of the chemical elements throughout the universe, the assumption that a surface, so subject to alteration as that of *Mars*, possesses a limited number of their combinations, and only those which have the same reflective action on violet and ultra-violet light does not seem especially attractive. One is therefore disposed to discard the explanation that the flattening out of detail in the violet and ultra-violet photographs of *Mars* is due to a chance matching in reflectivity at those colors of all the materials of which *Mars* is com-

¹¹ This should be especially true with respect to *Mars* because of the low surface gravity of the planet. The density of the supporting atmosphere varies with gravity, but the density of a solid particle remains constant. Dust of a size that floats in the Earth's atmosphere might therefore be expected to sink if the force of gravity were reduced.

¹² This happens to have been pretty nearly the case when the spot referred to was seen, the planetocentric angle between the Earth and Sun being, at the time, about 2°. But the argument relates equally well to the cloud of July 9, 1922, when this condition did not obtain.

posed, and to accept the obvious alternative that it is the effect of an overlying atmosphere.

Let us now revert to the temporary yellow markings, and consider specifically the one of July, 1922. The color of this object is variously described as "creamy white" and "yellow." Because of its changes in form and its mobility, the object in question was interpreted by W. H. Pickering, E. C. Slipher, and no doubt others as a cloud. Slipher photographed it by yellow-light, but it was invisible in his photographs taken by blue. In this respect it paralleled the behavior of the temporary yellow spot of November 2, 1926, referred to in preceding paragraphs, and we shall assume that the color reactions of the two objects were similar throughout the spectrum. There appears to be little reason to doubt that both of these objects are examples of the "yellowish clouds" which, though they are not usual features of the telescopic view, have in the course of years frequently been seen by all systematic observers of *Mars*. Therefore, while the following comments are made with the cloud of July, 1922, especially in mind, and under the assumption with respect to its color just stated, it is believed that they refer equally to all of that class of temporary yellow markings whose general behavior permits of their interpretation as clouds.

For reasons which have been sketched in previous paragraphs we shall assume that this object was not a dust cloud. If it was not that, what was it? So far as we can estimate probabilities in such a matter on the basis of terrestrial experience, I think we must regard as reasonable the assumption which observers of *Mars* seem usually to have made with respect to similar objects, that it was an aqueous cloud. The temperature of *Mars* appears to be favorable to this assumption. We shall therefore, for the purpose of getting on with our interpretation, take the ground that this was an aqueous cloud. Now aqueous clouds as we know them, are white, and it is hard to understand how one could appear yellow, unless it were viewed through the equivalent of a yellow screen. The implication of this comment is that the atmosphere of *Mars* is not homogeneously transparent to light of all colors, but that it has optical properties which make it the equivalent of such a screen.¹³ The yellowness cannot be due to selective absorption in the upper (shorter wave-length) part of the visible spectrum, because the

line absorption of *Mars* is, so far as has been found, a replica of that of the Sun. The only available explanation is in terms of a progressively increasing general absorption in the direction of shorter wave-lengths. Since several of the phenomena presented by color photographs of *Mars* have suggested a similar property on the part of its atmosphere, it seems worth while to entertain the suggestion, tentatively at least, as an hypothesis, for the purpose of inquiring whether it is capable of explaining any of the other curious things which the planet presents for our interpretation.

It is well at the outset to emphasize the seriousness of the assumption of this property on the part of an atmosphere. Such an hypothesis, as suggested in an earlier paper, has been criticized by a number of astronomers, notably Menzel,¹⁴ as not being a consequence of the phenomenon of light scattering which is usually taken to explain the yellow screening effect of our own atmosphere when, for example, we observe the Sun through it. It was pointed out by Langley that the Earth's atmosphere causes the Sun to appear to us yellower than it is, and a yellow tinge is consequently imparted to the landscape. This is relieved to a certain degree by blue illumination from the sky, though, since the latter undoubtedly returns blue light to space, the net effect on the color of the landscape, as seen by the terrestrial observer, is still yellow. An observer outside the atmosphere would see the blue sky projected against the landscape, and Menzel, by assuming that there is no true absorption in the atmosphere (as distinguished from depletion by scattering), shows that the net result should be to make the landscape appear, to such an observer, bluer than it actually is, not yellower. In order for the atmosphere to impart a yellow tinge it would seem necessary that it possess absorptive properties for which he is unable to account. Dr. Menzel's paper and others on the subject¹⁵ are undoubtedly valuable contributions to the theory of planetary atmospheres, and help in putting to rights certain matters of cause and effect, but they deal with the subject in an academic manner, and make use of assumptions which one does not feel sure are justified either by the state of our understanding of light scattering, or that of our knowledge of conditions to be expected in the atmosphere of a planet. In fact it seems questionable whether our present knowledge of the optical properties of atmospheres is sufficiently well grounded to serve as the basis of exact quantitative predictions of what will or will not occur under conditions which have not been realized, approximately at least, in observation. In taking this view I am perhaps not uninfluenced by the historical fact that the principal optical properties of the Earth's atmosphere,

¹³ Until this paper was practically ready for the press I did not know that Professor W. H. Pickering had offered such an explanation of the yellowness of Martian clouds. This was due entirely to my own negligence. The reference was caught in a search for a description of the color of the polar caps, undertaken in relation to a later part of this paper (see footnote 18). The explanation is such an obvious one that it must have occurred to others besides Professor Pickering. The possibility of a "yellow" atmosphere was also suggested in my paper of 1924. It has seemed best to let the present phrasing of the text stand, and make the acknowledgment to Professor Pickering in this way.

¹⁴ *Astrophys. Jour.*, **63**, pp. 48 and 58, 1926.

¹⁵ See also a paper by Fessenkoff, *A. N.*, **228**, 25, 1926.

excepting refraction, have been discovered, not through deduction and prediction from well established physical laws, but in spite of such ratiocinative processes. Knowledge of these properties has on each occasion come as a surprise, and in order to account for them, theory has had to be improved, or special assumptions made with respect to the constitution of the atmosphere. To illustrate, let us recall that the peculiar polarization of skylight observed by Tyndall was in his opinion out of harmony with the theory of polarization by reflection. After its detection the phenomenon was explained by Rayleigh. Another case is the opacity of the upper air to light of wave-length less than λ 2900. This would never have been suspected from known phenomena of the atmospheric region to which we have access. We explain it now by a layer of ozone at unexplored altitudes. Consider finally that remarkable property of the atmosphere which permits the transmission of signals, and of speech, over large areas of the curved surface of the Earth, through the agency of electro-magnetic radiation. Those who derive pleasure or profit from the art based on this property do so, it is said, by virtue of a layer of ionized gas inconveniently (or perhaps that is not the correct word) located in a region which forbids its being made the subject of direct study. Perhaps when we have more light on such matters as these, and can explain, among other things, the strong absorption phenomena in the spectra of the major planets, we shall know enough about atmospheres to support fairly secure inferences as to what a planet enclosed by one ought to look like from the outside. These comments are regarded as applying to all theoretical discussions of atmospheres, including the present one. Until such a time as we have a better understanding of such matters, conjecture, or plausible explanations of what we see, would seem to be permissible even though they may appear to run counter in some respects to the apparent implications of incompletely understood laws acting in assumed environments.

We shall admit that the hypothesis of an atmosphere with relatively great absorbing power for light of shorter wave-lengths requires further study in relation to its physical probability, but there can be no harm done if, in the emergency, we ticket it for consideration at some future time, and proceed to follow up some of its consequences. If these prove to be sufficiently consistent with phenomena observed on *Mars*, zest will be lent to an investigation of a possible property of its atmosphere which might otherwise seem of no especial consequence. Suggestion respecting a possible constituent of the atmosphere which might confer such a property is purposely avoided.

To sum up, the suggested hypothesis permits a qualitative explanation of:

- 1) Why the albedo of *Mars* in blue and violet light is low for a planet having so extensive an atmosphere as that indicated by the inequality in size of the violet and infra-red images. We get the light principally from the upper atmosphere. Incident light is reduced in strength on its way in, and being scattered at depth is again subject to absorption on its way out.

- 2) Why surface details cannot be photographed by light of the shorter wave-lengths.—That light is heavily absorbed and the little which is returned from the surface is masked by light scattered at high levels of the atmosphere.

- 3) Why the brighter parts of the surface (which give the planet its characteristic color) appear yellow.

- 4) Why the dark areas appear bluish green.—The areas are dark because they reflect little light of any kind. We therefore see the Martian sky projected upon them. Our own sky is blue, and would, presumably, appear that color from without if projected against an absolutely black area. But with such an atmosphere as has been assumed for *Mars* rays of somewhat longer wave-length would penetrate farther and therefore provide an effectively greater "thickness" of sky. This would displace the effective color toward the green. Furthermore it is not likely that the dark areas of *Mars* are absolutely black. By our assumption they probably return a small amount of light, which, by the time it reaches the outer atmosphere of the planet, may be assumed to be yellowish. This should effect an additional alteration of the sky color in the direction of longer wave-lengths, and emphasize the green tinge. The dark markings are subject to alteration in strength (a number of instances have already been referred to and are illustrated in Plate VIII). By such changes the content of yellow light from the planet would be altered, and the apparent color of these markings changed. Variation in the color of these markings has been reported, and is conceivably due to this cause; in fact a test of this explanation might be made by comparing observed color changes of a marking with changes in its strength. The darker the marking, the bluer one would expect to find it. Clouds, as a possible source of interference, would of course have to be considered in such a test, as every observer knows. It seems not unlikely that there are records of observations which might bear on this question. It is, of course, impossible to say whether the cause here suggested of the color is complete or contributory. That the effect of the Martian skylight on the color of the dark areas is far from negligible follows from the assumption made in this paper that the blue-violet photographs are due in greater part to the atmosphere. From this point of view most of the blue content of their light, which must unquestionably affect their apparent hue, is

atmospheric. The difficulty is not in accounting for as much blue light as is seen, but in explaining why there is not more of it if the atmosphere is so extensive as we are assuming it to be. To what extent the undoubted greenish cast of the markings may have its origin in the considerations sketched above it is impossible to estimate, especially in the absence of numerical photometric data. Their color is of especial interest in view of the fact that they are very commonly interpreted as areas covered with vegetation. The more critical observers of *Mars* appear to have recognized the influence of Martian skylight on the color of the markings, but the magnitude of the effect has, perhaps, not generally been appreciated. While there is, of course, nothing here to suggest that the markings are not actually green, it seems likely that their appearance to us is determined, very largely at least, by the atmospheric veil.

5) Polar caps. It may well be asked why the polar caps do not look yellow, since they are on the surface and are, as well, observed through a great thickness of this hypothetical yellow atmosphere. The explanation is that the caps are shown by photographs taken by violet and ultra-violet light to be surrounded by a halo, which is visible also by light of a somewhat less refrangible part of the spectrum. When the surface cap, as photographed by infra-red light, is absent, or extremely small, this halo is faintly visible in the orange photographs, and becomes progressively stronger in photographs of shorter wave-length. In partial verification of this see Plate VI, figures 3, 2, 6, 1 in order. There was at the time these photographs were taken an extremely small cap on infra-red images, but because of its eccentricity (with respect to the planet's pole) the presentation on figure 4 is unfavorable for showing it. However on 6 the presentation is still more unfavorable, but the halo shows plainly. What is here called the halo was referred to in the earlier papers as the "atmospheric cap," but the term halo has been adopted as both less awkward and more appropriate. The yellow of the surface cap is corrected by the bluish light of the halo, and produces the impression of white. In my own limited experience in the photography of *Mars* the south pole only has been in position for observation and the halo has persisted even when no record of a cap could be found on the infra-red images, that is to say, when the surface-cap was presumably not present, or was at least extremely small. Under such circumstances one would expect the color of the object seen in the telescope to tend toward blue. That a halo is not always present is shown by some photographs obtained in violet light by W. H. Pickering during the oppositions of 1888 and 1890.¹⁶ Some of these show no polar whitening,

that is to say, under the interpretation here adopted there was, at times, no halo. Had a surface cap existed under such circumstances it might have been expected to appear yellow, and generally speaking the apparent color of the cap would depend on the relative brightness of cap and halo. In this connection we are able to quote a terse description of the appearance of both caps and clouds by Professor Pickering:

"For some unknown reason the clouds of *Mars* are always of a light yellow color, while the planet's poles are sometimes yellow, sometimes pure white, and sometimes a light but vivid green."¹⁷

And later:

"On *Mars* the clouds are distinctly yellow, as has been noted by many observers. Dr. Lowell even went so far as to describe them as dust clouds. Nevertheless they are of all grades of whitish yellow and yellowish white. On rather rare occasions they appear to be as white as our own. The polar caps never appear as white as snow near at hand, although whiter than the snow upon distant mountains."¹⁸

There seems to be enough variation in the color of the cap as observed by this indefatigable student of *Mars* to allow for a great range in relative intensity of cap and halo. The green color mentioned by Pickering would be explained by the dominance of the halo, the yellow by that of the surface cap. Should a yellow cap again be observed it would be well worth while to take photographs of it by violet, and if possible by red or infra-red light. The explanation of color here given would be substantiated or disproved according as the halo were weak or strong.

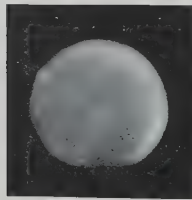
The presence of the polar halo seems to be quite positively established by the color photographs, but it is of interest to note that it affords an explanation of, and is therefore corroborated by, some observations relating to the color of the cap. As is generally known, the south polar cap is eccentrically situated, and is consequently carried to and from the limb by the rotation of the planet. The travel, at the two past oppositions, has been between limits of approximately 10° and 23° from the limb. When near the limb the cap, as photographed by infra-red light, becomes relatively faint; by violet light it suffers slight diminution, but not nearly so much as by infra-red. Trumpler has recently remarked upon the same effect in respect to photographs made by red and orange light.¹⁹ On approach to the limb the decrease in visibility is greater for the red than for the orange cap. In other words, as the cap approaches the limb the "effective wave-length" of its light becomes smaller; as it sets it gets "bluer." This is to be explained by the fact that the halo, which contributes the light of shorter wave-lengths, is high in the atmosphere, and its visibility does not suffer commensurably with that of the

¹⁷ *Ibid.*, p. 155.

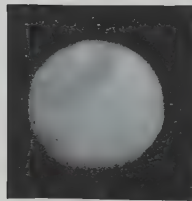
¹⁸ Report on *Mars* No. 30. *Pop. Astron.*, 33, 439, 1925.

¹⁹ *Lick Obs. Bull.*, 13, 43, 1927.

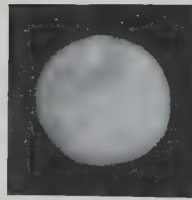
¹⁶ *Ann. Harv. Obs.*, 53, 170, 1905.



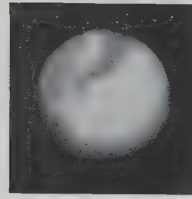
1 Ultra-violet



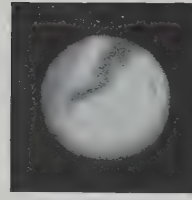
2 Green



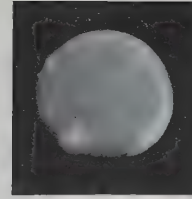
3 Orange



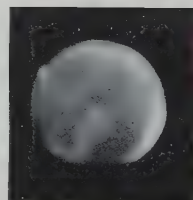
4 Red



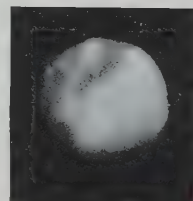
5 Infra-red



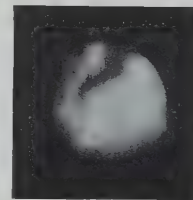
6 Violet



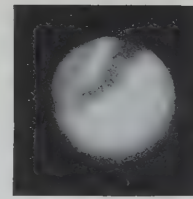
7 (High contrast)



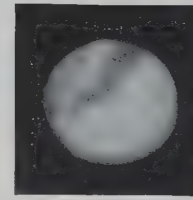
8 (High contrast)



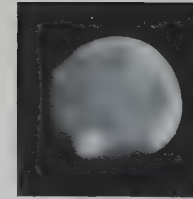
9 (High contrast)



10 Infra-red

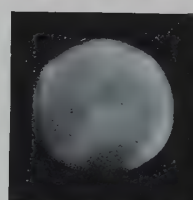


11 Red

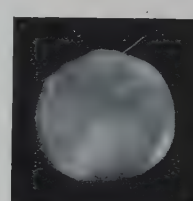


12 (High contrast)

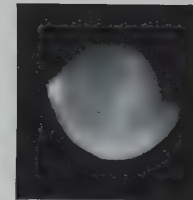
1926 October 16



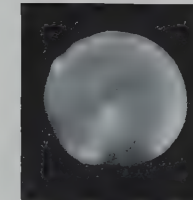
13 Ultra-violet



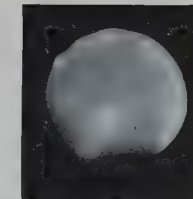
14 Violet



15 Violet



16 Ultra-violet



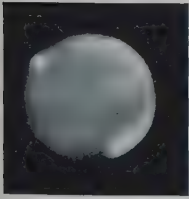
17 Violet

1926 October 14

1924 September 22

1926 October 15

Photographs of *Mars*



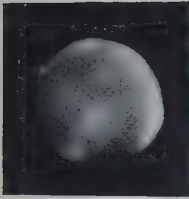
1 5^h 57^m



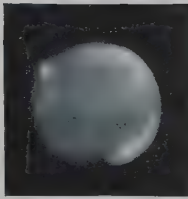
2 7^h 40^m



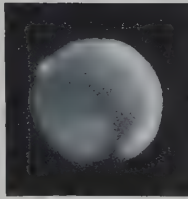
3 8^h 3^m



4 9^h 51^m



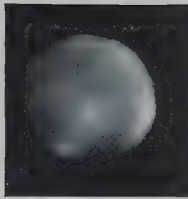
5 Violet



6 Ultra-violet



7 Violet

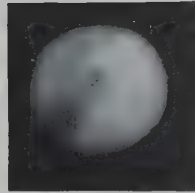


8 Violet

October 18

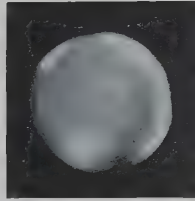


9 Violet

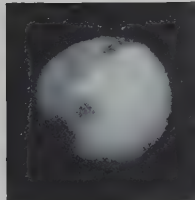


10 Infra-red

October 20

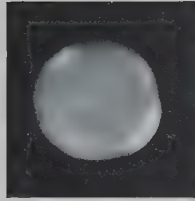


11 Violet

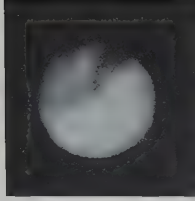


12 Red

October 23



13 Ultra-violet

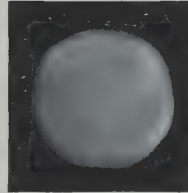


14 Infra-red

November 17

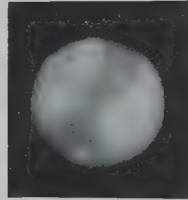


15 Infra-red



16 Ultra-violet

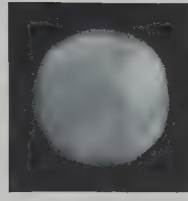
November 2



17 Orange

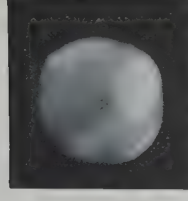


18 Infra-red



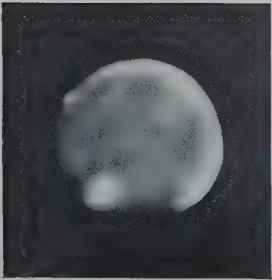
19 Violet

November 3

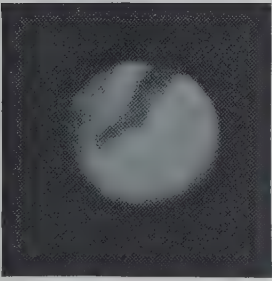


20 Violet (High Contrast)

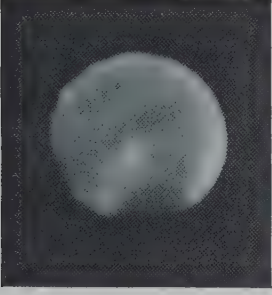
Photographs of *Mars*, 1926



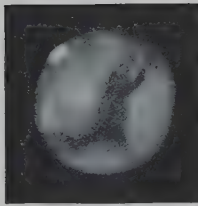
1 Violet
1926, October 16



2 Red
1926, October 16



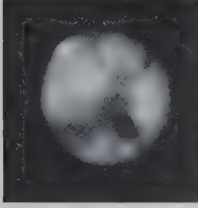
3 Ultra-violet
1926, October 15



4 Orange
1909, September 29

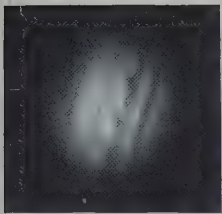


5 Infra-red
1926, November 2

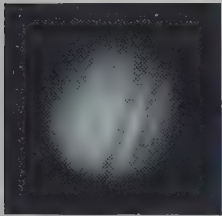


6 Infra-red
1924, September 12

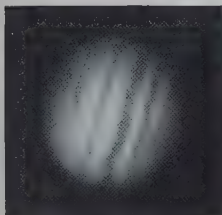
Photographs of *Mars*



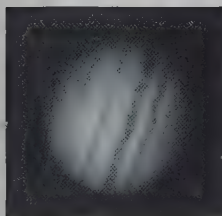
6 Infra-red



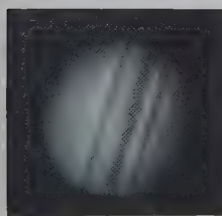
5 Red



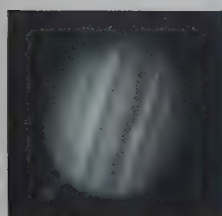
4 Orange



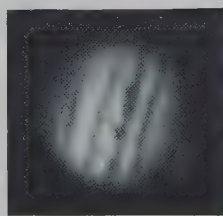
3 Green



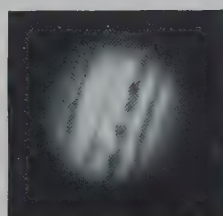
2 Violet



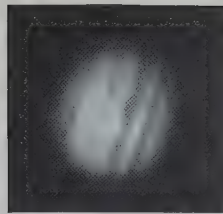
1 Ultra-violet



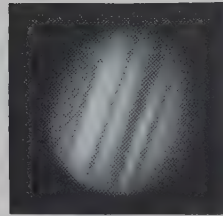
7 Infra-red
(Exaggerated contrast)



8 Infra-red

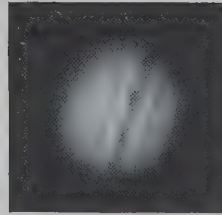


10 Infra-red

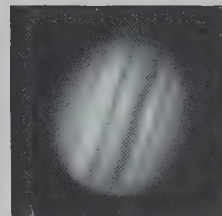


9 Ultra-violet

October 14



14 Infra-red



13 Ultra-violet

October 22



12 Infra-red



11 Violet

October 20

Photographs of *Jupiter*, 1926

surface cap, which must be seen through a greatly increased atmospheric thickness. It would be rather difficult to explain these observations on the assumption that the polar cap of *Mars* consists of a surface layer only. In order to do so it would seem necessary to invert our hypothesis regarding the absorption characteristics of the atmosphere and assume its transparency to be greater for violet than for red light. This would lead us to insuperable difficulties elsewhere. The change in relative strength of red and blue photographs is so marked that it seems likely the alteration in color would be perceptible to the eye. It would probably best be seen with a cap sufficiently small to clear the pole (12° in diameter) and yet large enough to insure that the surface cap (which appears to be smaller than the halo) would contribute a substantial share of the total light. The observation should be made when there is no interference from limb light (as in figure 14 of Plate VI). Possibly observations of this kind are already available. At the next opposition (1928-29), the south pole of *Mars* will be close to the limb, and the polar cap, if present at that season, will be carried over the edge: it will actually rise and set. It will be important to observe its color changes and record its visibility under these conditions.

6) Clouds. The clouds belonging to Class I described above (blue clouds) are interpreted as being due to finely divided material condensed out of the upper atmosphere. The particles of which they are composed must be smaller than the water droplets, or ice crystals, as the case may be, of terrestrial clouds because they appear to be visible by scattered light. The drops or crystals in terrestrial clouds refract and reflect light, and are altogether too large to serve as scatterers; consequently terrestrial clouds have a high reflectivity for light of all colors, including red light. The reason for believing the light of Martian clouds of Class I to be due to scattering lies in their high and progressively increasing visibility by light of short wave-lengths, and their seeming transparency to infra-red light. A position near the surface of the atmosphere is assigned to them because they are photographed by light to which the atmosphere is regarded as relatively opaque. When they are exceptionally bright, as they were, for example, during October, 1926, they are visible in the telescope, and are undoubtedly the rare "white" clouds mentioned by Pickering. As a matter of fact these clouds are not rare in their occurrence, they are frequently photographed, but, on account of their color, they are not easily seen. They are rarely bright enough to be seen in the telescope. The clouds observed by E. C. Slipher in the spring of 1920²⁰ appear from his general description of them to belong to this class.

Clouds of Class II. Yellow clouds are, according to this system of explanation, comparatively low lying affairs. They are regarded as aqueous clouds, like those of the Earth. It ought not, however, to be assumed that they occur invariably at the same level. The higher of them would be seen through less atmosphere, and might therefore be expected to appear lighter in color. This would account for the observed variations in shade reported by all observers. The difficulty in photographing them by violet light is occasioned by the assumed absorptive properties of the atmosphere.

In the foregoing paragraphs I have attempted to formulate a hypothesis suggested by the observed color of Martian clouds, and to sketch some of its consequences. In discussions of this kind one is frequently forced, in order to avoid the embarrassment of continual qualification, to treat assumptions as though they were facts, and an impression of assurance is sometimes conveyed which does not truly reflect the outlook of the author. It is a source of great regret to me that it is not possible at this time to apply numerical tests to some of the explanations offered in this paper. Qualitative reasoning, uncontrolled by quantitative check, is notoriously treacherous. It is hoped that photometric measurements of the photographs will eventually provide data of the kind desired. By measuring the visibility of a marking in its travel across the disk, information bearing on the amount of diffusion and absorption by the planet's atmosphere should be yielded. For this purpose the cinema pictures, taken at three-minute intervals throughout the night, may be of value. It would have been my preference to reserve comment on this whole subject until a digest of all of the available material had ripened it for discussion; but that would have occasioned a very great delay, and it is believed that the photographs possess a general interest, which warrants their prompt publication. Having decided to present them, it is difficult to avoid mentioning some of the obvious, and more or less plausible interpretations which differences in registry by light of different colors suggest.

METHOD OF MAKING THE PHOTOGRAPHS OF MARS AND JUPITER FOR ILLUSTRATION

The photographs were taken with the Crossley reflecting telescope, to which was attached an enlarging camera having an amplifying factor of approximately 5.4. The arrangement is the same as that previously described.²¹ The size of the image of *Mars*, at closest

²⁰ *Pop. Astron.*, 29, 69, 1921.

²¹ *Pub. Ast. Soc. Pac.*, 36, 239, 1924; *Lick Obs. Bull.*, 12, 43, 1925.

approach for the opposition, is 2.7 mm. and of *Jupiter*, at the time of taking most of the photographs, 5.7 mm. The further enlargement to the scale of the illustrations was secured in copying. Positive copies were made from the original negatives and secondary negatives were taken from these, in some instances by special methods presently to be described. The secondary negatives were used in making the prints supplied the engraver.

The special methods referred to are the outcome of reflections indulged in from time to time on the possible advantages of combining separate photographs of an astronomical subject for the purpose of suppressing or mitigating the effect of peculiarities of single images. Such peculiarities may be due to almost innumerable causes, but result principally from poor seeing, faulty following of the telescope, and the chance distribution of silver grains on the photographic plate. The idea of compositing photographs is a sufficiently familiar one, but I am not aware of its having been used in scientific observation, at least for the purpose here described.²² In the system here employed the positive enlargements of the pictures it is desired to combine are mounted in a manner which ensures accurate registry, and are successively photographed on a single plate, or film. A variation of this procedure, which augments contrast, is to superpose one positive on another and photograph through the combination thus secured. If the positives are on glass the number which can be so utilized is limited to two, and one of these must of course be "reversed," but if they are on thin films reversal is not necessary, and as many as desired may be placed in series. In some instances the above described methods have been combined, that is to say, several original photographs have been brought into one, of double, or of triple contrast as the case may be. If double contrast is desired and three pictures are to be used, they are combined in pairs in three different ways, and each pair is photographed, for a third of the total exposure. The technique of the process of registry and superposition is extremely simple, and need hardly be described here.

Experience with this method of combining photographs has, on the whole, been satisfactory. When three or four photographs of an equal degree of excellence are combined in normal contrast some advantage in detail may be expected to result, while photographic grain, which in an enlargement is displeasing to the eye, is appreciably reduced. If, however, one of the photographs is noticeably better than the others it is usually found that the advantages of combination are insufficient to compensate for the poorer quality of the companion pictures, although because of the modified

grain, the composite may be more sightly. Where increased contrast is sought there is a positive advantage in combination: faint shadings which might easily escape the eye can be built up until they dominate the picture. A check on the reality of such features can be provided, where it seems necessary, by building up two independent sets of photographs, and whenever this has been done the features in question have been verified.

It is a matter of common knowledge that contrast in a subject can be enhanced to a very great degree by purely photographic methods; for example, by copying on plates and papers of suitable quality, by control of exposure and development, and by repeated copying; and it may well be asked what advantage the procedure here described has over the more familiar methods. One advantage lies in the freedom from exaggerated grain, and from magnified imperfections of the single original plate; another in the fact that any desired contrast factor can be had in one simple operation. If the necessary number of original photographs is available the method is to be recommended in preference to the older one.

In order still further to eliminate the appearance of grain, experiments were tried with diffuse printing. This was done by placing a piece of glass between the negative and the paper. A more pleasing picture is sometimes secured in this way, and one which provides a better half-tone. (Compare figures 1 and 3 of Plate VIII; figure 3 is diffusely printed.)

Shading. A familiar difficulty which attends the enhancement of contrast in any pictorial subject is that detail in the fainter or stronger parts may be lost through insufficient latitude in the recording agency. In order to mitigate this as much as possible, artificial shading of a part of the picture has been resorted to in instances where it was believed it would not be deceptive. Where any of the above described devices, namely composition, reinforcement of contrast, diffuse printing, or shading, have been employed the fact is noted in Table 2.

Description of Table 2: This is a tabulation of data relating to illustrations. The first two columns require no explanation. Column 3 gives the number of the negative from which the illustration is taken. There are several images on each negative, and the numbers in column 4 are the distinguishing marks by which the ones used may be identified. Where more than one figure is recorded the illustration is a composite photograph. Column 5 gives the contrast factor of the negative used in printing. Where the printing negative is made from positives in single thickness the contrast is termed 1, where several positives are placed one over the other, the number used is taken to indicate the degree of contrast. For example, in Plate VI,

²² This comment refers to its general use. I have employed the method on several occasions, for example, in *Lick Obs. Bull.*, 7, Plate I, figure 2, 1912.

TABLE 2. DATA RELATING TO ILLUSTRATIONS

Plate	Fig.	Negative No.	Image No.	Contrast	Gr. Univ. Time 1926	Long. of* Cent. Merid.	Remarks
VI (Mars)	1	4679	1	1	Oct. 16, 8 ^h 35 ^m	127°	
	2	4687	4	1	9 8	134	
	3	4688	1	1	9 16	137	
	4	4689	1	1—	9 26	138	
	5	4691	4	1	10 46	159	
	6	4692	2	1	10 52	160	
	7	4679	1, 2, 3	2	8 41	128	a
	8	4687	2, 3, 4	3	9 7	134	
	9	4688	1, 1, 2	3	9 16	137	
	10	4690	5	1	10 33	155	
	11	4691	2, 3, 4	2—	10 46	159	
	12	4692	1, 2, 3	2—	10 52	160	c
	13	4665	1, 2	2	Oct. 14, 7 32	129	a
	14	4666	1, 2	2	7 49	133	a
	15	4409-10	3, 4, 7, 8	4		196	a, b
	16	4672	1, 2	2	Oct. 15, 8 21	132	a
	17	4673	3, 5	2	8 33	135	a
VII (Mars)	1	4699	1, 2, 3	3	Oct. 18, 5 57	70	a
	2	4700	1, 2	2	7 40	96	a
	3	4701	2, 3, 4, 5	2	8 3	101	a
	4	4706	1	2	9 51	127	a
	5 to 8						d
	9	4715	1, 2, 3, 4	4	Oct. 20, 7 43	78	a, c
	10	4716	2, 3, 4	2	7 49	79	e
	11	4731	1, 2, 4	3	11 22	106	a
	12	4730	1, 2	2	11 9	102	e
	13	4828	1, 2, 3	2	6 4	168	
	14	4829	1	1	6 8	168	
	15	4763	1	1	Nov. 2, 6 12	302	
	16	4764	1	2	6 57	313	
	17	4765	1, 2	2	7 31	321	
	18	4770	0, 1	2	Nov. 3, 5 2	276	
	19	4771	1	1	5 28	282	
	20	4771	2, 3, 4	3	5 32	283	a, c
VIII (Mars)	1	4692	1, 2, 3	3	Oct. 16, 10 53	160	a
	2	4691	2, 3, 4	2—	10 46	159	
	3	4672	1, 2	2	Oct. 15, 8 21	132	c
	4					301	f
	5	4763	1	1	Nov. 2, 6 12	302	
	6	4397	3	1		299	h
IX (Jupiter)	1	4693	1	1	Oct. 18, 3 9	339	
	2	4694	5	1	3 23	348	
	3	4695	3	1	3 31	352	
	4	4696	5	1	3 47	3	
	5	4697	3	1	3 57	8	
	6	4698	4	1	4 26	24	
	7	4698	3, 4	2	4 25	24	g
	8	4698	5, 6	2	4 30	28	g
	9	4660	4	1	Oct. 14, 4 56	159	
	10	4664	4	1	5 51	192	
	11	4709	3	1	Oct. 20, 4 37	331	
	12	4708	4	1	4 19	320	
	13	4717	3	1	Oct. 22, 3 45	241	
	14	4718	2	1	4 0	248	

* Not corrected for phase. Longitudes for *Jupiter* are based on System II, as tabulated in the *American Ephemeris*.

a Terminator shaded in printing.

b This is the photograph, rendered in higher contrast, which is shown in *Lick Obs. Bull.*, 12, Plate II, fig. 6. The date as given there, 1924, Sept. 21, 17^h 37^m (G.M.T.), corresponds to 1924, Sept. 22, 8^h 37^m (U.T.).

c Diffusely printed.

d Figures 5, 6, 7, and 8 are repetitions of those immediately above except that they are diffusely printed.

e Shaded on southern limb in printing.

f A photograph by Barnard, taken with the Yerkes 40-inch refractor; G.M.T. 1909, Sept. 28, 16^h 24^m=U.T. 1909, Sept. 29, 4^h 24^m.

g Shaded around edge to reduce fading.

h Photograph made during preceding opposition; 1924, Sept. 11, 18^h 30^m=U.T. Sept. 12, 6^h 30^m.

figure 7, three separate images, designated on the original plate as numbers 1, 2, and 3, were combined three ways in pairs, and a partial exposure made to each pair. In this way three images were brought into the illustration with contrast 2. In Plate VII, figure 16, double contrast was desired, but only one image of passable quality was available. Two positives were made from this image and placed one over the other. In a few instances prints were made on a soft grade of paper, and that fact is indicated in each case by the sign (-). The remainder of the table does not require explanation.

PHOTOGRAPHS OF JUPITER

The photographs of *Jupiter* were made rather casually, as time could be spared from the work on *Mars*. During the summer, while *Mars* was not within reach in the early evening, there was little interference of the one object with the other, but as *Mars* approached opposition it received the greater share of attention. Indeed, the observations of *Jupiter* were regarded as of secondary importance because the low declination of the planet placed it in an unfavorable position for observation. At the time of observation it had a zenith distance, when on the meridian, of about 52° . Better photographs should be obtainable in 1927, and the position will improve during succeeding years as the planet works its way northward.

Notwithstanding the unfavorable position, some of the photographs are of very good quality, and they show variations with change in the color by which the planet is pictured which are quite striking. In Plate IX there are reproduced a complete set of photographs taken in the six colors employed, ranging from ultra-violet to infra-red, and several pairs of photographs consisting in each case of an infra-red member and one taken either by violet or by ultra-violet light.

The complete set is given in the upper row of illustrations, the photographs being arranged in the spectral order of the colors used, which happens as well to be the time sequence in which they were taken. It will be noticed that there is a very considerable difference between the images at the extremities of the row, notably in the structure of the heavy lower belt, which in the right hand photograph exhibits curious inclined streaks, or filaments, as though a pen and ink drawing had, while still wet, been brushed in a direction upward to the right. The contrast between belt and background is rather low in the photographs taken in the longer-waved colors, and, in order to insure a representation in the half-tone engraving of the infra-red image, two prints made in exaggerated contrast are inserted just below the left-hand members of the series.

The means employed to magnify contrast are described in the section of this paper relating especially to the illustrations.

It is necessary to bear in mind, in comparing the several images of the series, that, because of the rapid rotation of *Jupiter*, approximately equivalent presentations are had only in successive photographs. The interval between the taking of the pictures at the extremities of the row was one hour and seventeen minutes, during which *Jupiter* turned 45° on its axis. The direction of motion due to rotation is from right to left on the paper, and the displacement is, of course, progressive through the series.

Most of the changes with color are so obvious that attention need hardly be directed to them. The remarkable difference in the structure of the heavy northern (lower) belt has already been alluded to. There are changes in the relative strengths of some of the other belts. A pronounced fading in the higher latitudes is shown by the images of short wave-lengths, and this imparts to those images an exaggerated appearance of ellipticity. The general edge fading, which is so marked a phenomenon in *Jupiter*, is much more pronounced in the long than in the short wave images; in fact the limb of the planet is quite invisible in infra-red photographs of normal exposure. It was therefore necessary, in making the two prints pictured on the second line in magnified contrast, to shade the edge strongly in order to prevent it from being completely lost. Limb darkening is hardly appreciable in the ultra-violet photographs. Possibly it might have been a little more pronounced had they been taken closer to opposition, though the relatively strong terminator indicates that it would not have approached that of the long-wave photographs. In respect to this fading my observations are altogether at variance with those of Professor R. W. Wood, who reported, on the basis of his pioneer work on the photography of *Jupiter* by light of different colors,²³ that the fading is more pronounced in the images of shorter wave-length. The present observations indicate that the change in limb fading is progressive as the color used in observation passes from one end of the spectrum to the other. Some of the changes here mentioned, as well as the invisibility of the red spot in colors of longer wave-length, were referred to in an earlier note.²⁴ The red spot is shown on the ultra-violet and violet images, just emerging on the disk at the upper right side. It was well on the disk when the infra-red pictures were taken.

Reverting to the inclined filaments, which in the infra-red images reach out from the strong north equatorial belt, it is to be noted that they are bordered

²³ *Astrophys. Jour.*, **43**, 317, 1916.

²⁴ *Publ. Astron. Soc. Pac.*, **38**, 334, 1926.

on their southern (upper) edges by patches of extreme brilliancy, which are, in fact, the brightest parts of the planet as photographed by infra-red light. The filaments are terminated by short dark lines, or cross bars, which lie approximately on the planet's equator. These terminal lines are probably regions of exceptional density in the faint, narrow, dark belt in that position. On the infra-red image of October 22 (figure 14) one of the stronger filaments divides into a broadly spread italic *Y*, one branch of which recurves sharply. The upper terminals of the *Y* merge either with the south equatorial belt, or with an extremely faint belt lying just below it. Most of the filaments are attached to the north equatorial belt and extend upward to the right in the illustrations. There are a few, however, reaching in the same direction from the south equatorial belt, and perhaps one extremely faint one directed upward to the left.

These filaments are unquestionably the wisps, or faint diagonal lacings, which have been seen by many observers with the telescope, but of which I had no knowledge when writing the above description. The reader's indulgence must be begged for the display here and elsewhere of very great ignorance regarding planetary matters. It seems well enough, however, to let the account stand as written, because it tallies so closely with the descriptions given by eye observers, and thus establishes conclusively the identity of the extremely strong markings of the infra-red photographs with the delicate tracings seen in the telescope. Attention has been especially directed to these wisps, as observed visually, by Scriven Bolton, W. F. Denning, and Rev. T. E. R. Phillips, though they appear to have been known since the early days of planetary observation. Excellent descriptions of them are available especially in the *Journal* and *Memoirs* of the British Astronomical Association, and in *Lowell Observatory Bulletin* No. 38. E. C. Slipher at Lowell Observatory and Pease at Mount Wilson have photographed them by yellow light. Their appearance in the yellow photographs is probably not very different from the telescopic aspect. They have been aptly described by one observer as resembling driven showers of rain. The curious thing about them is that, while to the eye they are not at all conspicuous, on the infra-red photographs they compare favorably in visibility with the belts themselves. Professor Pease has kindly sent me copies of some of his photographs taken with a yellow screen. On one of them the wisps reach downward to the right and in another upward to the right. Observers with the telescope describe them as favoring an inclination of approximately 45°, but as having no especial preference in the matter of quadrants; and they are occasionally reported as going straight across from one belt to another. The bright patches noted

as accompanying the filaments in the photographs have usually been found to be present in the telescopic view.

Wisp-like markings such as these were seen on *Saturn* in 1909 by Lowell and Slipher.²⁵ It is possible that infra-red light will prove as advantageous in observing them there as it is on *Jupiter*.

When one looks at *Jupiter* through the telescope the eye perceives, of course, the integrated effect of different designs in different colors. The multitude of shades, which make this planet colorful beyond all others, is therefore undoubtedly due in part to the crisscrossing of detail in the elementary color images, and the intricacy of the resulting pattern is largely provided by light of the redward end of the spectrum.

Regarding interpretation. It is feared that the following discussion does not justify the noun of its introductory phrase; speculation would have been a more suitable word and one of none too wide a connotation. The one used was chosen rather to make the form of this paper match that of others in which the observational data are perhaps better able to support a conclusion. The phenomena presented by these curious photographs of *Jupiter* appear to be so complex, the observational material is so limited, and the conditions on the great planet are so unlike those on the Earth, where our intuitions are bred, that I feel that a serious attempt at interpretation would now be premature. There is the further fact that the spectrum of *Jupiter* provides a powerful and as yet little utilized means of studying its atmosphere. In the case of *Mars*, where a tentative attempt at explanation has been made, we have a body obviously not very different from the Earth; in that of *Jupiter* the conditions are not at all comparable. The following paragraph, quoted from a note by Mr. Edwin Holmes, epitomizes some very important implications of the high superficial gravity of *Jupiter*:

"We must not forget that on *Jupiter* gravity doubles the pressure (or halves it) every 1.6 miles. M. Flammarion calculated that if his atmosphere had a depth of only 100 kilometers, or about 63 miles, the density at 37 miles from the surface would be the density of platinum, and 'yet there are 25 more doublings of density in the remaining descent.' He adds: 'It is perfectly monstrous.' The atmosphere at the surface would be 17 millions of times more dense than platinum. I think this amounts to telling us that long before we reached 37 miles down, the atmosphere would be compressed into a solid body, and would be in no reasonable sense an atmosphere at all. And the clouds in any case under the conditions must be extremely thin, and *Jupiter's* beauty only skin-deep."²⁶

For the purpose of illustrating how little agreement there is in views respecting conditions on *Jupiter* let us compare with the above paragraph the following excerpt from a paper by Dr. Harold Jeffreys:

²⁵ *Lowell Observatory Bulletin*, 1, 241, 1910.

²⁶ *Journal Brit. Astron. Assoc.*, 27, 85, 1916.

"To sum up, the available evidence regarding *Jupiter* and *Saturn* is consistent with the hypothesis that each is composed of a rocky core, surrounded by a thick layer of ice, which is covered by an atmosphere whose depth is in the one case about 9 per cent. and in the other 23 per cent. of the radius of the planet."²⁷

Nine per cent. of the radius of *Jupiter* is, in round numbers, 380 miles. There is evidently a divergence of opinion among authorities relative to the atmosphere of this enigmatical planet.

The problem presented by *Jupiter* is one of such surpassing difficulty that it is doubtful whether it can be reduced to simple terms without mustering to our service every possible means of attack. It may not, however, be out of place to trace the steps of a small reconnaissance undertaken along lines suggested by the color work on *Mars*. The approach adopted is through a consideration of the phenomenon of edge fading. Failing visibility at the limb in observations made by light of a given color may well be caused, or at least influenced, by the occurrence of Jovian atmospheric absorption bands within the range of color used. This is a consideration of vital consequence in interpreting any phenomenon of absorption. A comparison of the colors used and their respective ranges, with the spectrum of *Jupiter* as photographed by V. M. Slipher,²⁸ does not suggest serious interference by the selective absorption of these bands or rather broad lines, though a few weak Jovian absorption lines do occur in several of the regions used. Apparent freedom from such interference is also indicated by the continuity of the change in character of the image with change in color. It will be well, eventually, to examine the ultra-violet and infra-red sections of the spectrum, but it seems safe for the present to assume that, in the changes here described, we are dealing with a general color effect and not with the consequences of isolated or uncontinuous absorption.

That there is general atmospheric absorption in the lower spectrum of *Jupiter* will be taken as established by the pronounced edge fading in the red and infra-red photographs. If we assume that the general absorptive power at short wave-length is not less than in the red,²⁹ it follows from the almost negligible darkening at the limb of the violet photographs, that these depict on the whole, phenomena of *Jupiter's* upper atmosphere. Absence of edge fading in the case of a planet which receives its illumination from without and has an atmosphere of appreciable absorbing power does not necessarily imply that there is negligible penetration of the atmosphere. Possibly subject to certain

limitations discussed by Fessenkoff,³⁰ which are not trespassed upon here, it means that the light's effective penetration (into and out of the atmosphere) is not greatly in excess of the atmosphere's vertical thickness; in other words that we cannot see to the bottom.

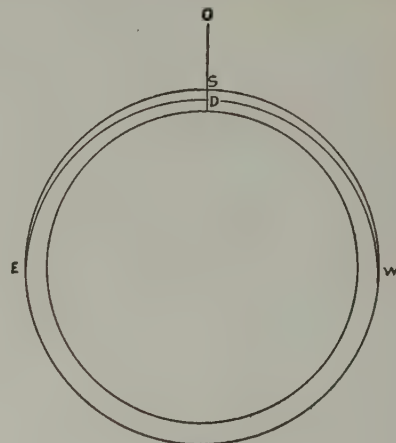


Figure 1

Figure 1 will be of convenience in illustrating this. It is supposed to represent a planet illuminated and observed from the direction 0. The atmosphere of the planet is assumed to be able to scatter and to absorb light. Let the outer circle represent the outer atmospheric layer of appreciable density, and the inner one the surface of the planet. We shall assume that violet rays are effective in photographing only to the depth *D*, while red rays have an effective penetration considerably in excess of the atmosphere's vertical thickness. The effective depth to which violet rays penetrate and return at points away from the center is indicated by the curve *EDW* and the part of the atmosphere photographed is the lune-like section *EDWSE*. To a degree of approximation which is sufficient for present purposes this limited volume of the atmosphere will be seen, or photographed, as a uniformly illuminated disk, provided only the ratio of scattering to absorption is constant throughout the volume considered.³¹ On the other hand the red rays, as a result of their greater penetration, will be able to take advantage of the less thickness of the atmosphere

³⁰ As the result of an elaborate analysis of the problem of planetary reflection, in which is introduced the assumption that atmospheric absorption occurs only through scattering, Fessenkoff (*A. N.*, 228, 30, 1926) concludes that a planet having an albedo of approximately 0.2 will present a disk of uniform brightness, whatever the atmospheric absorption. The analysis is quite complex, and will bear study. Whether or not the conclusion is correct, it does not apply in the present case because the photographic albedo of *Jupiter* (0.73 according to Russell) is greatly in excess of Fessenkoff's critical value.

³¹ Uniformity will result only when the planet is approximately at opposition. With illumination from one side there should be an increase in brightness from terminator to limb.

²⁷ *M. N. R. A. S.*, 84, 538, 1924.

²⁸ *Lowell Obs. Bull.*, 1, 231, 1909.

²⁹ The photographs taken by red and infra-red light in the one case, and by violet and ultra-violet light in the other, are so similar that in what follows the somewhat awkward prefixes will not be used.

near the middle of the disk, and will show the well known phenomenon of edge fading.

It is quite clear, on a little reflection, that the violet photographs cannot be explained on the unmodified assumption that they are due to scattered light. Generally speaking, light of that kind is not especially influenced by the composition of particles which do the scattering. By way of illustration let us consider the phenomenon of our own sky. The part of the light of the sky which is due to fine suspended dust, while it is undoubtedly less blue than that scattered by molecules of air, is, on the whole, not very different from it. A fairly uniform surface might therefore be expected in the ultra-violet photographs of *Jupiter*, even though the scattering particles might differ considerably in size and composition at different parts of the planet's surface. The observed fact is that the variations in intensity—the contrasts between belt and background—are quite as great in these short-wave photographs as in any of the others.

A path out of the difficulty is perhaps provided by the following considerations. The brightness of our hypothetical atmosphere of figure 1, for a given intensity of illumination, depends on the ratio of scattering efficiency to absorption. If therefore, we have a medium in which this ratio is low (as, for example, is suggested for *Mars*) a uniformly darker disk will result. If such a medium were distributed in zones, like the belts of *Jupiter*, it would seem to meet our requirements, for it would maintain constancy in brightness (or darkness) to the planet's limb. It is to be noted that the medium must extend clear to the surface of the atmosphere in order to fulfill the condition named, i.e., constancy of ratio of scattering to absorption, for that condition merely implies homogeneity. If we were to assume the contrary to be the case, namely that the belt material (as just interpreted) falls short of the atmospheric surface, the increased thickness of the free atmospheric layer through which we should have to look at the belt when near the limb would result in the superposition of light over the dark area and consequently in a reduction of contrast. To put the matter a little differently suppose the upper surface of the belt to lie between *D* and *S* (figure 1). (It must lie above *D* because that is the greatest depth to which one sees by violet light.) As the portion of the belt under consideration is carried toward *E* by the planet's rotation it will eventually fall below the curve *DE*, and therefore become invisible. Since the belts apparently persist to the limb with little alteration in strength it is necessary to assume that they extend clear to the surface *S*.

The statement here made that the belts persist in normal intensity to the edge is based only on casual inspection of the photographs. It may, of course, not

be more than approximately true. On photographs taken close to opposition (August 22 and 25), when the phase angle was less than 2° , the dark belts appear to be lost in a peripheral line of brightness at the limb. These were the first photographs taken, and they were not made under the best conditions. On October 14 (see Plate IX, figure 9), with a phase angle of 10° , the comparatively narrow South Equatorial and North Temperate belts apparently extend to the limb, but it is not certain that the wide North Equatorial belt reaches it on any of the photographs. The structure of this belt varies so from point to point that a true idea of the effect upon it of proximity to the limb can be had only by studying the appearance of one or more of its configurations in their diurnal travel across the disk and on their approach to the limb. This will be done it is hoped, at the coming opposition, with the aid of cinematography. For the present we shall have to content ourselves with the approximate statement that, relatively to their appearance on photographs taken by light of longer wave-length, the equatorial regions of the planet, as recorded on ultra-violet photographs, are free from limb effect, and from this fact the provisional inference of the preceding paragraph, respecting the upward extension of the belts, is drawn.

Several assumptions regarding the nature of our hypothetical belt medium seem to be permissible as far as the violet photographs are concerned. One is that it consists of small absorbing bodies homogeneously distributed in the atmosphere. If that be the case, one is tempted to assign to them a reddish color, for that is the color of the belts, and since they are near the surface it does not appear that their color can be affected by absorption. The red color accounts for falling off of contrast of the belts in the red images. If these particles have a density distribution homogeneous with that of the atmosphere in which they float, the optical conditions necessary for our belt medium are fulfilled. It is to be remarked that the high value of gravity on *Jupiter* would favor the flotation of solid particles of considerable size.

Our alternative hypothesis is that the medium of which the belts are composed is a gas whose ratio of scattering to absorbing power is low.

These two hypotheses have been examined rather sketchily in relation to the red images. The phenomena there are very complex and I should prefer not to embark on a discussion of them without the photometric data which the plates will eventually provide. The red images seem to favor the explanation in terms of an absorbing gas as against absorption by suspended particles, because darkening at the limb, which is strong in the red images and affects the belts as well as the bright surface, seems to be better taken care of by it.

In regard to the depth to which the violet photographs extend—the position of the point *D* in figure 1—a general inference seems possible. If we assume that the photographs 1 and 7–8 of Plate IX actually represent different average levels, and that 7 is lower than 1, it seems reasonable to assume that the inclined filaments of the latter photograph are completely below the level of figure 1, because there is no trace of them there. From this it is judged that the point *D* must be situated fairly well up in the atmosphere, in order to leave room for the development of these very conspicuous features below it. In this connection the rather puzzling behavior of the red spot may be mentioned. Diminished visibility of that object on the red photographs is to be expected, since it is a dark marking of reddish hue seen against a white background; but if it is a phenomenon of the upper levels, as its occurrence in the violet photographs would seem to indicate, we must be looking *through* it in the red photographs. One would, therefore, expect to find

some trace of it on them. The ultimate visibility of the characteristic markings of one color on photographs taken in the other, may eventually settle the question of which level is the higher.

It will be understood that none of the inferences which are here tentatively drawn are regarded in any sense as conclusions. These notes are mere comments accompanying what are believed to be unique photographs of *Jupiter*. Future interpretation, based on more observational material and competent photometric data, may lead in a very different direction than that taken in this exploratory ramble.

I am indebted to Mr. J. F. Chappell, Lick Observatory photographer, for his painstaking care in making the prints which illustrate this paper.

April, 1927.

Issued May 24, 1927.

LICK OBSERVATORY BULLETIN

NUMBER 390

THE SPACE DISTRIBUTION OF THE STARS OF DIFFERENT
SPECTRAL CLASSES AS DETERMINED FROM STUDIES
IN THE CYGNUS REGION OF THE MILKY WAY

By

ALLAN D. MAXWELL

I. INTRODUCTION

The absence of data concerning the distances of the fainter stars has been one of the most serious limitations encountered by statistical investigators. In view of recent work by Lundmark,¹ Lindblad² and others, indicating that a fair approximation of distance can be derived from a careful classification of spectra, the writer determined to make a spectroscopic study of some of the stars of Kapteyn's Selected Areas (systematic plan), with a view to studying the distribution in space of the stars in the Milky Way between *Vulpecula* and *Perseus* down to about magnitude 13.5 on the Harvard photographic scale. The slitless quartz spectrograph of the Crossley reflector seemed adapted to such work, but as the diameter of the field is rather small the study was restricted to twenty-four regions of about 20' diameter within six of the galactic Areas. The choice of galactic fields was made partly on account of instrumental limitations and partly because the galactic distribution is probably of greater interest than the non-galactic distribution of stars in space. For statistical purposes, of course, a maximum number of stars is imperative, and within a given interval of time this number is most readily obtained in the Milky Way. The Harvard-Gronigen *Durchmusterung of Selected Areas*³ together with systematic correc-

tions recently published by Seares and others⁴ gives us well-determined photographic magnitudes of all stars in these Areas down to magnitude about 16.5. A fairly uniform section of the Milky Way is found between galactic longitudes 30° and 110°, bordered by the constellations *Vulpecula* and *Perseus*. The six Areas in this region will be combined and discussed as a unit. The region has the additional advantage that it is easily observed over an interval of several months from Mount Hamilton.

The "Plan of Selected Areas," outlined by Kapteyn in 1906 in a booklet under that title, is a proposed method of attack on the problem of the structure of the sidereal system, which shall reduce the amount of work to a minimum. On the systematic plan⁵ 206 Areas are selected at intervals of fifteen degrees in declination from the north to the south pole and at intervals of one to four hours in right ascension according to the distance from the equator.

Kapteyn's plan embraced a *Durchmusterung* and a determination of the photographic magnitudes, visual magnitudes, parallaxes, proper motions, spectral classes and radial velocities of the fainter stars in these Areas. For details the reader is referred to the booklet just mentioned, and to subsequent reports on the progress of the Plan.

Spectra of stars brighter than magnitude 8.5 are classified in the *Henry Draper Catalogue*. For magnitudes 8.5 to 11.0, the Hamburger Sternwarte at Berge-

¹ *Publ. A. S. P.* **34**, 150, 1922.

² B. Lindblad gives methods of distinguishing high luminosity stars from stars of low luminosity and of the same spectral type. *Mt. Wilson Contr.* 228, 1922. See also "Spectrophotometric Determinations of Stellar Luminosities" (first paper), *Nova Acta Reg. Soc. Scient. Upsala*, Ser. IV, **6**, no. 5, 1925.

³ *H. A.* **101**, 1918; **102**, 1923; **103**, 1924.

⁴ *Mt. Wilson Contr.* no. 289, 1924.

⁵ In this paper, by "Selected Areas" we refer to the Systematic Plan and not to the Special Plan proposed by Kapteyn.

dorf is planning the spectral classification. Mr. Humason at Mount Wilson has undertaken the classification of some of the stars in northern Areas from magnitudes 11 to 12. The present classification will doubtless overlap the latter to some extent, but we shall in some cases be able to extend the limit of brightness by over 1.5 magnitudes for the special regions under consideration. Furthermore, for reasons which will appear in the text, a classification of spectra based partly on the ultra-violet region is especially desirable.

It has become quite customary in modern investigations to deal with what may be called a "simplified stellar system." Usually the galaxy is studied as a whole, local irregularities are neglected, and the variation of density of stars in space in terms of the distance from the Sun is represented by a mathematical formula called the "density law." Various forms of the density function have been proposed and each investigator has used that which he has found most nearly accordant with the facts. The luminosity law and the frequency function of apparent magnitudes are related through the density law. The forms of these laws are usually assumed and the constants are evaluated from the data available. The assumptions must be made so that the mean parallaxes of groups of stars obtained in different ways, are well represented. Charlier⁶ enumerates six hypotheses as to the form of the density law, which, except for notation, are as follows:

- I. $\Delta(\rho) = \text{constant}; M = \text{constant}$ (Herschel)
- II. $\Delta(\rho) = \text{constant}; \varphi(M)$ arbitrary (Schiaparelli)
- III. $\Delta(\rho) = \gamma \rho^{-s}$ ($s > 0$) (Seeliger)
- IV. $D(y) = \frac{N}{\sigma \sqrt{2\pi}} e^{-\frac{(y-y_0)^2}{2\sigma^2}}$ where $y = 5 \log_{10} \rho$ (Schwarzschild)
- V. $\Delta(\rho) = \frac{C}{\rho} e^{-k^2 \rho^2}$ (Dyson and Eddington)
- VI. $\Delta(\rho) = \Delta(0) e^{-\frac{\rho^2}{2k^2}}$ (Charlier)

where M is the absolute magnitude, $\varphi(M)$ is the luminosity law, ρ is the distance from the Sun, and $\gamma, s, y_0, k, \sigma$, and C are constants. The first and second lead to a constant magnitude-ratio of 3.981 and the third to a magnitude-ratio $10^{0.2(3-s)}$. The fourth leads to a normal frequency function for the frequency function of the apparent magnitudes if a normal frequency function for the luminosity law is assumed. This is the form adopted by Kapteyn, van Rhijn, Seares and others.

The density law will be derived empirically in the present paper using the distances of stars derived from spectral types. Thus with the assumed distance and the apparent magnitude of each star, we shall be

⁶ "The Motion and Distribution of the Stars," *Memoirs of the Univ. of Calif.*, 7, 97, 1924.

independent of specific forms of the function. Furthermore the results will be of a special nature, referring only to a definite section of the Milky Way. In this we depart, to a certain extent, from the simplified form of the stellar system.

The distances of stars in statistical discussions are usually derived from mean secular parallaxes, from parallaxes derived as functions of both proper motion and apparent magnitudes, or from spectroscopic parallaxes. Trigonometric parallaxes, Cepheid parallaxes, and moving cluster parallaxes are selective in character. Parallaxes derived from spectral types—to which we shall hereafter refer as "spectral" parallaxes—are possible, as shown by Lundmark, if the giant or dwarf character of the spectrum can be ascertained. This is a direct consequence of the discovery of Hertzsprung and Russell that the dispersion in absolute magnitudes among stars of a given sub-class is small.

The present study is based on the following data:

(1) 557 stars, mainly between photographic magnitudes 10.0 and 14.0, observed and classified for this purpose and contained in six of the Selected Areas.

(2) 347 stars from the *Henry Draper Catalogue* with photographic magnitudes 5.0 to 8.0 contained in six areas of approximately 44 square degrees each and concentric with the observed Selected Areas.

(3) 1156 stars in 83 square degrees having the *Cygnus* cloud in the center. The photographic magnitudes are from 8.0 to 10.0. This is the region covered by the first and part of the second installment of the *Henry Draper Extension* which contain 807 of the stars. The remainder are from the *Henry Draper Catalogue*.

The following problems will be attacked:

(1) To determine, for this part of the Milky Way, the density law of each spectral type. In this we shall be limited to those distances to which our observations, for stars of given luminosity, are complete.

(2) In the nearer regions of space, to find the relative frequencies of yellow-red giant stars and yellow-red dwarf stars. By this we mean stars of spectral types G and K.

(3) For the giant stars (including types A and B), to determine the proportion belonging to each spectral class and to find how this proportion varies with distance.

II. INSTRUMENTS AND METHOD OF OBSERVATION

The spectrograms were made with the two-prism quartz spectrograph in its slitless form, attached to the Crossley reflector. The instrument was designed by Mr. W. H. Wright and is described by him in *Lick*

Obs. Bull. 9, 52, 1917. The dispersion is such as to separate H β and H ϵ by 7.4 mm. The prisms are set for minimum deviation at λ 3600. Eastman speedway plates were used throughout. A field of about 22' in diameter is obtainable and excellent spectra are found within 7'5 of the center. To avoid risking a change in focus, only two plate holders, lettered "A" and "B," were used in this work.

A standardizing sensitometer designed by Wright⁷ and described in its present form by Yü⁸ was used to facilitate measures of the photographic density of some of the spectra, to be described later. Before each plate was placed in the spectrograph it was exposed in the sensitometer for 20 seconds to the light of a carbon lamp operating at 220 volts and using two amperes. After passing through a blue glass the light is cut off by small circular windows whose areas are in the proportion 1:4:8:32, and these windows are made to illuminate certain areas on the photographic plate. An accurate system of diaphragms prevents reflection from the sides, so that the intensities of the incident light on the photographic plate are in the above proportions. The instrument is provided with windows whose areas, on the same scale, are in the proportions 2 and 16, but these were not used in the present work, because of the danger of intercepting some of the spectra. As these measures are only for the purpose of separating certain giant stars from dwarfs of the same spectral type, the variation of development factor with wave-length, as well as the large disparity of exposure time, is neglected.

All plates used in this work were developed in two ounces of a ten per cent solution of *Rodinal* at 65° F. for ten minutes. Uniformity of development, intended for the facilitation of spectrophotometric measurements, is also important in the direct estimation of spectral type. Approximately uniform conditions of fixing and washing were obtained.

Each of the areas in Table I was divided into four square sections and the telescope was pointed, as nearly as possible, to the center of each section. In this way, four exposures of five hours each were made on each Selected Area. The work was duplicated with one-hour exposures to obtain the spectra of the brighter stars.

TABLE I

TABLE OF THE OBSERVED SELECTED AREAS

Sel. Area No.	Center (1900.0) ($x=0, y=0$)		Limits	
	α	δ	in x	in y
64	19 ^h 58 ^m	+30° 00'	74' to 14'	-20' to +20'
40	20 47	45 00	75 to 15	-20 to +20
18	21 24	60 10	83 to 23	-20 to +20
19	23 23	60 00	80 to 20	-20 to +20
8	1 00	60 10	77 to 17	-20 to +20
9	3 04	60 20	77 to 17	-20 to +20

⁷ *Publ. A. S. P.* 35, 281, 1923.

⁸ *Lick Obs. Bull.* 12, 109, 1926.

The Areas are arranged in order of the dates of observations. The columns contain data taken from *Harvard Annals* 101, giving the number of the area, the position of the origin of the rectangular coördinates x and y , and the limits of the areas in rectangular coördinates. When x is negative it has been increased by 100'. The extreme limits of galactic longitude for this set of Areas are 35° and 106° and those of galactic latitude, -2° and +6°. All of the Areas are of the same size (40' x 40').

Each plate gives forty to fifty very narrow spectra, depending on the richness of the region. The limit of completeness of the plates varies, but is about 13.5 photographic magnitude on the Harvard scale at the centers of the plates. Since fainter spectra are visible in the center than near the edge, the plates have arbitrarily been divided into two concentric zones, the outermost 22', and the inner zone 15' in diameter. For statistical purposes a certain limit of magnitude must be established, beyond which no stars may be included. By taking different limits for the two zones a greater number of stars may be included and the limit of magnitude is pushed farther for the central zone.

To facilitate spectral classification a list of comparison stars (given in Table II) was chosen from the Mount Wilson lists⁹ of spectroscopic parallax stars. The spectral types of these stars were then taken from the *Henry Draper Catalogue* and used as a basis of the classification in this paper. The same spectrograph was used as for stars in the Selected Areas and the standard squares of the sensitometer were impressed on each plate. Five spectra with exposures in the ratio 1:4:8:32, placed side by side on the plate, were secured for each star. The longest exposure was twice the star's computed normal exposure. The spectra were only very slightly broadened in order to be directly comparable with the spectra to be classified.

TABLE II

TABLE OF COMPARISON STARS

No.	H.D. Sp.	Star	Plate No.	Normal Exposure	α (1900.0)
1	B0	69 Cygni.....	M228B	14 ^a	21 ^b 21 ^c 7
2	B1	ϵ Persei.....	M248B	3 ^a	3 51.1
3	B2	12 Lacertae.....	M206B	7 ^a	22 37.0
4	B3	16 Pegasi.....	M204B	6 ^a	21 48.5
5	B3	6 Lacertae.....	M205A	3 ^a	22 26.2
6	B5	Brad. 325.....	M249A	20 ^a	2 14.2
7	B8	Groomb. 244.....	M215A	11 ^a	1 02.4
8	B8	ϕ Andromedae.....	M214B	2 $\frac{1}{2}$ ^a	1 03.7
9	B8	Pulk. 2680.....	M201A	15 ^a	18 54.6
10	B8	14 Cygni.....	M217A	10 ^a	19 36.2
11	B9	Brad. 138.....	M241B	11 ^a	1 05.0
12	A0	7 Trianguli.....	M252A	10 ^a	2 10.0

⁹ Stars of types F, G, K, and M were taken from *Mt. Wilson Contr.* no. 199; those of type A from *Mt. Wilson Contr.* no. 244; and the B-type stars from *Mt. Wilson Contr.* no. 262.

TABLE II—(Continued)

No.	H.D. Sp.	Star	Plate No.	Normal Exposure	α (1900.0)
13	A0	29 Cygni.....	M203A	3½ ^s	20 ^h 10 ^m 8
14	A2	ι Delphini.....	M218B	12 ^s	20 33.0
15	A5	δ Cassiopeiae.....	M243A	½ ^s	1 19.3
16	A5	ω Aquilae.....	M238A	10 ^s	19 13.1
17	F0	22 Andromedae....	M209A	12 ^s	0 05.1
18	F0	28 Andromedae....	M210B	12 ^s	0 24.8
19	F0	ρ Piscium.....	M232B	14 ^s	1 20.9
20	F0	7 Andromedae.....	M207A	6 ^s	23 08.0
21	F2	13 Pegasi.....	M229A	15 ^s	21 45.4
22	F5	Pi. 171.....	M226B	22 ^s	1 43.0
23	F5	ν Persei.....	M247A	3 ^s	3 38.4
24	F5	25 Monocerotis....	M253B	15 ^s	7 32.3
25	F5	Pulk. 2677.....	M227A	17 ^s	18 53.8
26	gG5	Pi. 128.....	M211A	31 ^s	0 32.0
27	gG5	ε Andromedae.....	M222B	10 ^s	0 33.3
28	gG5	η Piscium.....	M225A	3½ ^s	1 26.1
29	gK0	ζ Andromedae.....	M213A	10 ^s	0 42.0
30	gK0	η Ceti.....	M236B	4 ^s	1 03.6
31	gK0	χ Piscium.....	M224B	19 ^s	1 06.1
32	gK0	ξ Andromedae.....	M242A	20 ^s	1 16.4
33	gK5	δ Piscium.....	M223A	15 ^s	0 43.5
34	gK5	31 Lyncis.....	M255B	12 ^s	8 16.0
35	gK5	ν Pegasi.....	M219A	22 ^s	22 00.6
36	gMa	χ Pegasi.....	M220B	28 ^s	0 09.4
37	gMa	ρ Ursae Majoris...	M254B	31 ^s	8 53.5
38	gMb	47 Piscium.....	M221A	43 ^s	0 22.8
39	dG0	9 Ceti.....	M235A	60 ^s	0 17.7
40	dG0	Lal. 1966.....	M240A	5 ^m	1 03.3
41	dG0	Lal. 3153.....	M233A	3 ^m	1 37.4
42	dG0	δ Trianguli.....	M246A	14 ^s	2 10.9
43	dG0	τ Persei.....	M250A	4 ^s	2 47.2
44	dG5	μ Cassiopeiae.....	M216B	20 ^s	1 01.6
45	dK0	W.B. 1 ^b 161.....	M237A	10 ^m	1 13.5
46	dK0	τ Ceti.....	M234B	4 ^s	1 39.4
47	dK0	Pi. 159.....	M245A	43 ^s	1 40.5
48	dK0	A0 25685.....	M230B	2 ^m 25 ^s	23 26.5
49	dK2	Bu 12740S.....	M231A	20 ^s	1 36.8
50	dK2	A0 1867.....	M244B	12 ^m	1 36.8
51	dK2	Brad. 3077.....	M208B	43 ^s	23 98.5
52	dMa	Groomb. 34.....	M239B	16 ^m	0 12.7

Column two of Table II gives the spectral type in the *Henry Draper Catalogue*, giant and dwarf stars being indicated by prefixes "g" and "d" respectively to the spectral type, in accordance with the international notation. The giant or dwarf character is taken from the Mount Wilson spectroscopic absolute magnitudes. The absence of a few spectral subdivisions arises from making the choice of stars with regard to the Mount Wilson "estimated" type rather than the Harvard type, and from the fact that certain subtypes of the Harvard sequence such as A3, G2 and late M-types, are not recognized in our classification.

III. SCHEME OF SPECTRAL CLASSIFICATION

The principal points of spectral classification as applied to the particular dispersion and character of the spectra obtained are given in the following scheme:

Principal Criteria for Spectral Classification

H.D. Type

- B0-B1 Spectrum is long and of nearly uniform density. The hydrogen lines are faint.
- B2-B3 The hydrogen lines are stronger, and the helium line $\lambda 3820$ is conspicuous.
- B5-B8 Marked by increasing strength of the hydrogen lines, weakening of $\lambda 3820$, and the beginning of continuous absorption in the ultra-violet.
- A0 The hydrogen lines reach a maximum and the K-line is occasionally visible.
- A2 The K-line, though faint, is easily seen.
- A5 The K-line is nearly as strong as H δ or H ζ .
- F0 The K-line is somewhat stronger than H δ or H ζ .
- F2 The lines H and K are very conspicuous. The G-band begins to appear.
- F5 The G-band is nearly as strong as H γ . The hydrogen lines are rather weak, and the lines H and K are very strong.
- G0 The G-band is stronger than H γ . The hydrogen lines, though faint, are easily seen.
- G5 The G-band is quite pronounced. There is some contrast at the G-band. H γ is still visible and the doublet $\lambda\lambda 4383-85$ is just visible.
- K0 There is strong contrast at the G-band, the doublet $\lambda\lambda 4383-85$ is strong, the hydrogen lines are generally invisible, and in dwarfs $\lambda 4226$ is occasionally seen.
- K2 The line $\lambda 4226$ is easily seen, the triplet $\lambda\lambda 4030, 4033, 4034$ and the iron line $\lambda 4045$ are easily visible.
- K5 Same as K2, except that $\lambda\lambda 4226, 4030-34, 4045$ are stronger.
- Ma Spectrum is banded, $\lambda 4226$ is extremely intense.
- Mb The banded character of the spectrum is very conspicuous; $\lambda 4226$ is broad and strong.

The distinction between giants and dwarfs was made at type G0 and later. Only two criteria have been used for this purpose, (1) the relative strength of the cyanogen band at $\lambda 3883$ and (2) the fact that giants and dwarfs of the same spectral type have a somewhat different color index and hence a different distribution of spectral intensity. The cyanogen band $\lambda 3883$ is nearly absent in dwarf stars of type K0, but in giants of the same type it is usually very strong. This contrast diminishes as we go toward G0 or Ma,

reaching a maximum at K0. The extreme sensitive-ness of this region of the spectrum to changes in absolute magnitude was pointed out by Lindblad.¹⁰ The effect is still present at type G0, though less pronouncedly. We shall see later that certain conclusions drawn from this paper depend mainly on an accurate division of K-type stars into giants and dwarfs. Thus the greatest accuracy is present at the point of greatest need.

The color index criterion was used only for class M stars and those stars of type K whose spectra are intercepted in such a manner as to render the application of the first criterion impossible. It is true that, for the former, the ordinary color index change is rather small; yet when properly chosen regions of the spectrum are photometrically examined it is easily possible to distinguish the dwarfs from the giants. In the next section we shall briefly describe the method by which this is effected. It should be noted that it has been applied only to stars whose classification was already accomplished by means of the spectrum lines.

IV. METHOD OF MEASUREMENT OF SPECTRAL INTENSITIES

To distinguish between giants and dwarfs in the case of a few late type stars, two regions of the spectra were measured with the Hartmann¹¹ equalizing wedge microphotometer. The standard squares of known relative intensity, already mentioned, were available on all plates, both of comparison star spectra and of spectra in the Selected Areas. These were used to calibrate the wedge readings whence the relative spectral intensities could be obtained. If, as stated above, we neglect variation of the development factor with wave length and neglect the effect of the wide disparity of exposure times, a given reading of the wedge may be converted into a number which signifies the photographic intensity in terms of the faintest standard square. Dividing this number for one wave-length by the same number for another wave-length we get a ratio which, when found for different spectrograms, will give us some indication of the giant or dwarf character of a late-type star if its spectral class is obtained from the lines and if the spectral regions are properly chosen. Dividing this ratio for one star by the same for another star, we eliminate, to a first approximation, the effects of atmospheric transmission, reflectivity of the silver mirror, transmission of the quartz spectrograph, dispersion, and sensitivity of the Eastman speedway plates.

A word may be said concerning the method of reduction. By plotting the intensities of the standard

squares expressed in terms of the faintest square against the wedge reading, a "reduction curve" is obtained from which the wedge readings on the spectra give the spectral intensities. In order, however, to reduce the errors of graphical work to a minimum, use was made of the fact that the reduction curve has, as a first approximation, the form:

$$I_0 = 10^{1.505 \frac{D-D_1}{D_8-D_1}}$$

where I_0 is a first approximation to the intensity of any unknown region, D is the wedge reading for the region, and D_1 and D_8 are the wedge readings for the faintest and brightest standard squares respectively. The small difference $I-I_0$ was then plotted for other standard squares, and the correction from this reduction curve was applied to the unknown region. The disadvantage is an increase in the amount of computation, but the advantage is the gain in accuracy and saving of care in the graphical part of the work. Figure 1 shows the reduction curve for plate M250A.

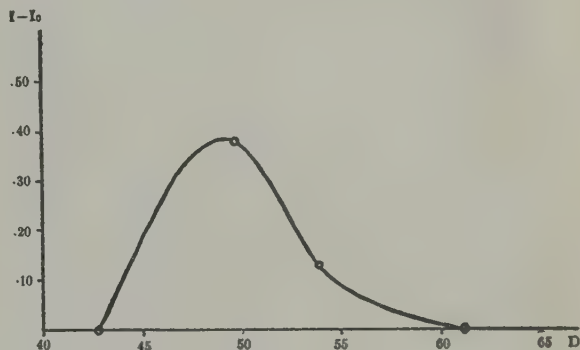


Fig. 1. Reduction curve for Plate No. M250A.

After some experimenting the regions in the neighborhood of the wave-lengths $\lambda\lambda 4360$ and 4050 were found most suitable for the giant and dwarf classification. The several exposures on each of eighteen plates

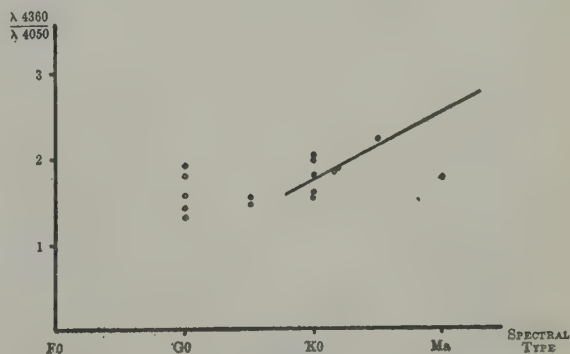


Fig 2. Intensity ratio $\frac{\lambda 4360}{\lambda 4050}$ plotted against spectral type.

¹⁰ *Mt. Wilson Contr.* no. 228, 20, 1922.

¹¹ J. Hartmann, *Zeitschr. für Instrumentenkunde*, 19, 97, 1899.

of giant and dwarf comparison stars were measured.

The intensity ratio $\frac{\lambda 4360}{\lambda 4050}$ was formed for each exposure and means were taken for all the exposures on a single plate. These means were then plotted against the spectral type. Figure 2 gives the resulting distribution. All stars above the inclined line are giants and all below are dwarfs.

By repeating the procedure for spectra whose complete classification was desired, the position on the diagram was used to determine the order of luminosity. Stars so classified are marked "measured" in the column headed "Remarks" of the Catalogue which is to follow. Eleven stars in the Selected Areas were so classified.

V. CLASSIFICATION OF STARS IN THE SELECTED AREAS

As already stated, each plate of the spectra of stars in the Selected Areas, as soon as the center was determined, was divided into two concentric zones. We shall, for convenience, call the innermost zone, Zone I, and the other, Zone II. Zone I is 7.5 in radius and Zone II is bounded by Zone I on the inside, and a concentric circle of radius 11' on the outside. A limit of photographic magnitude on the Harvard-Groningen scale (*H. A.* 101) was determined for each zone of each plate. Every effort was then made to classify accurately all of the spectra of stars brighter than that magnitude. This "limiting magnitude" will be defined as the mean of the magnitudes of the faintest star included in the classification and of the brightest star not classified, taken to the nearest tenth of a magnitude.

Each spectrum was classified at least twice independently. If the agreement was satisfactory the mean of the two classifications was adopted; if not, the plate was examined a third time. In the first classification, after each area had been covered, the plates of the comparison stars, after thorough mixing, were examined and classified. The same process was repeated after the completion of each pair of Areas in the second classification. Thus nine complete classifications of the comparison stars were made at well-distributed intervals throughout the classification of the unknown spectra.

In order to study the systematic error of classification the means of all the Harvard types classified as a given type were taken. These means represent the Harvard type corresponding to the given estimated type. The intention was to correct the estimates accordingly, but in all cases except types dK5 and gK2 the means agreed exactly with the estimates. As these two exceptions are not represented among the com-

parison star spectra, the disagreement has little or no meaning, and the correction was neglected.

Mount Wilson⁴ investigators have shown that the photographic magnitudes given in the Harvard-Groningen *Durchmusterung of Selected Areas* have a systematic scale error as well as a large color equation when compared with the Mount Wilson system. We shall here attempt to reduce the magnitudes to the Harvard photographic scale which is, by definition, the same as the Harvard visual scale for a star of sixth magnitude and class A0, and which is greater by one magnitude than the Harvard visual scale when the star is of class K0 and the sixth visual magnitude.¹²

Let

HPg = the Harvard photographic magnitude.

HV = the Harvard visual magnitude, used in *Harvard Annals* 71, and on the same scale as that of the *Henry Draper Catalogue*.

Pv_W = the Mount Wilson photo-visual magnitude.

Pg_W = the Mount Wilson photographic magnitude.

DM = the Harvard - Groningen magnitudes, given in *Harvard Annals* 101 for stars in the Selected Areas.

C_H = the Harvard color index.

C_W = the Mount Wilson color index.

In Table I of *Mount Wilson Contribution* 287, 1925, we find for a star brighter than magnitude 6.5 and class K0, $C_W = +1.48$. Since for the same star, $C_H = +1.00$, it follows that

$$C_H = \frac{1}{1.48} C_W \quad (1)$$

$$\text{By definition of } C_H, HPg = HV + C_H \quad (2)$$

Searse¹³ gives, as the color-equation of the Harvard visual system

$$HV = Pv_W + 0.14 C_W \quad (3)$$

if we neglect the scale correction, which should not affect the photographic scales. By definition of C_W

$$Pv_W = Pg_W - C_W \quad (4)$$

The color equation of the Harvard-Groningen *Durchmusterung* relative to the Mount Wilson photographic system is:⁴

$$Pg_W = DM + 0.06(DM - 6) + 0.21 C_W \quad (5)$$

⁴ *Mt. Wilson Contr.* no. 289, 1924.

¹² *H. A.* 80, no. 9, 1917.

¹³ *Trans. of Int. Ast. Union*, 1, 78, 1922.

A combination of these five equations yields the desired equation:

$$HPg = DM + 0.06(DM - 6) + 0.026 C_w$$

The coefficient of C_w is so small that it will here be neglected, and our working equation is:

$$HPg = DM + 0.06(DM - 6) \quad (6)$$

Thus there is no color correction and we need only correct for scale. The equation is only applicable to the brighter stars ($m \lesssim 10$). For the fainter stars we shall use the approximately constant correction of $+0^m.25$ given in column 10, Table IV, *Mt. Wilson Contr.* 289. That no discontinuity might be introduced at magnitude 10, the values for the different magnitudes were plotted and the corrections were read from a smoothed curve. The following table gives the corrections adopted:

DM Mag.	Scale Corr.	DM Mag.	Scale Corr.
7.0	0.06	11.0	0.25
7.5	.09	11.5	.25
8.0	.13	12.0	.25
8.5	.16	12.5	.25
9.0	.19	13.0	.25
9.5	.22	13.5	.25
10.0	.23	14.0	.25
10.5	.24		

The first column in Table III gives the Groningen number from *Harvard Annals* 101, 1918. The second column gives the zone on the photographic plate, defined above. The third column gives the Harvard photographic magnitude as just defined, and the fourth column gives the estimated spectral type. Although the original classification was limited to the types outlined in the scheme, in taking the means intermediate types were introduced. Thus a star classified first as A0 and then as A2 would be called A1.

Sections of the Areas are designated NE, NW, SE or SW according to their positions. Thus 64 NE is used to designate the north-east section of Area 64. The center of each plate is given in rectangular coördinates. The origins of the coördinate systems are defined by Table I. The date of exposure of each plate is also given.

TABLE III

CATALOGUE OF STARS IN THE SELECTED AREAS WHOSE SPECTRA ARE CLASSIFIED

Plate M111A, 1926, Aug. 5. Center of Plate $\begin{cases} x+100' = 98'6 \\ y = 7.1 \end{cases}$
Area 64 NE

Limiting photographic $\begin{cases} \text{Zone I } 13^m.5 \\ \text{magnitudes } \text{Zone II } 12.8 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
853	II	11.88	gK0	Overlaps No. 1358
1275	II	11.08	A0	Plate M113A
1298	II	11.48	A0	Plate M113A

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
1307	II	11.52	A2	Plate M113A
1320	II	12.39	A0	Plate M113A
1324	I	11.64	F5	Plate M113A
1328	I	11.88	A0	Plate M113A
1344	II	12.68	gG0	
1352	I	12.99	A0	
1353	I	13.09	F5	
1357	I	13.09	dG0	
1358	II	12.73	A0	Overlaps No. 853
1360	I	13.44	F5	Faint
1374	I	10.02	gK0	BD+29° 3860
1382	I	12.99	A2	
1387	I	12.68	F5	
1389	I	13.23	F5	
1394	I	11.49	A5	
1395	I	12.83	A0	
1412	I	12.13	F5	} Overlap
1414	I	11.18	F2	
1427	I	13.26	A5	
1429	I	12.26	gK0	
1434	I	12.68	gK0	
1435	II	12.62	dG8	
1446	I	13.44	dG2	
1451	I	12.68	dG5	
1467	II	12.68	gK0	Measured
1469	II	12.68	A8	
1483	II	11.88	B3	
1485	II	12.26	dG0	
1778	II	12.26		} Overlap BD+30° 3857
1779	II	10.98		
1781	II	12.00	F8	
1783	II	12.68	Pec.	Two spectra ?
1823	II	12.39	F5	
1838	I	11.39	A0	
1840	I	13.13	gK5	Spectrum is intercepted
1859	I	12.99	gK0	
1862	I	12.62	dG0	
1875	II	12.26	dG2	
1889	II	12.68	dG2	

Plate M113A, 1926, Aug. 7. Center of Plate $\begin{cases} x+100' = 82'2 \\ y = 4.3 \end{cases}$
Area 64 NW

Limiting photographic $\begin{cases} \text{Zone I } 13^m.5 \\ \text{magnitudes } \text{Zone II } 12.9 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
535	II	12.53	A0	
581	II	12.53	A2	Plate M117A
609	I	12.26	gK0	
646	II	11.30	gK0	
677	I	12.99	A5	
706	II	10.68	A5	Plate M117A; BD+29° 3847
733	II	11.29	A0	Plate M117A
749	I	12.68	dG2	
778	II	12.83	A0	
779	II	11.20	A2	Plate M117A; BD+29° 3850
788	II	12.83	A0	
808	II	12.83	A1	
1048	II	12.13	F8	
1062	II	12.39	A2	
1063	I	12.99	F2	
1074	II	12.68	gK0	Faint
1103	I	12.68	A8	
1104	I	11.76	A2	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
1113	I	12.53	A0	
1114	I	13.44	F0	
1174	I	10.53	A2	BD+29° 3842
1176	I	12.15	A2	
1179	I	11.40	A5	
1186	I	11.12	A0	
1188	I	11.11	A1	
1203	I	13.09	dG5	
1210	I	12.99		Intercepted by No. 1665
1218	I	11.26	B9	
1232	I	13.16	F2	
1260	I	12.26	dK0	
1263	I	13.26	dK1	Faint
1265	I	12.83	A0	Lines are faint
1275	I	11.08	A0	Plate M111A
1280	I	13.26		Intercepted by No. 1282
1282	I	12.00	A2	
1288	I	13.16	A0	Lines are faint
1290	I	12.99	F2	
1298	II	11.48	A0	Plate M111A
1307	II	11.52	A2	Plate M111A
1320	II	12.39	A0	Plate M111A
1324	II	11.64	F5	Plate M111A
1328	II	11.88	A0	Plate M111A
1550	II	12.83		Spectra overlap
1553	II	12.83		
1645	II	11.39	gK0	BD+30° 3852
1665	I	12.74		Intercepted by No. 1210
1670	II	12.83		Intercepted by No. 1218
1680	II	12.68	A1	
1704	II	12.00	A1	

Plate M115A, 1926, Aug. 8. Center of Plate $\begin{cases} x = 0.9 \\ y + 60' = 48.2 \end{cases}$
Area 64 SE
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 12^m75 \\ \text{Zone II } 12.26 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
209	II	12.13	A2	Plate M117A
225	II	10.46	A2	Plate M117A
239	II	9.53	gK8	Plate M117A
263	I	12.53	gK0	
296	I	12.68	F2	
347	I	12.13	F8	
375	I	11.76	A0	
382	I	10.91	dK1	BD+29° 3865
383	II	11.39	gG5	Spectrum overlaps No. 382
384	I	12.39	dG0	
396	I	11.76	F5	
404	II	11.25	gK1	BD+29° 3868
834	II	8.26	gG2	HD 189943, type K0, BD+29° 3857
841	I	12.39	gK0	
867	I	11.14	gK0	
911	I	12.39	A5	
936	I	9.27	A1	BD+29° 3864
1007	II	10.60	A0	

Plate M117A, 1926, Aug. 9. Center of Plate $\begin{cases} x + 100' = 84.0 \\ y + 60 = 48.0 \end{cases}$
Area 64 SW
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 12^m9 \\ \text{Zone II } 12.5 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
34	II	12.39	F5	Spectrum is intercepted

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
64	I	12.26	F0	
87	I	12.83	F0	
95	I	11.76	A2	
102	I	12.53	gG8	
110	I	12.39	F4	Intercepted by No. 111
111	I	10.89	A2	BD+29° 3843
122	I	10.72	A1	BD+29° 3846
127	I	12.83	F8	
143	I	11.76	A2	
148	I	12.68	gK0	
149	I	11.09	gK0	Intercepted by No. 148; measured.
174	I	10.87	A5	
189	I	10.87	A2	
204	II	9.25	A2	BD+29° 3852
209	I	12.13	A2	Plate M115A
213	II	11.29	A2	
222	II	11.88	F0	
224	II	11.18	A5	
225	II	10.46	A2	Plate M115A; BD+29° 3853
239	II	9.53	gK8	Plate M115A; BD+29° 3855
543	II	12.39	dG5	
581	I	12.53	A2	Plate M113A
583	I	12.00	A0	
646	I	11.30	gK0	
706	II	10.68	A5	Plate M113A; BD+29° 3847
710	I	12.83	A0	
733	II	11.29	A0	Plate M113A
748	I	12.83	A0	
767	II	11.88	dG0	
772	I	11.88	A5	
779	II	11.20	A2	Plate M113A; BD+29° 3850
782	II	11.88		Intercepted by No. 779
790	II	12.39	A8	
809	II	12.39	gG2	Faint
826	II	12.45	F8	Faint

Plate M119A, 1926, Aug. 12. Center of Plate $\begin{cases} x = 1.2 \\ y = 6.5 \end{cases}$
Area 40 NE
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 14^m1 \\ \text{Zone II } 13.4 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
203	II	9.75	gK0	BD+44° 3596
208	II	13.20	dK0	Faint
216	I	11.46	A5	
224	II	13.14		Overlap; Plate M126B
225	II	12.19	dK1	
235	II	13.30	A3	Faint, intercepted; Plate M126B
401	II	11.43	A8	
406	II	13.14	dK0	Faint
417	I	12.19	A0	
420	I	11.71	B9	
421	I	12.19	F5	
425	I	12.97	Nb	Henry Draper classification
428	I	12.81	B6	
444	I	13.97	F8	
448	I	14.04	A0	
451	I	12.34	dG5	
466	I	13.30	A1	
470	I	13.62	dG0	
475	II	12.97	dG8	
480	II	13.14	F8	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
495	II	11.38	gG0	
679	II	12.19	B9	
683	II	12.34	A0	
686	II	13.14	F5	
721	I	14.04	dG2	Faint
729	I	13.30	B9	

Plate M121A, 1926, Aug. 13. Center of Plate $\begin{cases} x+100'=79'.6 \\ y'=11.8 \end{cases}$
 Area 40 NW
 Limiting photographic magnitudes $\begin{cases} \text{Zone I } 12^m.9 \\ \text{Zone II } 12.6 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
294	I	12.81	F8	
295	II	11.39	gK0	
300	I	8.02	A0	BD+44° 3590
306	II	12.19	dG5	
323	I	12.04	F8	
337	II	12.49	A0	
516	I	12.49	gG5	
521	I	12.04	gK2	
529	I	12.19	A1	
537	I	12.19	A3	
548	I	9.53	dG2	BD+45° 3288
553	I	12.81	dG5	
564	I	12.81	dG5	Faint
582	I	12.65	dK2	Intercepted; measured
587	I	9.19	A2	BD+45° 3290
598	I	10.95	A5	
640	II	12.19	dG5	

Plate M126B, 1926, Aug. 15. Center of Plate $\begin{cases} x=3'.7 \\ y+60'=48.0 \end{cases}$
 Area 40 SE
 Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.6 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
64	II	12.34	F5	
67	II	12.34		Intercepted by No. 197
70	II	11.05	F0	
71	II	13.14	F8	
75	II	10.90	gK0	BD+44° 3598
76	I	13.14	gG0	
81	I	13.46	dG5	
82	I	13.63	dG2	Faint
84	I	13.30	B9	
88	I	12.97	F2	
95	I	13.63	F0	Faint
96	I	12.81	F5	
101	I	13.46	dG0	
105	II	10.14	gK4	BD+44° 3602
108	I	13.30	gG5	Faint
112	I	13.46	dG8	
117	II	11.38	F5	
196	II	13.14		Intercepted by No. 67
197	II	12.19	F4	
205	II	12.34	gK0	
224	II	13.14		} Overlap; Plate M119A
225	II	12.19	dK1	
232	I	12.19	dG0	
235	II	13.30	A3	Plate M119A
248	I	13.46	A0	
254	I	12.34	dG0	
259	II	11.64	B8	
262	II	12.81	gG5	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
265	II	9.95	$\begin{cases} \text{F0} \\ \text{F2} \end{cases}$	BD+44° 3603

Plate M124B, 1926, Aug. 14. Center of Plate $\begin{cases} x+100'=80'.8 \\ y+60=46.6 \end{cases}$
 Area 40 SW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.9 \\ \text{Zone II } 13.5 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
6	I	12.34	dG8	
10	I	12.81	gK0	
25	I	12.65	A0	
38	I	13.80	dG5	
43	I	12.19	F1	
55	II	11.43	F4	
121	I	13.53	A0	Faint
122	II	12.81	dG0	
126	I	13.46		Intercepted
135	II	13.14	gG8	
142	I	10.08	F0	BD+44° 3592
145	I	13.73		Intercepted by No. 142
150	I	10.87	F1	BD+44° 3593
152	I	13.14	dG0	
175	II	12.49	dG8	Faint

Plate M136A, 1926, Sept. 8. Center of Plate $\begin{cases} x=14'.0 \\ y+10'=23.0 \end{cases}$
 Area 18 NE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.2 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
959	II	13.00	dG2	Faint
972	I	13.56	gK0	Faint
976	I	13.75	A0	
1030	II	11.78	F0	
1232	II	10.23	B8	Plate M130A; BD+60° 2243
1254	I	13.75	F1	Faint
1257	I	12.81	A0	
1258	I	13.75	A0	
1263	I	13.75	F4	Faint
1273	I	13.37	A1	
1277	I	12.26	gK0	BD+60° 2244
1284	I	11.63	gK0	BD+60° 2245
1286	I	11.22	gK0	BD+60° 2246
1309	I	11.56	gK4	BD+60° 2248
1313	I	11.79	A3	
1317	I	10.90	A3	BD+60° 2249
1324	I	13.18	F0	
1332	I	12.81	dG2	
1343	II	12.63	A2	
1347	I	13.00	A8	
1359	I	12.04	A0	

Plate M130A, 1926, Sept. 3. Center of Plate $\begin{cases} x+100'=95'.5 \\ y+10=24.0 \end{cases}$
 Area 18 NW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.3 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
747	II	11.86	dG0	
759	I	11.39	F2	
777	II	12.18	A5	
838	II	12.44	F2	
844	II	12.04	F5	
862	II	12.81	A3	Faint
1082	II	12.63		Intercepted

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
1116	II	13.18	gG0	Faint
1118	I	13.37	gG2	
1124	I	13.56	A1	
1130	I	11.07	A0	BD+60° 2238
1131	I	13.75		Intercepted
1150	I	13.00	dG2	
1154	I	13.18	dG8	Faint
1164	I	13.75	A3	
1165	I	13.18	A0	
1173	I	11.51	A0	
1175	I	13.56	dG5	Faint
1178	I	13.56	dG5	
1185	I	13.18	F5	
1189	I	13.00	gMa	Measured
1191	I	13.18	dG0	
1205	I	11.67	F0	
1209	I	12.63	F1	
1232	II	10.23	B8	Plate M136A; BD+60° 2243

Plate M132A, 1926, Sept. 4. Center of Plate $\begin{cases} x = 11.9 \\ y + 70' = 56.3 \end{cases}$
Area 18 SE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 14^m0 \\ \text{Zone II } 13.5 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
154	II	10.30	F0	Plate M134A; BD+59° 2379
171	II	11.64	dG4	Plate M134A
177	II	10.45	A1	Plate M134A; BD+59° 2382
206	I	12.44	A2	
222	I	11.85	gK2	
243	I	12.81	F1	
246	I	13.00	dK1	
259	I	13.56	dG5	Faint
265	I	13.37	dG0	
275	I	14.05	A0	
277	I	13.00	gK0	
279	I	13.37	F5	
289	I	13.75	F5	
296	I	13.75	dG2	
298	I	13.93	dG0	
299	I	13.56	F2	Double ?
313	I	13.93	A1	
330	I	12.81	gK8	Measured
341	II	12.81	gK2	
348	II	13.37	A5	Faint
349	II	12.44	A5	
374	II	8.98	B4	
606	I	13.18	A1	BD+59° 2387
607	I	13.93		} Overlap
618	I	13.37	F4	
664	II	11.79	gK1	

Plate M134A, 1926, Sept. 5. Center of Plate $\begin{cases} x + 100' = 94.9 \\ y + 70 = 59.8 \end{cases}$
Area 18 SW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m6 \\ \text{Zone II } 13.3 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
22	II	13.00	dG8	Measured
27	II	12.63	A2	
40	II	10.06	gK1	BD+59° 2374
43	I	12.03	A0	
44	II	12.26	A0	
50	I	12.06	A8	
53	I	13.18	A3	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
60	I	11.23	A0	
80	I	12.07	F0	
96	I	11.95	A2	
112	I	13.07	gK0	
154	II	10.30	F0	Plate M132A; BD+59° 2379
171	II	11.64	dG4	Plate M132A
177	II	10.46	A1	Plate M132A; BD+59° 2382
412	II	12.44	A2	} Overlap
413	II	13.00		
429	I	13.00		Intercepted by No. 43
470	I	11.63	gK1	
492	I	12.26	F8	
508	II	12.81	dK1	Measured
513	I	13.00	F4	
540	II	9.90	gK4	BD+59° 2380

Plate M138A, 1926, Sept. 10. Center of Plate $\begin{cases} x = 10.5 \\ y = 12.9 \end{cases}$
Area 19 NE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 14^m1 \\ \text{Zone II } 13.5 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
623	II	12.79	gG8	Faint; Plate M140A
625	II	13.12	A5	Faint; Plate M140A
626	II	13.22	F5	Faint
632	II	13.12	A5	Plate M140A
635	II	12.46	A2	
639	II	13.12	A1	
650	I	14.06	gG2	Faint
653	II	12.79	A2	
669	II	13.29	dK0	
674	I	13.95	dG8	Faint
677	I	12.30	A5	
685	I	13.95	F0	Faint
709	I	13.29	A2	
736	II	13.29	gK2	Faint
911	II	12.46	A2	Plate M140A
925	II	10.21	A5	BD+60° 2546
927	II	13.29	F8	Faint
929	I	12.46	A2	
933	I	12.13	dG0	
940	I	10.02	A2	BD+60° 2552
952	I	12.46	A0	
964	I	14.06	A8	
965	I	10.19	F4	BD+60° 2554
976	I	12.30	A1	
981	I	13.29	A0	
983	I	12.96	B6	
986	I	11.33	A0	
987	I	13.88	A0	
988	I	13.45	gK0	
993	I	13.39	F4	
1003	I	13.55	A0	
1008	I	11.97	F1	
1053	I	12.79	A5	
1084	II	13.22	gG8	

Plate M140A, 1926, Sept. 11. Center of Plate $\begin{cases} x + 100' = 92.2 \\ y = 13.8 \end{cases}$
Area 19 NW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 14^m0 \\ \text{Zone II } 13.2 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
515	II	12.13	gK0	
521	II	11.64	B5	BD+59° 2719

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
544	I	12.96	A5	
563	I	13.62	dK0	
567	II	12.46	gK1	Plate M145A
571	II	10.83	F1	Plate M145A; BD+59° 2723
581	II	13.12	B6	Plate M145A
584	II	12.13	A2	Plate M145A
588	I	13.95	dG0	Faint
614	II	12.63	gK1	
615	II	12.96	F8	
618	II	12.96		Faint
623	II	12.79	gG8	Plate M138A
625	II	13.12	A5	Plate M138A
632	II	13.12	A5	Plate M138A
774	II	12.46	A5	
776	II	12.96	A3	
791	II	12.63	A1	
795	I	9.77	dG0	BD+60° 2538
798	I	11.22	gK1	BD+59° 2721
801	I	13.62	F2	
803	I	12.63	A1	
807	I	12.79	A5	
814	I	13.12	F5	
831	I	11.10	A0	
832	I	13.95	dK0	
839	I	14.02	A5	
848	I	13.95	A3	
850	I	13.45	B9	
854	I	13.95	A0	
856	I	13.95	A3	
862	I	13.62	F5	
870	I	13.45	F4	Faint
871	I	13.29	F5	
879	I	13.85	A0	
880	I	14.02	gG2	
883	I	12.96	A0	
888	I	11.97	F1	
899	I	13.12	dG0	
901	I	13.12	dG0	Faint
911	II	12.46	A2	Plate M138A

Plate M142A, 1926, Sept. 12. Center of Plate $\begin{cases} x = 10.3 \\ y + 60' = 51.1 \end{cases}$
Area 19 SE

Limits of photographic magnitudes $\begin{cases} \text{Zone I } 14.0 \\ \text{Zone II } 13.6 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
130	II	12.96	F4	
144	I	12.63	A0	
145	I	13.39	dK0	Intercepted; measured
147	I	13.72	dK5	Measured
179	II	11.80	A0	
187	II	11.43	F2	BD+59° 2730
196	II	12.96	A3	
208	I	13.95	dG5	
209	II	13.29	gG8	
213	I	10.33	A0	BD+59° 2734
219	II	12.63	F1	
224	II	13.55	F8	
235	II	12.30	A2	
237	II	13.45	A0	
354	II	10.44	A0	Plate M145A; BD+59° 2727
388	II	11.27	dG0	
391	I	13.85	dK0	Faint
393	I	13.29	gK0	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
398	I	13.39	gG0	
400	I	13.29	gMb	
414	I	12.79	A0	
415	I	11.97	dK0	
424	I	12.96	F8	
428	I	11.26	gK0	BD+59° 2731
431	I	13.12	gK2	
442	II	11.64	A2	
665	II	11.97	A5	

Plate M145A, 1926, Oct. 5. Center of Plate $\begin{cases} x + 100' = 93.4 \\ y + 60 = 55.4 \end{cases}$
Area 19 SW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13.8 \\ \text{Zone II } 13.3 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
81	II	10.74	A0	BD+59° 2725
86	I	12.96	A8	
95	II	13.22	A0	
101	II	12.30	A5	
262	II	11.97	dG2	
277	II	12.96	A1	
280	I	12.79	A0	
284	I	13.72	gG0	Faint
293	I	13.55	dK5	Faint
308	I	11.97	A0	
311	I	12.63	A0	
312	I	13.62	A8	Faint
314	I	12.46	A2	
332	I	10.85	gK4	BD+59° 2724
333	I	12.30	gK0	
335	I	13.55	dG8	
345	I	14.02	A2	
347	I	13.78	A0	
354	I	10.44	A0	Plate M142A; BD+59° 2727
362	I	13.72		Intercepted by No. 621
363	I	13.78	A0	
567	II	12.46	gK1	Plate M140A
571	II	10.83	F1	Plate M140A; BD+59° 2723
581	II	13.12	B6	Faint; Plate M140A
584	II	12.13	A2	Plate M140A
606	I	11.80	gK1	
621	II	9.14	A2	BD+59° 2728
628	II	13.12	F5	

Plate M148A, 1926, Nov. 4. Center of Plate $\begin{cases} x = 6.1 \\ y + 10' = 22.3 \end{cases}$
Area 8 NE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13.6 \\ \text{Zone II } 13.1 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
580	I	11.27	A2	
583	I	12.30	F0	
599	I	13.32	A2	
604	I	11.73	F0	
607	II	12.60	F5	Plate M155A
619	II	11.73	A8	Plate M155A
622	I	12.38	F8	
625	II	12.80	dG0	Faint
629	II	11.73	A1	
639	II	12.60	gG0	Faint
718	II	11.20	F8	
724	I	11.20	A2	
731	I	13.49	A2	
734	I	12.68	F8	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
738	I	11.73	A8	
748	I	13.58	dK0	
758	I	11.54	gK1	
766	I	11.92	F0	
767	I	13.41	A8	Faint
768	I	12.00	A1	
774	I	12.60	A3	
777	I	13.49	A3	
781	II	12.11	A0	
783	I	12.95	A5	
786	I	12.11	A2	
788	II	11.39	A5	
800	II	12.68	F2	

Plate M150A, 1926, Nov. 6. Center of Plate $\begin{cases} x+100'=83'.3 \\ y+10=22.2 \end{cases}$
Area 8 NW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.1 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
470	II	10.63	gK2	BD+60° 155
472	I	12.76	dG5	
480	I	13.41	gK0	Faint
486	I	13.49	dG2	Faint
488	II	12.76	A0	
493	I	12.87	A3	
495	I	12.30	A0	
499	I	11.73	A2	
518	I	13.63	A0	
523	I	12.95	A0	
545	II	11.73	A1	Plate M156A
644	I	10.81	A1	
650	I	12.49	A0	
659	I	12.80	F8	
671	I	13.58		Intercepted by No. 499
675	I	12.87	A2	
678	I	11.23	A0	
702	II	11.13	gK1	
707	II	12.11	dG2	
711	II	12.30	A2	

Plate M155A, 1926, Dec. 27. Center of Plate $\begin{cases} x = 4'.3 \\ y+10'=4.6 \end{cases}$
Area 8 SE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.8 \\ \text{Zone II } 13.6 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
153	II	12.87	A5	
154	II	11.73	A0	
163	II	13.41	dG0	
166	I	13.72	A0	
184	I	13.41	gG0	
186	II	13.58	A5	
189	II	11.92	gK1	
190	I	13.11	F8	
237	II	12.76	F5	
380	I	10.19	A0	BD+59° 179; Plate M156A
387	I	12.68	gMa	Measured
391	I	11.45	F8	
398	I	12.30	A0	
399	I	12.45	A1	
402	I	12.91	A2	
404	I	13.58	F5	
407	I	13.07	F0	
409	I	11.04	A1	BD+59° 182

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
413	I	11.39	A3	
414	I	13.20	A2	
415	I	12.60	B5	
422	I	10.96	B3	} Overlap; BD+59° 184
423	I	13.58		
430	I	12.99	A0	
433	I	13.20	F4	
437	II	13.24	gK0	
438	I	13.80	gG0	
444	II	13.20	A0	
446	II	10.59	B3	BD+59° 186
452	II	12.80	A0	
573	II	11.92	F8	
584	I	12.87	A5	
607	II	12.60	F5	Plate M148A
619	II	11.73	A8	Plate M148A
621	II	12.41	F5	

Plate M156A, 1926, Dec. 28. Center of Plate $\begin{cases} x+100'=88'.0 \\ y+10=4.4 \end{cases}$
Area 8 SW

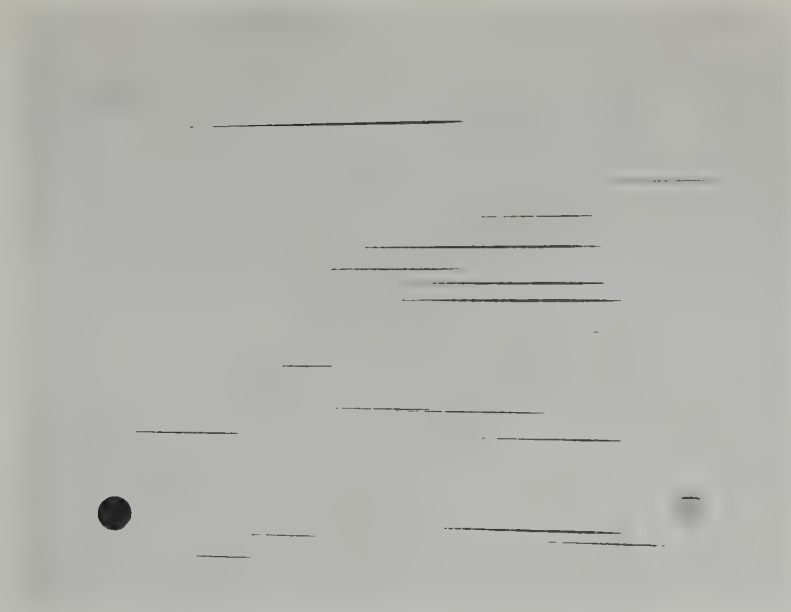
Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.7 \\ \text{Zone II } 13.4 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
11	II	10.36	F0	} Two spectra; BD+59° 171
			F1	
31	II	12.11	A0	
63	I	13.41	A0	
65	I	13.49	gK5	Faint
72	I	11.92	A0	
88	II	12.80	gG5	
91	I	11.11	F5	
94	I	10.23	A1	BD+59° 177
105	II	13.28	F0	Faint
110	I	13.63	F8	
117	II	12.30	F0	
121	II	11.04	A5	
272	II	13.15	gK0	
283	I	12.11	gK8	Measured; BD+59° 173
284	II	12.95	gK2	Faint
287	I	13.63	gG5	Faint
291	I	13.41	dK0	Faint
293	I	12.41	dK0	
312	I	13.63	gG8	Faint
326	I	12.41	A0	
333	I	13.63	A8	
352	I	13.36	F1	
353	I	12.45	A5	
356	I	13.49	gG2	Faint
380	II	10.19	A0	BD+59° 179; Plate M155A
484	II	13.32	F8	Faint
488	II	12.76	A0	
516	II	13.40	A0	Faint
539	II	13.20	A5	
545	II	11.73	A1	Plate M150A
550	II	13.40	A8	

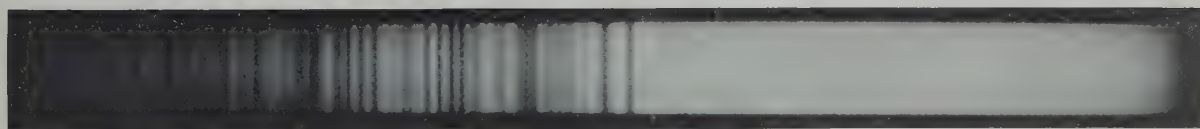
Plate M158A, 1926, Dec. 29. Center of Plate $\begin{cases} x = 5'.6 \\ y+20'=32.4 \end{cases}$
Area 9 NE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.6 \\ \text{Zone II } 12.9 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
246	I	13.47	gG5	
250	I	12.25	F2	



Two-fold Enlargment of a Plate of Selected Area SSE



Type dK0



Type gK0

Typical Giant and Dwarf Spectra of Class K0

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
252	I	12.47	A8	
253	I	8.86	A0	HD 19653, Type A0
264	I	12.80	A2	
265	II	12.80	F5	
275	II	12.80	F5	Faint
281	II	9.67	gK1	BD+60° 644
341	II	12.80	gK0	Faint
347	I	10.40	gK2	BD+60° 639
350	I	12.96	A0	
351	I	12.80	gK0	
355	I	12.96	A2	
359	I	12.09	A0	
361	I	9.93	A5	BD+60° 642
363	I	12.96	F4	} Overlap
364	I	13.30		
377	I	12.96	dG0	
379	II	12.47	dG2	Faint

Plate M160A, 1926, Dec. 30. Center of Plate $\begin{cases} x+100'=84'.0 \\ y+20'=29.2 \end{cases}$
Area 9 NW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.6 \\ \text{Zone II } 12.8 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
199	I	12.19	A5	
203	II	12.27	dG5	Faint
207	I	11.11	A0	Plate M169A
210	I	11.81	A2	
214	I	13.47	dG5	
219	I	12.80	dG2	Faint
221	II	12.63	gG5	
300	II	12.63	dG8	
320	II	8.10	A5	$\begin{cases} \text{BD}+60^\circ 633; \text{HD } 19501, \\ \text{Type A2} \end{cases}$

Plate M164A, 1927, Jan. 3. Center of Plate $\begin{cases} x = 5'.1 \\ y+20'=11.5 \end{cases}$
Area 9 SE

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.7 \\ \text{Zone II } 13.0 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
46	I	12.41	dG0	Plate M169A
57	I	12.63	A3	
128	II	11.84	A5	Plate M169A
130	II	7.09	A1	$\begin{cases} \text{BD}+60^\circ 636; \text{HD } 19536, \\ \text{Type A0; Plate M169A} \end{cases}$
139	II	11.62	F5	
145	I	9.86	A5	BD+59° 604
152	I	13.47		} Overlap
153	I	12.96	dK1	
163	I	11.91	A2	
164	I	13.65	F2	
169	II	11.09	F0	BD+60° 641
183	I	11.81	dG0	
184	II	12.80	dG8	
262	II	11.50	A4	

Plate M169A, 1927, Jan. 23. Center of Plate $\begin{cases} x+100'=88'.4 \\ y+20=11.6 \end{cases}$
Area 9 SW

Limiting photographic magnitudes $\begin{cases} \text{Zone I } 13^m.6 \\ \text{Zone II } 12.9 \end{cases}$

Gron. No.	Zone	H Pg	Sp. Type	Remarks
9	II	9.44	A0	BD+59° 597
10	I	13.47	gK0	

TABLE III—(Continued)

Gron. No.	Zone	H Pg	Sp. Type	Remarks
15	I	12.41	gK0	
16	I	12.27	A2	
23	I	10.07	gK1	BD+59° 600
30	II	11.37	F5	
31	I	13.13	dG8	
32	I	10.22	gK1	BD+59° 602
38	I	13.47	gG2	
46	II	12.41	dG0	Faint; Plate M164A
104	II	10.90	A3	
107	I	11.91	F2	
117	I	13.13	dG5	
128	II	11.84	A5	Plate M164A
130	II	7.09	A2	$\begin{cases} \text{BD}+60^\circ 636; \text{HD } 19536, \\ \text{Type A0, Plate M164A} \end{cases}$
207	II	11.11	A0	Plate M160A

Only one star, No. 425 of Area 40, was of type N. The classification is from the *Henry Draper Catalogue* in this case. The spectrum of No. 1783 of Area 64 is called "peculiar." The hydrogen lines $H\beta$, $H\gamma$ and $H\delta$ appear bright but the spectrum is too faint to warrant conclusions. The observed effect may be due to two overlapping spectra. Spectra which could not be classified because of overlapping are marked "intercepted." Such overlapping should not depend on spectrum type nor, to any great extent, on magnitude but only on the accidental position of the star in the sky. Thus we may safely omit these spectra without prejudicing our results. In one or two instances the star was omitted when too faint to classify. In these cases, however, the spectrum was much fainter than would be indicated by the photographic magnitude.

In seven instances, stars here classified were contained in the *Henry Draper Catalogue* and in twelve

TABLE IV

A COMPARISON OF THESE ESTIMATIONS WITH OTHER CLASSIFICATIONS

Area	Gron. No.	Plate Nos.	Lind.	Sp. Type HD	Est.
64	535	M113A.....	A0		A0
64	581	M113A, M117A.....	B9		A2
64	583	M117A.....	A0		A0
64	646	M113A.....	gG8		gK0
64	677	M113A.....	A0		A5
64	706	M113A, M117A.....	A2		A5
64	710	M117A.....	A0		A0
64	733	M113A, M117A.....	A0		A0
64	748	M117A.....	A0		A0
64	767	M117A.....	dG5		dG0
64	834	M115A.....		K0	gG2
64	1007	M115A.....	A0		A0
64	1104	M113A.....	dA2		A2
40	300	M121A.....		A0	A0
40	587	M121A.....		B5	A2
18	374	M132A.....		B8	B4
9	130	M164A.....		A0	A1
9	253	M158A.....		A0	A0
9	320	M160A.....		A2	A5

cases the star has been classified at Mount Wilson by Lindblad. A comparison of these with the estimations in this paper is shown in Table IV. The latter are marked "Est."

The classification in the spectral sequence, as will appear later, is probably least accurate from G0 to K0 and most accurate from A0 to A5. In order to express the accuracy in a quantitative manner, the probable error of a pair of classifications was computed from the results for the standard comparison stars. This probable error, plotted in terms of spectral type, yielded the curve of Fig. 3. Thus the maximum probable error

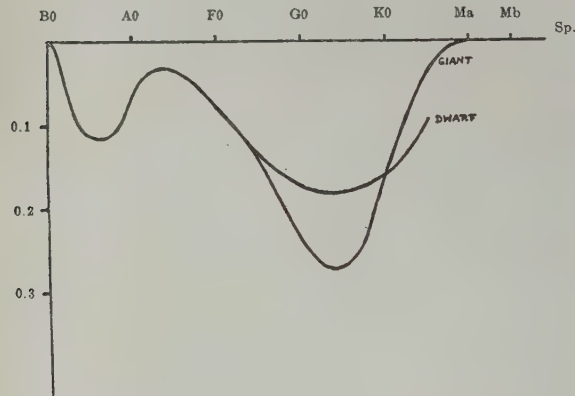


Fig. 3. The computed probable error of a pair of estimates of spectral class, plotted as a function of the spectral type. The ordinates are given in decimals of a spectral division.

is well under 0.3 of a spectral division. In the A-type region the strength of the K-line gives a sharp criterion. It was here noted, however, that stars within a single Harvard sub-group seem to differ somewhat in type. This is probably due to the difference between the scheme used in this work and the Harvard scheme of classification. The extension of the spectrum into the ultra-violet has enabled us to use criteria not available to the Harvard observers.

VI. TREATMENT OF THE MATERIAL

The total area covered by the plates included 584 stars within the limiting magnitude of the plates. Due to overlapping, the spectra of 25 stars remain unclassified. Two stars have spectra not contained in the ordinary sequence, and eight stars are below magnitude 14.0 and will not be included in the discussion. There remain 557 stars of which 538 are between magnitudes 10.00 and 14.00. The distribution in spectral types is shown in Table V.

The limiting magnitude and the area covered on each plate being known, it was possible to reduce the results to stars per square degree. For this purpose it was necessary to group the stars into tenths of a magnitude and multiply each group by a factor depend-

TABLE V

B3-B8.....	11
B9-A5.....	221
A8-F5.....	107
gG0-gG5.....	24
gG8-gK5.....	73
dF8-dG5.....	82
dG8-dK5.....	33
dK8, gMa, gMb.....	6
Total.....	557

ing on the area covered to that magnitude. Care was taken to avoid counting overlapping areas twice, and to include each star but once. When the reduced frequencies were derived they were collected into classes of an entire magnitude. Stars whose spectra, for reasons purely accidental, could not be classified, were distributed over the spectral classes in the same proportion as the observed stars of that magnitude. They merely contributed to the total number of stars and not to the relative frequencies of spectral types.

The following table gives the number of stars per square degree for each main spectral division:

TABLE VI

Sp. \ m_{ph}	10.01-11.00	11.01-12.00	12.01-13.00	13.01-14.00
B3- B8	1.41	1.49	1.68	0.64
B9- A5	9.40	25.06	49.23	62.54
A8- F5	3.29	11.83	21.91	37.12
gF8-gG5		0.94	3.93	16.57
gG8-gK5	4.23	9.48	12.98	12.57
dF8-dG5		4.31	20.98	46.13
dG8-dK5	0.94	0.47	7.73	25.39
gK8, gMa, gMb			2.24	0.70
Peculiar			1.09	
Total	19.27	53.58	121.77	201.66

In the discussion which follows we shall make the customary assumption that there is no extinction of light in space. It admits of the use of the equation connecting distance in parsecs ρ , apparent magnitude m , and absolute magnitude at ten parsecs M :

$$5 \log \rho = m - M + 5 \quad (7)$$

without further discussion. We shall compute ρ for each star according to this formula. For M we can substitute the mean absolute magnitude corresponding to a given spectral type. Several investigators have given such values, derived in various ways. It seemed best, however, for the sake of uniformity, to confine ourselves to a single set of results. The Mount Wilson spectroscopic absolute magnitudes were accordingly chosen. Since the spectral types in this paper are on the Harvard system it is necessary to obtain the absolute magnitudes corresponding to Harvard spectral types.

The relations between the spectroscopic classifications of A and B type stars of Harvard and Mount Wilson are discussed by Lindblad¹⁴ and also by Miss Fairfield.¹⁵ The latter paper contains the mean absolute magnitudes, spectroscopically determined, for every sub-type of the Harvard classification and is based on a total of 666 stars. Means of the values for "nebulous" and "sharp" line spectra were adopted here. For stars of types F-M, the absolute magnitudes of Adams and Joy¹⁶ were averaged for each sub-type of the Henry Draper classification. The line of division between the giant and dwarf groups was estimated from Curtis's diagram.¹⁷ Stars above the line were grouped with giants and those below were grouped with dwarfs. Only stars in the *Henry Draper Catalogue* were used and stars falling on the line of division into giants and dwarfs as well as stars of types not considered in this paper, were omitted. The result is a set of mean absolute visual magnitudes for each sub-type of the *Henry Draper Catalogue* within the limits of types included in our discussion. For the late types 1501 stars were used, making 2167 stars in all. A few of these, however, are probably duplicates.

In order to coördinate the results obtained in the manner just described, the mean absolute magnitudes were plotted against spectral type, according to the Hertzsprung-Russell method, and a smooth curve was drawn to give greater weights to points representing greater numbers of stars. The result is the continuous curve of Figure 4.

As the limits of completeness of our catalogue of spectral types are referred to photographic magnitudes rather than to visual magnitudes a correction of the curve for color index was necessary. The color indices adopted for the spectral types are taken from the *Henry Draper Catalogue* for types earlier than G0, and for the giants and dwarfs of type G0 and later they are taken as proportional to the values given by Seares.¹⁸ In order that a giant K0 star may have a color index +1.0, the values of Seares were multiplied by $\frac{1}{1.48}$. The curve for the Harvard photographic

magnitudes is the dotted curve of Figure 4. With the known spectral type the absolute photographic magnitude is taken from the curve and the distance in parsecs derived by equation (7), with the help of the apparent photographic magnitude. For those few stars whose spectra were found to be double, the luminosity has been divided equally between the components. This is partly justified by the fact that in

all such cases the intensities of the component spectra were found to be about equal.

A selection factor enters into the derivation of our curve, inasmuch as the stars upon which it was based are mostly brighter than magnitude 6.0. Malmquist¹⁹ has shown that if a simple exponential law of increase is assumed for the apparent magnitudes this correction is independent of the magnitude. Since the mean absolute magnitudes just derived are to be applied to stars selected with reference to magnitude, this effect on our own stars must be of second order.

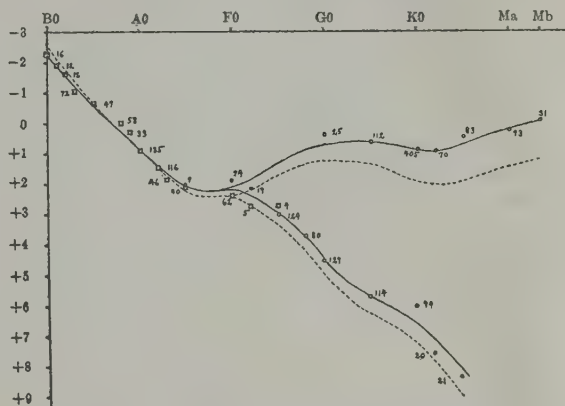


Fig. 4. Adopted Hertzsprung-Russell Diagram.

Ordinates: Absolute magnitude from the Mount Wilson spectroscopic parallaxes.

Abcissae: Harvard spectral type.

□ Mean of "n" and "s" values for the absolute magnitude given by Miss Fairfield.

○ Mean of absolute magnitudes from the Mount Wilson list of 1646 stars.

The continuous curve is on the Harvard visual system, and the dotted curve on the Harvard photographic system.

The absolute magnitudes just derived agree fairly well with Lundmark's¹ values except for the type G0 for which Lundmark adopts a higher luminosity than that given by our curve. The same remark applies to the values derived by Lindblad²⁰ but to a lesser extent. Malmquist,²¹ basing his study on much the same material as the above, and including considerations of proper motion in his discussion, agrees very well with our adopted luminosities. Luyten²² finds a value -0.8 for the absolute magnitude of a giant M-type star,—considerably brighter than any of the foregoing authorities. For the B-stars, Gerasimovic²³ finds a somewhat higher luminosity than the above. These latter types, however, are not important in the study of the observa-

¹ *Publ. A. S. P.* 34, 150, 1922.

¹⁹ "A Contribution to the Problem of Determining the Distribution in Space of the Stars," *Lunds Medd. B.* 19A, no. 106, 1925.

²⁰ "Researches based on Determination of Stellar Luminosities" (second paper), *Medd. fran. Ast. Obs. Upsala*, no. 11, 15, 1926.

²¹ *Lunds Medd. Ser. III*, 1, no. 32, 1923.

²² *Harv. Circ.* 262, 1924.

²³ *Vjs. d. Ast. Ges.* 61, 219, 1926.

¹⁴ *Astroph. Jour.* 59, 305, 1924.

¹⁵ *Harv. Circ.* no. 264, 1924; See Table V.

¹⁶ *Mt. Wilson Contr.* no. 199, 1921.

¹⁷ "Stellar Luminosities," *Publ. A. S. P.* 34, 33, 1922.

¹⁸ *Mt. Wilson Contr.* no. 226, Table XII, 1921.

tional material of this paper, as they occur very infrequently in the list of spectral types here presented. It must be admitted, however, that uncertainties in luminosities at the ends of the spectral series may have an effect on the studies derived from spectral types of brighter stars.

Errors in the distances may be introduced in two ways:

- (1) From mis-classification of the spectra.
- (2) From an actual dispersion in absolute magnitude within each spectral sub-division.

The combined effect of the two errors, however, will be taken care of if we find the probable error resulting from the dispersion within a Harvard spectral sub-type. This was done for the stars upon which our Hertzsprung-Russell diagram was based. The results plotted in terms in spectral type form the curve of Figure 5.

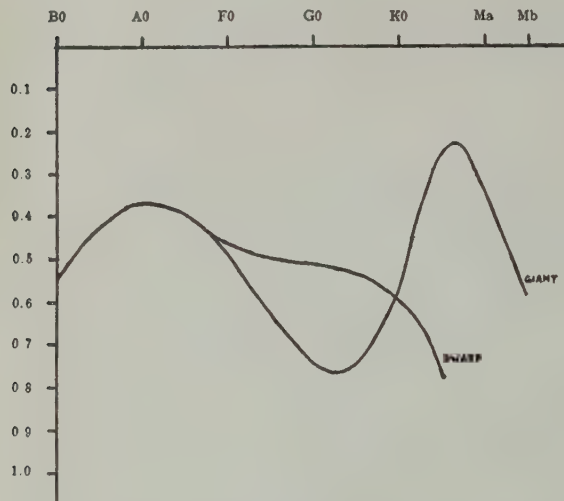


Fig. 5. Ordinates are probable errors in absolute magnitude, computed from the dispersion in absolute magnitude within each sub-type as derived from the Mount Wilson spectroscopic parallax data. Abscissae are spectral types.

The small variations in the curve of Figure 5 are probably of accidental origin. The adopted probable error was 0.45 for types B-F and the dwarf branch. A weighted solution for all giant stars together gave 0.54. These correspond to dispersions σ_m , 0.67 and 0.80 respectively. The corresponding percentage errors in the distances are 0.21 and 0.25, fairly comparable with the accuracy of the spectroscopic parallaxes themselves, which are estimated at 18 per cent for types F-M.¹⁶ Systematic errors in the spectroscopic parallaxes affect our values to the same extent, of course, as these are based directly on the latter.

¹⁶ *Mt. Wilson Contr.* no. 199, 1921.

VII. MATERIAL FROM THE HENRY DRAPER CATALOGUE AND EXTENSION

To derive the densities of high luminosity stars near the Sun, data were taken from the *Henry Draper Catalogue* and the *Henry Draper Extension*. Areas concentric with the Selected Areas were chosen in such a way that they would be free from galactic irregularities and, for convenience, would be bounded by hour circles and declination parallels. The boundaries are as follows:

TABLE VII

Central Sel. Area:	Limits in α (1900.0)		Limits in δ (1900.0)	
64	19 ^h 42 ^m 4	to 20 ^h 12 ^m 8	+26° 42' to +33° 18'	
40	20	10.7	21 22.7	+43 17 +46 43
18	20	57.7	21 50.7	+56 52 +63 28
19	22	56.6	23 49.4	+56 42 +63 18
8	0	33.3	1 26.3	+56 52 +63 28
9	2	37.1	3 30.5	+57 02 +63 38

From these areas 347 stars whose photographic magnitudes are between 5.01 and 8.00 were taken from the *Henry Draper Catalogue*. The photographic magnitudes were computed from the measured visual magnitudes and mean color index of the spectral type. When given only to tenths of a magnitude, the division at the magnitude boundaries was made on the basis of an interpolation to hundredths. The following table gives the frequencies of the spectral sub-types reduced to stars per 10,000 square degrees:

TABLE VIII

Sp. \ HPg Mag	5.01-6.00	6.01-7.00	7.01-8.00
B0.....	76	249	131
B1.....		38	38
B2.....	76	38	152
B3.....	59	397	570
B5.....		215	490
B8.....	76	380	388
B9.....	76	325	730
A0.....		574	2586
A2.....	38	304	308
A3.....	76	97	194
A5.....	38	76	228
F0.....	38	38	114
F2.....		59	55
F5.....	59	169	498
F8.....	76	38	173
G0.....			152
G5.....			401
K0.....	59	245	574
K2.....			249
K5.....	76	38	152
Ma.....	38	21	93
Mb.....			38
Pec. (O, N, etc.).....	76	38	
Total.....	937	3339	8314

TABLE IX
REDUCTION TABLE FOR gK-STARS

$m \backslash y$	13.01	13.51	14.01	14.51	15.01	15.51	16.01	16.51	Total
>10.00	-13.50	-14.00	-14.50	-15.00	-15.50	-16.00	-16.50	-17.00	
10.01-10.50	28200								28200
10.51-11.00		9400	4700						14100
11.01-11.50			42300	9400					51700
11.51-12.00				43100					43100
12.01-12.50					38800	9800			48600
12.51-13.00						64400	18700		83100
13.01-13.50							55000	48000	103000
13.51-14.00								25500	25500
>14.00									
Total	28200	9400	47000	52500	38800	74200	73700	73500	397300

For the magnitudes 8.01 to 10.00, the region covered by nos. 225301-229300 of the first and second installments of the *Henry Draper Extension*,²⁴ was used. The *Henry Draper Catalogue* and *Extension* are complete in this region to about magnitude 10.75 on the photographic scale, according to Shapley.²⁵ The actual number of stars in this aggregation is not used. Instead, the total number of stars was interpolated from the foregoing group and the observed group by assuming a normal frequency distribution for the apparent magnitudes:

$$a(m) = \frac{N}{\alpha\sqrt{2\pi}} e^{-\frac{(m-m_0)^2}{2\alpha^2}} \quad (8)$$

where N , α , and m_0 are constants and $a(m) dm$ is the number of stars per 10,000 square degrees whose apparent photographic magnitude lies between m and $m+dm$. A weighted least square solution gave:

$$\text{Log}_{10} a(m) = -.00587 m^2 + 0.534 m + 0.263 \quad (9)$$

from which the number for the interval 8.01-10.00 was computed. By thus using relative frequencies of spectral types for this region, instead of actual frequencies, we eliminate the abnormal richness of the area covered by the *Extension*.

Systematic corrections to the spectral types of the *Extension*, to reduce them to the same system as the *Catalogue*, are given by Shapley²⁵ and have been applied to the former. No changes have been made in the magnitudes. The two groups were now combined, and in what follows, they will be discussed as a unit.

VIII. DISCUSSION AND RESULTS

Method of Derivation of Density Curves for the Observed Stars

Individual distances were derived for the observed stars according to the method already described. A table was then formed for each main spectral division,

using m and $y = 5 \text{ Log}_{10} \rho = m - M + 5$ as arguments. As an example, we reproduce herewith the table for the stars of type gK (gG8-gK5). The discussion in these paragraphs refers to only one main spectral division.

The frequencies are given in stars per 10,000 square degrees. The intervals in m and y are taken as 0.5 throughout, for the observed stars. If the table were constructed for but one sub-type the numbers would all fall on one diagonal corresponding to $m-y = \text{const.}$ The inclusion of several sub-types widens this to a band.

At the lower right hand end the transfer from completeness in *magnitude* to completeness in *distance* was effected by a partial extrapolation of the numbers, when necessary. This was not necessary for giant stars, however.

If the intervals were infinitely small and if the dispersion within a sub-type were zero the sums taken from left to right would give us $a(m)dm$ and those from top to bottom $n(y)dy$, the number of stars of all apparent magnitudes whose moduli of distance (y) lie between y and $y+dy$. In order to obtain this number law $n(y)$ two corrections must therefore be applied:

- (1) Correction for dispersion.
- (2) Correction to the median value of y .

The first might be applied by distributing each frequency over a normal error curve in the horizontal direction and taking the revised sums at the bottom. It is simpler, however, to make an approximate correction on the basis of an assumed law of increase. For this purpose let $\varphi_m(M)$ be the frequency function of the absolute magnitudes within a given apparent magnitude. We shall assume this function to be a normal error function for each sub-type. The adopted dispersion is to be that found earlier in this paper, viz., 0.67 for dwarfs and types A and B, and 0.80 for giants. Thus $\varphi_m(m-y)$ is a fraction by which each number in the diagonal is to be multiplied to give its effect on a neighboring column. The number of stars between

²⁴ *H. A.* 100, nos. 1, 2, 1925.

²⁵ *Harv. Circ.* no. 278, 1925.

apparent magnitudes m and $m+dm$ and simultaneously between distances y and $y+dy$, we shall denote by $\Psi(m,y)dmdy$. The analytical expression of the latter is, then:

$$\Psi(m,y) = a(m) \varphi_m(m-y+5) \quad (10)$$

which is the relation from which the left-hand member is to be computed. Assuming for the first factor, the form

$$a(m) = ce^{\beta m} \quad (11)$$

and for the second a normal frequency distribution, it is easy to show that:

$$n(y) = \int_{-\infty}^{+\infty} \Psi(m,y) dm = e^{\frac{\sigma_m^2 \beta^2}{2}} a(M_m + y - 5) \quad (12)$$

where σ_m and M_m are dispersion and mode respectively of the frequency function $\varphi_m(M)$. Since y is originally computed from $y = m - M_m + 5$ the expression $a(M_m + y - 5)$ is the number originally tabulated, expressed in terms of y . According to (12), then, the effect of the dispersion is to multiply each of the original

tabular values by $e^{\frac{\sigma_m^2 \beta^2}{2}}$. We shall simply multiply totals of each column by this factor.

Since our intervals are finite, the sums at the bottom of the table do not give the number law $n(y)$ corresponding to the median value of y , but are actually:

$$n_{\Delta y}(y) = \int_{y-\frac{1}{2}\Delta y}^{y+\frac{1}{2}\Delta y} n(y) dy \quad (13)$$

where Δy is the interval and y in the limits is the median y . Expanding in a Taylor's series and omitting higher order terms the above reduces to:

$$n_{\Delta y}(y) = n(y) \left[1 + \frac{(\Delta y)^2}{24} \frac{d^2 n(y)}{dy^2} \right] \Delta y \quad (14)$$

Assuming Seeliger's Law of density²⁶ over the interval for the evaluation of the corrective term, we find:

$$n(y) = \frac{n_{\Delta y}(y)}{\Delta y \left[1 + \frac{\beta^2}{24} (\Delta y)^2 \right]} \quad (15)$$

It is well known that the number of stars per 10,000 square degrees whose distance lies between ρ and $\rho+d\rho$ parsecs and whose apparent magnitude is between m and $m+dm$ is given by:

$$\psi(m,\rho)dmd\rho = \left(\frac{\pi}{1.8} \right)^2 \rho^2 \Delta(\rho) \varphi(m+5-5 \log \rho) dmd\rho \quad (16)$$

where $\Delta(\rho)d\rho$ is the number of stars per cubic parsec whose distance lies between ρ and $\rho+d\rho$ divided by

²⁶ It is well known that Seeliger's Law leads directly to the exponential law of increase $a(m) = ce^{\beta m}$ where $\beta = \frac{0.2}{\log_{10} e} (3-s)$. For a demonstration, see Charlier, "The Motion and Distribution of the Stars," *Loc. Cit.*

the corresponding number in the neighborhood of the Sun, and $\varphi(M)dM$ is the number of stars per cubic parsec in the neighborhood of the Sun having absolute magnitudes M to $M+dM$. We shall also define N_0 by:

$$N_0 = \int_{-\infty}^{+\infty} \varphi(M) dM \quad (17)$$

or, in other words, the total number of stars in the neighborhood of the Sun, of all absolute magnitudes.

In place of ρ in (16), substituting

$$\rho = 10^{0.2y}$$

and $D(y) = \Delta(\rho)$, $\Psi(m,y) = \psi(m,\rho)$

the latter becomes:

$$\Psi(m,y) = 10^{0.147+0.6y} D(y) \varphi(m-y+5) \quad (18)$$

Integration with respect to m gives us the number law in terms of the density law:

$$n(y) = N_0 D(y) 10^{0.147+0.6y}$$

whence

$$N_0 D(y) = n(y) 10^{-0.147-0.6y} \quad (19)$$

Including, now, our two corrections we get, as our working equation:

$$N_0 D(y) = \frac{e^{\frac{\beta^2 \sigma_m^2}{2}} 10^{-0.147-0.6y} n_{\Delta y}(y)}{\Delta y \left[1 + \frac{\beta^2 (\Delta y)^2}{24} \right]} \quad (20)$$

Referring to equation (9) we see that β may be computed for all types together by:

$$\beta = \frac{d \log_e a(m)}{dm} = -0.027m + 1.23$$

Assuming that the mixture of spectral types does not vary greatly in going to fainter magnitudes, this value will probably be accurate enough for our corrective terms. We have, in fact, used only two values of β , one for the observed stars and another for the stars from the *Henry Draper Catalogue* and *Extension* group. The maximum value of the corrective term is about 33 per cent.

Dwarf Types.—We consider here two groups of stars classified as dwarfs, namely dG stars or stars with spectral types dF8–dG5, and dK stars or stars with spectral types dG8–dK5. Of the 82 stars in the former group 8 are outside of the limits of distance for this group:

$$10.51 < y < 14.00$$

and are therefore excluded. Similarly only 24 of the 33 dwarf K stars could be utilized. The loss of available material is a necessary consequence of the steep slope of the Hertzsprung-Russell diagram in these types.

Densities for the dwarf types were computed by formula (20) and plotted in terms of the modulus of distance, y . These densities represent nearly all of the available material for the dwarfs, except in the immediate neighborhood of the Sun. The dG stars, however, are well enough distributed to show a drop in the density occurring at about 250 parsecs. The points on the downward slope, however, are somewhat uncertain, as extrapolated values have entered with increasing influence.

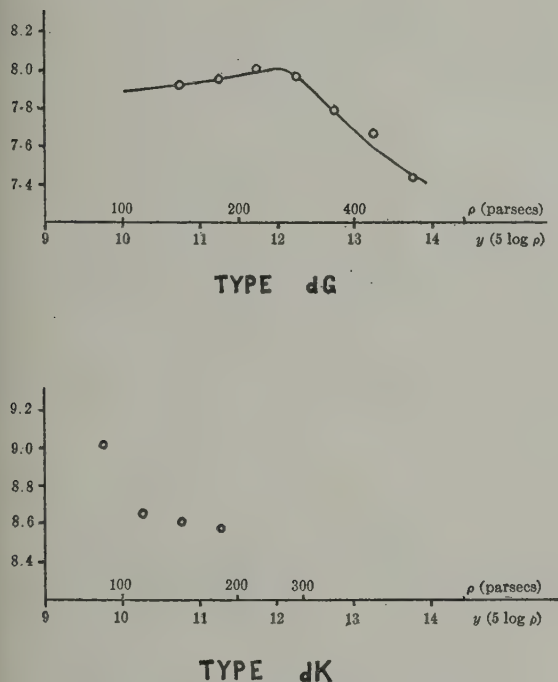


Fig. 6. Density curves for the dwarf types.

Ordinates: $\log_{10} N_0 D(y) + 10$.

Abscissae: $y = 5 \log_{10} \rho$.

The dK-stars show no effect of density on distance, the data being too meagre for this purpose. We have, however, some idea of the relative frequencies of these types in a limited region of space. Thus the region from 125 to 200 parsecs distant is common to all types but type B in this work. At the risk of some anticipation the number of stars per 100,000 cubic parsecs are given below for this region:

TABLE X

dK.....	3800
dG.....	870
A.....	63
gK.....	44
F.....	19
gG.....	15
gM.....	3
B.....	3

Thus the dK-type is by far the most frequent in space of any in this discussion. It is interesting to note, in this connection, that Seares²⁷ and van Rhijn²⁸ find evidence of a continually increasing frequency curve as the luminosity is taken at successively smaller values. In any case, the vast majority of stars are left completely out of the present discussion.

To separate the dwarfs from the giants among the brighter stars a constant density was assumed for both of the foregoing types. Under this assumption, it is known that the number of stars $A(m)$ brighter than magnitude m is expressible by:

$$A(m) = A(0) 10^{+0.6m}$$

A weighted solution for $A(0)$ gave:

$$A(0) = .00312 \text{ for the dG-type}$$

and $A(0) = .00133$ for the dK-type

At the sixth visual magnitude this would give, for the whole sky, 102 and 87 dG and dK stars, respectively. According to Malmquist²¹ there are, respectively, 91 and 39 dG and dK stars brighter than visual magnitude 6.00. In this direction, therefore, we seem to have a greater density than Malmquist has found for all directions combined. Errors in the assumed density law are not serious, unless large, as the whole number of dwarfs to be subtracted is rather small. The above formula, therefore, was retained for this purpose.

Giant Stars and Early-Type Stars.—The stars from the observed list have been treated individually in deriving the density curves. The interval in both m and y is 0.5 magnitude. For the brighter stars (i.e. stars from the *Henry Draper Catalogue and Extension*) an interval of 1.0 magnitude was used and individual distances were not derived. Care was taken, however, to deal rigorously with each sub-type in the derivation of the density. The densities of each sub-type, referring originally to different values of y , were reduced to an adopted value; (i.e., 9.50, 10.50, 11.50, etc., for A-type stars) and then added to form the densities for a whole group. These were then plotted in the same way as the data derived from the observations of the fainter stars. The resulting curves are reproduced herewith, in Figs. 7 and 8. Except for type B the curves begin as early as 65 parsecs, or less. Thus the early part of the curves are really in the solar neighborhood.

The ordinates were chosen as the common logarithms of the density and the abscissae as five times the logarithms of the distances in parsecs. On the hypotheses of Herschel and Schiaparelli we would

²¹ *Lunds Medd.* Ser. III, 1, no. 32, 1923.

²⁷ *Mt. Wilson Contr.* no. 273, 1924.

²⁸ *Gron. Publ.* no. 38, 1925.

have, on this system, a horizontal straight line; Seeliger's hypothesis would give a straight line with a negative slope, Schwarzschild's an inverted parabola and Charlier's an exponential curve.

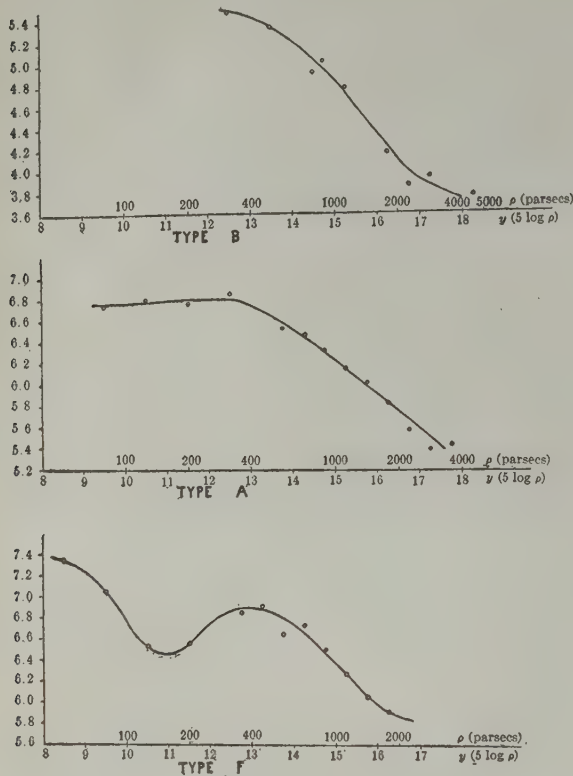


Fig. 7. Density curves for the early types.
Ordinates: $\log_{10} N_0 D(y) + 10$.
Abcissae: $y = 5 \log_{10} \rho$.

All of the curves show a drop in density at about 250 parsecs. The irregularities of the F-type curve are probably accidental. There are two reasons why this type should not be accurately represented:—the Hertzsprung-Russell diagram is steep at this point, and no subdivision into giants and dwarfs has been made. A more important reason may lie in a possible tendency of the *Henry Draper Extension* to avoid the F-type, putting the stars either into the A or the G types. The giant K stars show a more rapid drop than do the A-type stars. We shall revert to this point.

On the basis of the density curves of the different spectral types a total density curve for the types B, A, gG, gK, and gM was derived and is included herewith. Type F was omitted partly because of its uncertain nature and predominating influence, and partly because it belongs more properly with the dwarf group. An examination of the curve shows a dropping off of density of the giant stars at 200 to 400 parsecs. Pannekoek²⁹ from a study of the frequency gradient of

²⁹ B. A. N. 1, 54, 1922.

apparent magnitudes of all spectral types together in the *Cygnus* cloud, finds an agglomeration of stars at 200 to 600 parsecs. Although our curves show a falling off in density beyond this distance, they cannot be

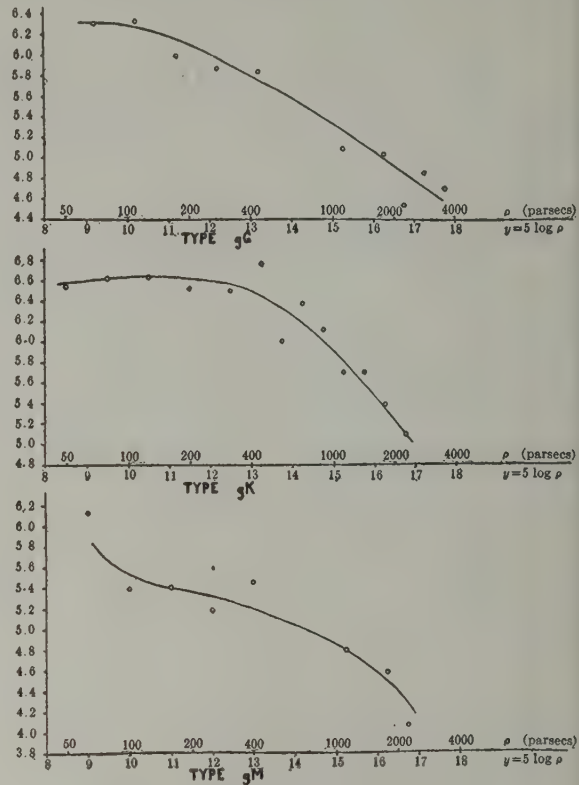


Fig. 8. Density curves for the late giants.
Ordinates: $\log_{10} N_0 D(y) + 10$.
Abcissae: $y = 5 \log_{10} \rho$.

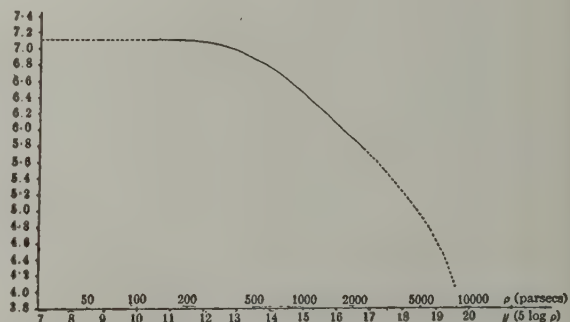


Fig. 9. Density curve of types B, A, gG, gK, and gM combined. The dotted portion is extrapolated.

Ordinates: $\log_{10} N_0 D(y) + 10$.
Abcissae: $y = 5 \log_{10} \rho$.

said to show any effect of such an agglomeration, in view of the fact that within this distance the density seems to be fairly constant. Of the local cluster hypothesis we shall speak later.

The preponderance of dwarf stars over giants is shown by Table X in which the relative numbers at 125 to 200 parsecs is given. The increase of this preponderance with spectral type is greater than that which is generally adopted. For instance, Malmquist²¹ gives for the solar neighborhood about 730 G dwarfs and 1840 K dwarfs per 100,000 cubic parsecs while the corresponding figures from our tables are 870 and 3800 respectively. It will be shown later, by deriving a provisional luminosity curve, that the same statement applies to a comparison with van Rhijn's values. As to the cause of this discrepancy we can only speculate. It may be due to a marked change in the giant-and-dwarf mixture with galactic latitude. This, however, seems unlikely in view of the fact that the mean color index of the Milky Way, as shown by Shajn,³⁰ is less than that of high galactic latitudes. The dwarfs are presumably of large color index and any great excess of them would make the effect opposite to that which is observed. A more probable explanation may lie in the supposition that dwarf stars have been very incompletely observed. This is in accord with the trend of results of recent investigators. Dwarf stars are usually picked up by their large proper motion. This is known to be a somewhat selective criterion, as stars whose cross-motion is parallel to the solar motion will be omitted. Incompleteness of material is also to be taken into account in discussions based on spectroscopic parallaxes.

The relative frequencies of giants of the different spectral types is readily obtainable from the density curves for the types and the total giant density curve. We give in Fig. 10 the curves of relative frequency in terms of the modulus of distance, y . The scale is such that the sum of all the ordinates at any point is equal to unity. Although many of the minor irregularities are probably accidental, one feature stands out so prominently that there can be little question of its reality. This is the increase of percentage of A-type stars with distance and the corresponding diminution of giant K-type stars. This was already noted in reference to the rapid drop in the gK-type density curve. The relative densities of the different types at the beginning of the curves ($y=10$) may be compared with Malmquist's²¹ values for the solar neighborhood:

Type	Malmquist	Density curves ($y=10$)
B	0.054	0.027
A	0.355	0.459
gG	0.064	0.149
gK	0.484	0.340
gM	0.043	0.025

The behavior of A and K types is in agreement with Shapley's³¹ observation that class A predominates in

low galactic latitudes and class K in high latitudes, since the results of Malmquist are without reference to galactic latitude. Another possible influence of galactic latitude, even at these small distances, is suggested by the fact that our total number for these

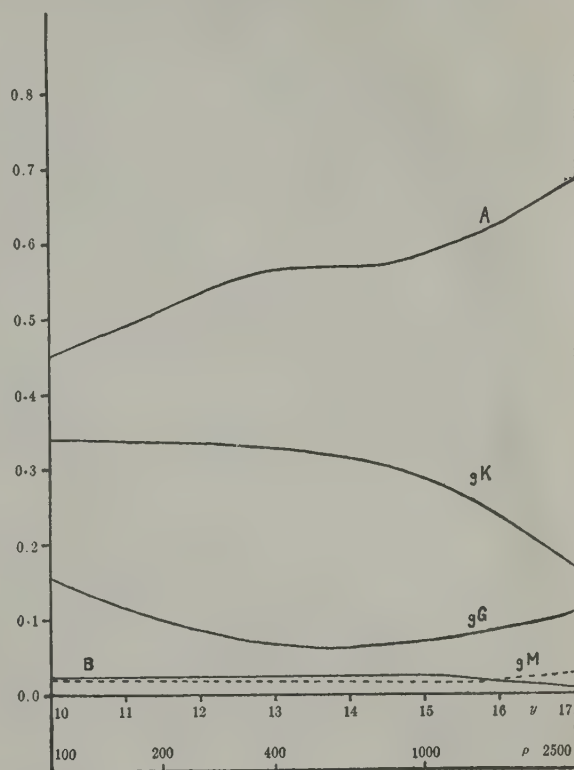


Fig. 10. Curves of Relative Densities of Types B, A, gG, gK, gM.

Ordinates: fraction of total density; abscissae: $y = 5 \log_{10} \rho$.

types is 1312 stars per 100,000 cubic parsecs, while Malmquist's value reduces to 1056 stars. That our density for giant stars is larger than that found for all galactic latitudes together is confirmed by a comparison with van Rhijn's luminosity curve to be made presently. The former is systematically larger than the latter for giant stars. Van Rhijn²⁸ seems inclined to doubt the reality of any change in the luminosity curve with galactic latitude, but leaves the matter open to question. It must be remembered, too, that the absolutely nearest stars are left out of our derivation.

IX. MINOR RESULTS

A derivation of the luminosity curve for stars from 125 to 200 parsecs does not properly belong to the major results of this paper, for several reasons. The intention was not to deal with the luminosity function

²¹ *Lunds Medd. Ser. III*, 1, no. 32, 1923.

³⁰ *Mon. Not. R. A. S.* 86, 382, 1926.

³¹ *Harv. Circ.* no. 248.

²⁸ *Gron. Publ.* no. 38, 1925.

except as a means of getting the density curves. Thus for each spectral sub-type we have really assumed a luminosity function and a solution for it would lead to our primary assumptions. We may, however, for the limited regions just mentioned, derive a luminosity curve for all spectral types together. This is equivalent to multiplying that for each spectral type by a certain coefficient depending on the density and summing over all types. These coefficients are now available for the region from 125–200 parsecs for absolute magnitudes as faint as that corresponding to dK5. This makes it of interest to perform the derivation, bearing in mind that the material is rather limited and that too much emphasis should not be placed on the smaller details of the result.

We shall now briefly describe the method of deriving the luminosity curve. Results have already been presented for the relative frequencies of different spectral types. Let us examine only the spectral sequence beginning with B0 and ending on the dwarf branch with dK5. Summing the relative frequencies along the series we obtain:

$$\int_{B0}^{Sp} F(Sp) dSp$$

where $F(Sp)$ is the frequency of a spectral type denoted by Sp . The lower limit corresponds to B0, the beginning of our spectral sequence and the upper limit to the division points between main spectral types, viz., B8, A5, F5, dG5 and dK5. This integrated function could now be plotted in terms of the spectral type and five points obtained from which a curve could be drawn. Since the Hertzsprung-Russell diagram, however, for this group, is uni-directional in its variation, the abscissae may be replaced by corresponding absolute magnitudes, whence the ordinates would become

$$\int_{-\infty}^M \varphi(M) dM$$

if we neglect the number of stars brighter than average stars of type B0. Actually the common logarithms of the above were plotted and found to lie quite close to a smooth curve. By differentiation and multiplication of the derivative by the integral itself and by $\log_{10} e$ we get at once the desired function $\varphi(M)$. For the giants a separate and similar treatment was applied. The logarithm of the resulting luminosity curve is given by the continuous curve in Fig. 11. The luminosity curve of van Rhijn²⁸ is the dotted curve in the figure. The latter was derived by interpolating a fraction 1:1.48 between van Rhijn's photographic and visual curves. This is necessary because the latter's photographic curve is on the Mount Wilson system while ours is on the Harvard photographic system.

²⁸ *Gron. Publ.* no. 38, 1925.

A comparison of the two curves brings out two facts already anticipated; namely, the derived luminosity curve is systematically larger than van Rhijn's, and the increase in the dwarf branch is enormously more pronounced. In addition to the explanation already offered, the first of these may be partially accounted for, in the giant part, by assuming a systematic error in the assumptions of the mean absolute magnitudes of the giant spectral types. It should be stated that one such correction has already been

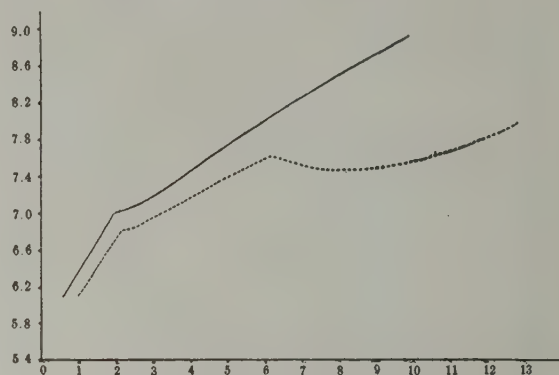


Fig. 11. The Luminosity Curve.

Ordinates: $\log_{10} \phi(M) + 10$.

Abscissae: M .

The continuous curve is derived from stars in our particular field of the Galaxy with distances from 125 to 200 parsecs. The dotted curve is that of Van Rhijn after reduction to the Harvard photographic system.

applied. The factor of selection in going from the mean absolute magnitude of a given apparent magnitude to the mean absolute magnitude in a given volume of space is, according to Malmquist,¹⁹ $\beta\sigma^2$, on the assumption of a simple exponential law of increase of the apparent magnitudes. This correction, on our assumed dispersion, is +0.68 for the giants and +0.48 for the dwarfs. It may be that this correction is slightly too large, but the discrepancy is more probably due to the fact that our curve is derived for the galactic regions, while that of van Rhijn is for all galactic latitudes together. Greater richness of the former would account for the difference.

The second discrepancy between the curves has, to some extent, already received our attention. It will here, however, play an important role in an attempt to decide on the bearing of our results on the problem of the galactic structure.

There are two prevailing theories as to this structure:

- (1) The older theory of a sidereal system of rather conservative dimensions, still re-

¹⁹ "A Contribution to the Problem of Determining the Distribution in Space of the Stars." *Lunds Medd. B.* 19A, no. 106, 1925.

tained by many astronomers. According to Seeliger, it has the form of an ellipsoid of revolution gradually diminishing in density. Seeliger considers the equatorial diameter to be about 20,000 light years³² (6100 parsecs).

- (2) The theory of a very extensive stellar system, brought about by the studies of Shapley on the globular clusters and assuming, moreover, that the globular clusters are intimately related to the stellar system. The dimensions of the system are about 15 times those of Seeliger's system. According to the proponents of this view the Sun is situated in a local cluster of stars.

The followers of the former hypothesis would interpret our density decrease as an actual and final fading away of the stellar system, in this particular direction of space. Those following the latter theory would interpret it as a falling off in density near the boundaries of the "local cluster" only, insisting that there are further condensations as we go into the supposedly denser and more distant galactic star clouds.

Assuming, for the moment, the latter idea to be the correct one, let us examine the situation in its relation to the results of other investigators. Our results for the density laws require, then, such an extent of the local cluster of giants that in the *Cygnus* direction the density does not begin to fall off appreciably until a distance of 200 to 400 parsecs is reached. The cluster derived by Charlier³³ for the B-type stars is probably identifiable with our supposed cluster. Shapley and Cannon³⁴ have found the same clustering for the A-type stars, the former³⁵ believing this B- and A-type cluster not related to the general structure of the Milky Way. The dimensions of the galactic system as given by Shapley and confirmed by Pannekoek³⁶ are many times those of the local cluster. The latter derived a distance of 40,000 parsecs for the *Cygnus* cloud but later corrected it to 18,000 parsecs.²⁹ Our density curves, therefore, would show no effect of the Milky Way as even for the B-type stars our limit is about 5000 parsecs. It is obvious that a sample, such as this, from one small region of space and so limited in extent as regards distance can produce no decisive evidence on this question.

If, on the other hand, the older theory is accepted, our limits must be almost comparable to the limits of

the galactic system. It is, therefore, natural to put the question "Is this density law of the giant stars, assuming the same law for the dwarfs, capable of representing the number of stars of very faint magnitude, which have been counted on photographic plates of long exposure?" More correctly, we want to know if it can represent the *order* of the number of stars counted. The Mount Wilson astronomers³⁷ have counted stars down to photographic magnitude 18.5 in the *Mount Wilson Catalogue of Selected Areas*. Other sources are utilized for the brighter stars. To test the above question a representation of this magnitude was attempted. The number law $n(y)$ is readily derivable from the density law. The number of stars brighter than photographic magnitude m is given by:

$$A(m) = \frac{1}{N_0} \int_{-\infty}^{+\infty} \Phi(m-y+5)n(y)dy$$

where $\Phi(M) = \int_{-\infty}^M \varphi(t)dt$

The number therefore depends on the luminosity law. At first that of van Rhijn was used and found to yield a number far too small. After the derivation of our luminosity curve it was used and the density curve was drawn, rather arbitrarily, asymptotic to the line $y=20$ as shown by the dotted part of the curve in Fig. 9. The stellar system is thus arbitrarily limited to 10,000 parsecs in this direction of space. Numerical integration, based on these two curves gave:

$$A(18.5) = 1.31 \times 10^8$$

The value from the Mount Wilson counts is:

$$A(18.5) = 1.46 \times 10^8 \quad (\text{gal. lat. } 0^\circ)$$

In this comparison we have neglected the difference between the photographic system which we are using and the Mount Wilson system. The comparison shows, however, that it is possible to represent the number of galactic stars down to 18.5 magnitude. Assuming, therefore, the validity of the luminosity law here derived and that the density diminishes gradually to zero at 10,000 parsecs, we may say that there seems to be no direct evidence that further condensations do exist.

An attempt was made, incidentally, to represent the number of twenty-first magnitude stars as extrapolated by the Mount Wilson investigators. Assuming the luminosity curve to continue to increase as rapidly as it does at the last observed point, the representation was found to fall short of the extrapolated values.

The mean color index of both galactic and non-galactic stars is known to increase with apparent magnitude. This increase is usually represented as a

²⁹ *B. A. N.* 1, 54, 1922.

³² *München Ber.* 87-144, 1920.

³³ "The Distances and Distribution in Space of Stars of Spectral Type B," *Lunds Medd.* IV, 4, no. 7, 1916. Since the writing of this paper an improved study of the B-stars by Charlier has appeared. See *Lunds Medd.* no. 34, 1926.

³⁴ *Harv. Circ.* no. 229, 1922.

³⁵ *Mt. Wilson Contr.* no. 157, 1918.

³⁶ *Mon. Not. R. A. S.* 79, 500, 1919.

³⁷ *Mt. Wilson Contr.* no. 301, 1925.

linear equation between C and m . The spectra of the stars enable us to derive such an equation under the assumption that the color index is directly given by the spectral type. Such a method is considerably improved if, moreover, our classification distinguishes between giant and dwarf stars. For the stars discussed in this paper, we have derived the mean color index of each magnitude from 5.01 to 14.00, the grouping remaining on the same system of photographic magnitudes as we have heretofore used. The following table gives the mean color index on the Mount Wilson system, as determined from the spectral type. The values of Seares¹⁸ for each spectral type are used. For magnitudes 5.01–11.00 the constant density law for dwarfs was assumed. The weights are proportional to the number of stars used.

Med. m.	C_w	Wt.
5.5	0.277	1
6.5	0.131	4
7.5	0.300	9
8.5	0.269	10
9.5	0.334	31
10.5	0.498	2
11.5	0.476	4
12.5	0.525	9
13.5	0.565	3

A least square solution gave:

$$C_w = -0.16 + 0.054 m$$

Seares¹⁸ gives the linear relation according to two groupings—visual and photographic. As, however, our photographic system is between these two Mount Wilson systems the equation with which to compare our results was interpolated from the two equations of Seares, giving:

$$C_w = +0.04 + 0.057 m \quad (\text{all gal. lat.})$$

$$C_w = +0.06 + 0.040 m \quad (\text{gal. lat. } 5^\circ)$$

It is thus seen that although there is some uncertainty in the zero point, the rate of change with m agrees very well. Shajn²⁰ gives 0.05 as the coefficient of m , both for the Milky Way and for high galactic latitudes. From the occurrence of the phenomenon among faint stars, the latter concludes that the proportion of late type giants is small in remote as well as in the nearer regions of space.

SUMMARY OF RESULTS

(1) From a classification of spectra of stars between magnitudes 10 and 14 in the *Cygnus* region of the Milky Way, together with data for the brighter

stars from the *Henry Draper Catalogue and Extension*, the density curves of the several spectral types extending to a maximum distance of 5000 parsecs, are derived for this part of space. All show a dropping off in density beginning at a distance of about 200 to 600 parsecs.

(2) The predominance of late type dwarfs over giants of the same spectral class seems to be greater than usually adopted for the nearer regions of space.

(3) The mixture of giant stars appears to vary with distance, the A-type stars showing an increase, while a corresponding relative diminution is shown by the gK-type stars.

(4) A provisional luminosity curve, derived for stars of all types between 125 and 200 parsecs distant, shows, in addition to a systematically higher density, an increase of frequency with absolute magnitude which for the intrinsically faint stars is more rapid than that given by other investigators.

(5) Although such limited observations are incompetent to make possible a decision on the problem of the structure of the galactic system, it is shown that the number of faint stars could be accounted for without exceeding dimensions of the order of 10,000 parsecs, provided that the frequency of dwarfs given by our luminosity curve is accepted.

(6) The mean color index of stars in this region increases with the apparent magnitude, the coefficient of increase being about 0.05 and therefore agreeing with results already adopted for the Milky Way as a whole.

ACKNOWLEDGMENTS

It is a pleasure to express my appreciation to Director W. W. Campbell and Associate Director R. G. Aitken for according me the use of the observatory equipment. I am indebted to Mr. W. H. Wright for instruction in the use of the Crossley reflector and quartz spectrograph, and to Dr. J. H. Moore and Dr. Aitken for kindly consenting to read this paper in manuscript form and offering friendly criticisms. My thanks are due to Mr. M. L. Humason of the Mount Wilson Observatory for useful suggestions received from him in a private letter. My deepest sense of obligation, however, I wish to express to Dr. R. J. Trumpler who suggested and outlined this problem and who has continually, with unlimited patience, offered suggestions and assisted in difficult situations, all of which required much sacrifice of time. Finally, this work was made possible by a grant of the James M. Goewey Fellowship in the University of California.

March, 1927.

Issued July 11, 1927.

¹⁸ *Mt. Wilson Contr.* no. 226, Table XII, 1921.

²⁰ *Mon. Not. R. A. S.* **86**, 382, 1926.

²¹ *Mt. Wilson Contr.* no. 287, 1925. Recently Kreiken (*Mon. Not. R. A. S.* **87**, 196, 1927) has found a somewhat larger coefficient in a few special regions near the Galaxy.

LICK OBSERVATORY BULLETIN

NUMBER 391

MERIDIAN CIRCLE OBSERVATIONS OF EROS STARS

BY

R. H. TUCKER

These observations of stars selected for comparison with the positions of *Eros*, at the next close opposition in 1930, were made at the Lick Observatory in the period November 5, 1925 to March 21, 1926. Owing to the exceptionally good winter season for observing it was possible to secure at least two observations of each star, one clamp east and one clamp west.

The observations number 1830, on 58 nights, an average of 12 nights per month. These were distributed; November, 9, December, 12, January, 21, February, 7, and March, 9. There were 419 stars in the list, and several extra stars have been included.

Eight fundamental stars, two pairs of circumpolar stars and two nadirs were observed each night. These observations, about 800 in number, form nearly forty-four per cent of the total.

The places from the system of Auwers, as tabulated in the *Berliner Jahrbuch*, were originally used for the

ascension. The second group included 135 stars, to $9^h 37^m$, and the third group included 138 stars, to $10^h 34^m$. The last hour of the list averages more than twice as many stars per hour as the first four hours, and in parts is relatively much denser than this proportion. In securing two observations throughout this last hour it was possible to observe nearly one-half of the stars four times, without extending the periods of observation. Such additional observations do not add greatly to the time required for computing. In general the most southern stars have the smaller number of observations in this hour. All the right ascensions have been corrected for my magnitude equation, the values adopted being $-0^s.016$ for stars facing north, and $-0^s.008$ for stars facing south. The Lick observations, reduced with the *B. J.* places, give the following systematic corrections to the old A. G. zone catalogues, and the average residuals for one star.

Cat.	Decl.	Mean	$\Delta\alpha$	v	$\Delta\delta$	v	Stars
Bonn	$+40^\circ+50^\circ$	45.9	$-0^s.060$	$\pm 0^s.10$	$-0^s.74$	$\pm 1^s.0$	271
Lund	$+35+40$	37.6	-0.163	± 0.07	-1.03	± 0.9	35
Leiden	$+30+35$	32.2	-0.083	± 0.08	-0.54	± 0.8	35
Cambridge	$+25+30$	27.3	-0.131	± 0.10	$+0.32$	± 0.9	23
Berlin	$+20+25$	22.0	-0.045	± 0.08	$+0.02$	± 0.8	21
Mean (5)			-0.096		-0.39		
Average (5)				± 0.09		± 0.9	

reductions. The observed places of the Eros stars have been corrected with the quantities tabulated by Dr. A. Kopff, in number 5481 of the *Astronomische Nachrichten*.

The Eros stars have been observed in three groups, the first two facing north, and the third facing south. The appropriate groups of fundamental stars are registered as A, B, and C, below. The first group included 146 stars, between $5^h 22^m$ and $7^h 30^m$, right

The same 35 stars as above, observed in the new *Lund Catalogue*, epoch 1925.0, have the following systematic differences and residuals:

Lick-Lund: $\Delta\alpha, -0^s.027$, $v, \pm 0^s.028$, $\Delta\delta, -0^s.05$, $v, \pm 0^s.38$.

The places of the new *Lund Catalogue* have about twice the weight of those of the old catalogue. The means of the proper motions, tabulated in the new catalogue for these stars are $-0^s.0018$ in right ascension, and

-0".014 in declination. The systematic corrections to the old *Lund Catalogue* are evidently largely due to proper motion.

There are a sufficiently large number of stars in the Bonn comparison for a detailed discussion, and the variation with declination has been summarized below.

DECLINATION GROUPS, BONN A.G.			
Stars	Decl.	$\Delta\alpha$	$\Delta\delta$
14	40°.3	-0.080	-0".56
9	41.4	-0.078	-1.01
9	42.5	-0.076	-1.02
19	43.5	-0.080	-0.20
34	44.5	-0.081	-0.50
48	45.5	-0.081	-0.65
52	46.5	-0.051	-0.70
66	47.4	-0.036	-0.80
20	48.2	-0.030	-1.07

The peculiarity of this comparison is the evident change in $\Delta\alpha$ at 46° declination. The latitude of Bonn is 50° 43', and the stars would all lie south of the zenith at that observatory. There is no evident systematic variation in $\Delta\delta$. These stars were all observed facing north, at Lick. Since the Lick right ascensions have been corrected for magnitude equation, the right ascension differences have been summed up below in grades of magnitude, for a test of the Bonn right ascensions. The computed magnitude equation of Bonn is +0".020, and this value has been applied in the sixth column below as the corrected $\Delta\alpha$.

Stars	Limits	Mean	$\Delta\alpha$	σ	Corr. $\Delta\alpha$	σ
41	Brighter than 8	7.6	-0.079	-0.017	-0.063	-0.001
64	8.0 to 8.4	8.2	-0.087	-0.025	-0.083	-0.021
91	8.5 to 8.9	8.7	-0.034	+0.028	-0.040	+0.022
74	9.0 and below	9.1	-0.050	+0.012	-0.064	-0.002
	Mean (4)	8.4	-0.062		-0.062	
	Average (4)			± 0.020		± 0.011

If the Bonn right ascensions are compared with those of Lick, uncorrected for magnitude equation, the resulting differential value for Bonn would be +0".036 per unit of magnitude.

The following comparisons have been made with six recent meridian catalogues, including the average residuals per star. The differences are based upon the *B. J.* places for the Lick observations.

Cat.	Stars	Decl.	$\Delta\alpha$	σ	$\Delta\delta$	σ
Bonn X.....	58	40°.8	0.000	± 0.058	-0".40	± 0.60
Bonn XVI.....	24	45.2	-0.005	± 0.076	-0.40	± 0.67
Abbadia 1915..	20	21.9	-0.017	± 0.038	+0.36	± 0.84
Green. 1910.....	37	28.3	+0.040	± 0.052	-0.12	± 0.45
Ber. Bab. IV....	50	35.9	+0.011	± 0.032	-0.04	± 0.42
Ber. Bab. V.....	11	43.6	-0.012	± 0.039	+0.27	± 0.25
	Mean (6)		+0.003		-0.05	
	Average (6)		± 0.014	± 0.049	± 0.26	± 0.54

The probable errors of observation have been computed first from the differences of pairs, one observation in each position of the instrument, clamp east and

clamp west. The mean differences, east minus west, have been included as part of the accidental errors. These mean differences were -0".017 in right ascension, and +0".22 in declination, for 281 stars observed facing north; and +0".024, and +0".08 for 138 stars observed facing south. The graduation errors are included in the declination errors, in this computation.

For the stars with four observations, the probable errors of single observations have been computed from the differences of pairs in the same position of the instrument. The errors of graduation are not included in this method of dealing with declination observations. Comparison of the two results gives $\pm 0".15$ as the average graduation error in one position.

The probable errors of one observation, including graduation errors in declination are:

Group	Stars	Decl.	R.A. Errors	Decl. Errors
A	146	46°.3	$\pm 0.028 = \pm 0.019 \text{ sec } \delta$	$\pm 0".27$
B	135	45.3	$\pm 0.035 = \pm 0.024 \text{ sec } \delta$	± 0.31
C	71	27.7	$\pm 0.033 = \pm 0.030 \text{ sec } \delta$	± 0.29
	Mean (3)		$\pm 0.024 \text{ sec } \delta$	± 0.29

The probable errors of one observation, exclusive of graduation errors in declination are:

Group	Stars	Decl.	R.A. Errors	Decl. Errors
C	67	35°.8	$\pm 0.030 = \pm 0.024 \text{ sec } \delta$	$\pm 0".25$

The resulting probable errors for the means of two observations are:

Right Ascension $\pm 0.017 \text{ sec } \delta = \pm 0".25$
Declination $\pm 0".20$, including graduation errors.

The Fundamental Stars.—The observed places and the observed corrections to the fundamental stars and the circumpolar stars are summarized below. The corrections to the fundamental stars are derived from the means of each group of eight stars, and refer to the tabulated places of the *Berliner Jahrbuch*. The right ascension corrections to the circumpolar stars have been derived with clock corrections from the corresponding groups of fundamental stars. The mean of the three groups is +0".023, which is close to the mean of the corrections to the fundamental right ascensions derived by Dr. Kopff, for 21 of these stars. His mean correction to the *B. J.* right ascensions is -0".030.

The declination corrections to the circumpolar stars have been derived here from the means of the latitude observations of the same stars. The mean of the three groups is +0".11.

All of the latitude observations have also been employed here for mean corrections to the declination system of the fundamental stars. The mean of the three systematic corrections is +0".04. This is close to the value, +0".07, which was derived by Dr. Kopff for these fundamental declinations.

The probable errors of a single observation of $\Delta\alpha$ are ± 0.025 in groups A and B, and ± 0.022 in group C.

The probable errors of an observed mean correction would be ± 0.007 for the first two groups, and ± 0.004 for the third group. These mean errors of observation must be considered in accepting the individual observed corrections to the *B. J.* right ascen-

sions, which average ± 0.016 . The probable error of a single observation of $\Delta\delta$ is $\pm 0''.21$, exclusive of graduation errors. The probable error of a mean $\Delta\delta$ is close to $\pm 0''.12$, including graduation errors, and the mean observed values of $\Delta\delta$ average $\pm 0''.22$.

B. J. FUNDAMENTAL STARS 1926.0 A							
Star	Mag.	Obs. R.A.		Corr.	Obs. Decl.		No. φ
1 Cam.	6.3	4 ^h	26 ^m	9.657	-0.019	53° 45' 6".33	+0.06 12 25.54
ε Aurig.	(3.2)	4	56	39.301	+0.005	43 42 55.98	+0.62 12 24.98
η Aurig.	3.3	5	1	19.326	-0.009	41 8 9.80	+0.22 12 25.38
μ Aurig.	5.1	5	8	21.734	+0.030	38 23 53.86	-0.38 12 25.98
26 Lyncis	5.7	7	49	19.801	-0.034	47 45 28.49	-0.10 12 25.70
27 Lyncis	4.6	8	2	53.975	-0.012	51 43 17.27	-0.28 12 25.88
31 Lyncis	4.4	8	17	46.580	-0.010	43 25 36.51	+0.09 12 25.51
Gr. 1450	6.3	8	28	6.748	+0.050	38 16 16.98	-0.25 11 25.80
Mean	4.9	6	36			44 50	25.60
Average				± 0.021			± 0.25

B. J. CIRCUMPOLAR STARS							
19 Urs. Min. L.C.	5.8	16 ^h	12 ^m	54.631	-0.015	76° 3' 52".22	-0.04 12 25.59
Gr. 848 U.C.	6.2	4	38	50.653	+0.005	75 48 34.46	+0.13 12 25.50
Br. 1147 U.C.	5.8	8	10	17.460	+0.070	75 59 7.37	+0.07 12 25.56
73 Draco. L.C.	5.3	20	32	30.293	+0.067	74 42 4.87	+0.24 12 25.87
Mean				+0.032			25.63
Correction to declinations of fundamental stars							+0.03

B. J. FUNDAMENTAL STARS 1926.0 B							
Star	Mag.	Obs. R.A.		Corr.	Obs. Decl.		No. φ
ψ5 Aurig.	5.5	6 ^h	41 ^m	24.482	-0.017	43° 39' 9".87	+0.28 14 25.02
63 Aurig.	5.0	7	6	34.128	-0.004	39 26 33.63	-0.63 14 25.93
64 Aurig.	6.0	7	12	53.724	-0.014	41 0 58.86	+0.39 14 24.91
19 Lyncis	5.5	7	16	50.230	+0.037	55 25 21.54	+0.02 14 25.28
19 Leon. Min.	5.2	9	53	9.615	+0.025	41 24 31.53	-0.14 14 25.44
λ Urs. Maj.	3.4	10	12	38.495	-0.012	43 17 4.46	+0.30 13 25.00
μ Urs. Maj.	3.0	10	17	55.671	-0.017	41 52 20.01	+0.03 9 25.26
31 Leon. Min.	4.2	10	23	36.656	+0.010	37 5 13.02	-0.13 9 25.41
Mean	4.7	8	38			42 54	25.28
Average				± 0.017			± 0.24

B. J. CIRCUMPOLAR STARS							
χ Draco. L.C.	3.6	18 ^h	22 ^m	23.549	+0.001	72° 42' 4".54	+0.46 14 25.80
23 H. Cam. U.C.	5.6	6	33	38.120	+0.010	79 38 55.44	+0.82 14 24.52
24 Ceph. L.C.	4.8	22	8	23.336	+0.023	71 58 35.30	+0.05 13 25.39
9 H. Draco. U.C.	4.9	10	28	51.175	+0.036	76 5 41.64	-0.30 9 25.64
Mean				+0.017			25.34
Correction to declinations of fundamental stars							+0.06

B. J. FUNDAMENTAL STARS 1926.0 C							
Star	Mag.	Obs. R.A.		Corr.	Obs. Decl.		No. φ
η Cancri	5.6	8 ^h	28 ^m	25.966	-0.001	20° 41' 37".33	+0.21 32 25.43
ι Cancri	4.1	8	42	13.418	-0.008	29 1 54.02	-0.03 32 25.67
σ2 Cancri	5.6	8	49	44.081	+0.015	30 51 38.41	-0.05 32 25.69
40 Lyncis	3.2	9	16	33.163	+0.007	34 42 23.19	+0.13 32 25.51
42 Leon. Min.	5.3	10	41	45.329	+0.012	31 4 20.66	-0.51 31 26.11
46 Leon. Min.	3.9	10	49	10.729	-0.012	34 36 51.23	+0.02 32 25.62
δ Leonis	2.4	11	10	10.554	+0.003	20 55 46.04	+0.26 32 25.38
ν Urs. Maj.	3.4	11	14	29.195	-0.017	33 29 53.75	-0.07 32 25.71
Mean	4.2	9	54			29 40	25.64
Average				± 0.009			± 0.16

B. J. CIRCUMPOLAR STARS 1926.0							
73 Draco. L.C.	5.3	20 ^h	32 ^m	30.222	-0.004	74° 42' 4".57	-0.06 32 25.62
d Urs. Maj. U.C.	4.5	9	27	58.260	+0.021	70 9 24.88	-0.07 27 25.75
π Ceph. L.C.	4.5	23	5	32.371	+0.034	74 59 14.25	0.00 31 25.68
λ Draco. U.C.	3.6	11	27	1.895	+0.024	69 44 22.74	0.00 32 25.68
Mean				+0.019			25.68
Correction to declinations of fundamental stars							+0.04

The Latitude.—Every observation of a fundamental and circumpolar star has been used for the computation of a value of the astronomical latitude. The results are summarized below by nights, in eight groups of nearly equal weights. Each group mean is the half sum of the latitudes from eight fundamental stars and from two pairs of circumpolar stars. Bisection corrections have been applied, the value adopted being $+0''.06$ for stars observed facing north. The corrections are eliminated in the results from group C.

MERIDIAN CIRCLE LATITUDES

Group	Clamp	Nights	Temp. F.	Epoch	φ	φ_0	E-W
A	W	6	44.3	1925.86	25.39	25.31	
B	W	7	47.3	1925.91	25.06	24.96	
A	E	6	48.5	1925.97	25.83	25.73	$+0''.42$
C	W	8	46.4	1926.02	25.57	25.48	
B	E	7	43.9	1926.03	25.48	25.39	$+0.43$
C	E	8	40.3	1926.05	25.84	25.75	$+0.27$
C	E	8	51.0	1926.15	25.74	25.68	$+0.22$
C	W	8	51.2	1926.20	25.50	25.46	
Mean (8)			46.6	1926.02	25.55	25.47	
Mean A (12)			46.4	1925.92	25.61	25.52	
Mean B (14)			45.6	1925.97	25.27	25.18	
Mean C (16)			43.4	1926.03	25.70	25.61	
Mean C (16)			51.1	1926.17	25.62	25.57	
Mean (4)					25.55	25.47	$+0.34$

The correction to the Pulkova refractions, derived from our observations of the past thirty-three years, $-0''.20 \tan Z.D.$, have been applied to all these observed zenith distances. The corrections for the variation of latitude, $\varphi - \varphi_0$, have been computed from the results at the international zenith telescope stations, during the past thirty years. The mean clamp difference, $(E-W) = +0''.34$, is in good agreement with our latitude results of recent years.¹ The probable error of an observed latitude for a single night, computed from the individual groups of the same stars, in the same position of the instrument, is $\pm 0''.19$. The probable error of the mean value of φ_0 is $\pm 0''.06$. This is larger than might be anticipated for a mean value from 58 nights, because of the existence of various systematic errors of observation and of the adopted declinations. The results of group B are exceptionally low, and the declinations of the group are probably subject to sensibly large corrections. The residual difference for this group is evidently not dependent upon epoch, nor upon the temperatures of the observing nights.

The Eros Stars.—The first column of magnitudes contains the grade assigned in the original list of stars, selected by Dr. Kopff, and published in No. 5375 of the *Astronomische Nachrichten*. The magnitude observed in the course of these observations follows. The general agreement of types is good, but some cases of stars which disagree as much as half a grade deserve comment, both for the benefit of other observers, and

because some of these stars may prove to be variables. The following list of cases gives the *B. D.* magnitude, and that from the *A. G.* catalogues, as well as the other two. The *B. D.* estimates of Argelander are generally consistent and reliable to his grade 9.4; but his grade 9.5 includes everything that he observed fainter than 9.4, and is often as faint as visual 10. With the modern meridian circle, of six inch aperture, stars of visual magnitude 9.5 can be observed with precision. The older type of meridian circle, with eight or nine inch aperture, should reach three quarters of a grade lower. Stars of tenth magnitude can be observed with this instrument, but rarely any that are fainter, and only with lack of precision.

No.	List	B.D.	A.G.	L.O.
14	8.7	8.7	9.3	9.4
26	9.0	9.0	9.0	9.6
48	9.0	8.4	9.1	9.4
56	9.2	9.0	8.8	8.6
71	9.0	9.0	8.7	8.4
213	7.8	7.8	8.1	8.5
230	8.6	8.5	7.9	7.9
233	8.6	8.5	8.6	8.0
241	9.5	9.5		10
249	9.4	9.0	9.2	9.0
252	8.7	8.5	8.2	7.8
269	9.5	9.5		10.5
271	9.6	9.2	9.1	9.1
281	9.5	9.5		10
293	9.3	9.3	9.5	9.9
308	9.5	9.0	8.8	9.0
374	9.6	9.0	9.0	8.6
382	9.3	9.3		10
390	9.6	9.0	8.8	8.5
397	9.5	9.5	9.5	10.4
399	9.2	8.8	8.5	8.5
400	9.5	9.5	9.5	10.2
408	9.6	9.0	9.0	9.0
416	9.7	9.1	9.0	8.8

The tabulated precessions have been brought forward from the *A. G.* catalogues, as a rule, and may serve as checks upon the computation of accurate values.

The *Notes* contain references to near-by stars, useful in identifying the star of the list. Some mistakes in catalogues have been noted. Proper motion has been questioned, when the differences from an *A. G.* catalogue indicate that there may be annual motions as large as $0''.007$ in right ascension, or $0''.07$ in declination. Generally no attempt has been made to compute proper motions by reference to any other authorities, as this will form a part of the final discussion of the star places, with other observations included. In a few cases modern meridian catalogues appear to confirm a sensibly large proper motion. The adoption of individual proper motions must depend largely upon exhaustive discussion of systematic differences, and of the allowable limits of accidental errors of observation.

¹ *Publ. Lick Obs.* 15, 14, 1925.

OBSERVATIONS OF EROS STARS. 1926.0

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
1	8.2	8.4	5 ^h 21 ^m 40.973	4.328	43° 35' 17.70	+3".34	25.89	
2	7.8	7.8	22 24.983	4.351	44 6 13.00	3.28	94	
3	7.7	8.2	22 33.075	4.385	44 50 58.20	3.27	90	
4	8.7	8.5	24 59.211	4.319	43 18 55.86	3.06	93	
5	8.9	8.9	25 16.616	4.380	44 40 44.66	3.03	94	P.M.(?)
6	7.6	8.0	27 12.253	4.365	44 18 23.06	2.87	89	N. fol.
7	8.6	9.0	27 18.793	4.335	43 39 51.19	2.86	90	+1' Bonn A.G.
8	8.4	8.5	27 50.664	4.406	45 11 12.26	2.81	90	
9	7.8	7.8	28 26.124	4.385	44 44 2.16	2.76	94	S. prec. Dup.
10	8.8	8.8	29 5.491	4.324	43 21 55.59	2.70	93	
11	9.0	9.0	30 6.098	4.407	45 10 59.25	2.61	94	
12	8.8	8.6	30 58.006	4.373	44 25 58.18	2.54	89	
13	8.2	7.9	31 52.938	4.337	43 36 4.46	2.46	90	
14	8.7	9.4	32 0.119	4.391	44 48 0.50	2.45	90	
15	8.9	9.0	32 39.555	4.379	44 32 15.51	2.39	94	S. fol. S. fol.
16	8.1	8.2	33 33.955	4.420	45 24 27.42	2.31	93	P.M. (?)
17	7.9	8.2	34 1.230	4.341	43 40 2.82	2.27	94	
18	9.0	9.0	34 57.155	4.370	44 18 35.89	2.19	89	
19	8.3	8.3	35 3.567	4.328	43 21 35.94	2.18	94	
20	9.1	9.1	37 10.395	4.426	45 29 36.82	2.00	90	
21	8.5	9.1	37 16.902	4.385	44 35 38.37	1.99	90	
22	8.8	8.6	39 0.884	4.376	44 23 20.41	1.84	93	P.M. (?)
23	8.2	8.4	39 22.792	4.354	43 55 11.96	1.81	89	Mistake Prec. Bonn A.G.
24	7.8	7.9	40 46.309	4.394	44 45 13.16	1.69	94	
25	8.6	9.0	41 6.322	4.374	44 18 37.53	1.66	90	
26	9.0	9.6	41 9.737	4.417	45 14 33.81	1.65	90	
27	8.0	8.5	41 33.962	4.440	45 43 47.52	1.62	94	
28	8.0	8.1	43 16.959	4.416	45 12 45.16	1.47	93	
29	9.0	9.2	44 4.979	4.373	44 15 30.17	1.40	89	
30	9.0	9.1	44 13.283	4.438	45 39 46.20	1.39	94	
31	8.7	8.6	45 31.820	4.404	44 56 10.93	1.27	90	
32	8.6	9.1	46 15.206	4.421	45 16 47.26	1.21	90	
33	7.9	7.6	47 23.592	4.379	44 21 37.59	1.11	93	
34	8.6	8.7	48 23.047	4.445	45 46 37.67	1.02	89	
35	8.6	8.3	49 17.047	4.385	44 29 15.88	0.94	94	
36	8.6	8.5	49 21.057	4.424	45 19 48.24	0.94	94	
37	9.0	9.2	50 32.157	4.455	45 58 29.00	0.83	90	
38	8.5	9.0	50 33.710	4.389	44 34 15.51	0.83	90	
39	8.2	7.8	52 32.018	4.414	45 6 5.09	0.66	94	
40	7.8	8.0	53 29.944	4.450	45 51 25.70	0.58	93	
41	8.9	8.9	55 0.806	4.436	45 32 43.00	0.44	90	N. fol.
42	8.6	8.7	55 1.179	4.399	44 46 28.56	0.44	90	
43	9.2	9.0	55 36.216	4.464	46 8 48.54	0.39	94	S. fol.
44	8.8	9.0	56 53.694	4.437	45 34 54.59	0.28	89	S. fol.
45	8.4	8.1	56 56.407	4.489	46 38 36.55	0.27	94	N. prec.
46	9.0	9.1	57 30.968	4.412	45 2 26.63	0.22	93	N. fol. P.M. (?)
47	8.7	9.0	58 1.967	4.395	44 40 44.82	0.18	94	
48	9.0	9.4	59 3.394	4.485	46 33 51.01	0.09	90	S. prec.
49	8.0	8.0	59 36.573	4.433	45 29 31.87	0.04	90	S. fol.
50	8.6	8.4	59 54.668	4.450	45 50 24.70	+0.01	94	S. fol.
51	8.9	8.5	6 0 40.043	4.464	46 8 10.94	-0.05	89	S. prec.
52	9.0	8.6	1 11.301	4.408	44 57 50.78	0.10	94	
53	8.2	8.5	2 27.948	4.491	46 40 24.54	0.21	93	
54	8.4	8.0	2 42.575	4.413	45 3 41.95	0.23	94	
55	8.8	9.0	3 14.905	4.441	45 39 15.09	0.28	90	

OBSERVATIONS OF EROS STARS. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
56	9.2	8.6	6 ^h 3 ^m 27.218	4.466	46° 9' 49".81	-0°30	25.90	
57	8.0	8.1	5 29.205	4.501	46 53 59.96	0.47	89	
58	8.6	8.6	6 50.597	4.413	45 3 54.47	0.59	93	
59	9.1	8.9	7 43.624	4.461	46 5 43.31	0.67	90	
60	9.2	8.8	8 29.457	4.485	46 34 20.02	0.74	90	
61	8.0	8.0	10 6.851	4.439	45 38 1.87	0.88	89	
62	9.0	8.9	11 8.939	4.510	47 5 43.57	0.97	94	P.M. (?)
63	8.9	9.0	11 41.980	4.425	45 21 6.21	1.02	90	
64	7.7	7.5	11 52.647	4.450	45 53 14.21	1.03	94	
65	8.4	8.7	12 26.282	4.432	45 30 34.60	1.08	90	
66	8.6	8.4	13 39.430	4.514	47 11 52.42	1.19	94	
67	9.1	8.9	14 4.359	4.467	46 16 20.76	1.22	89	
68	8.2	8.1	14 37.588	4.488	46 40 41.11	1.27	94	
69	9.0	9.0	15 32.591	4.517	47 15 29.15	1.35	93	
70	7.4	7.6	16 17.003	4.437	45 38 44.16	1.42	90	
71	9.0	8.4	16 22.256	4.486	46 39 35.32	1.42	96	
72	8.8	8.4	18 40.746	4.449	45 54 34.12	1.63	89	
73	7.8	7.5	18 57.710	4.510	47 9 36.52	1.65	90	
74	8.2	8.0	19 54.840	4.480	46 34 27.45	1.73	93	
75	8.4	8.4	20 50.447	4.493	46 50 51.88	1.81	90	
76	9.2	9.0	22 5.817	4.457	46 6 54.98	1.92	94	
77	9.0	8.6	22 9.840	4.511	47 12 16.10	1.93	94	
78	8.4	7.9	23 12.144	4.470	46 23 57.68	2.02	89	
79	8.9	8.6	23 50.553	4.441	45 48 54.74	2.08	90	
80	9.2	9.1	26 24.710	4.437	45 44 56.16	2.30	90	N. fol. Dup.
81	9.1	8.9	26 31.229	4.533	47 41 40.08	2.31	93	
82	8.0	8.0	27 41.507	4.478	46 37 45.67	2.42	89	S. fol.
83	8.0	7.9	27 46.136	4.510	47 16 22.70	2.42	94	
84	9.0	9.2	28 23.901	4.538	47 50 8.94	2.47	90	
85	9.4	8.9	29 8.106	4.453	46 8 28.87	2.54	94	
86	9.4	9.0	31 12.983	4.486	46 50 34.04	2.72	90	
87	9.1	9.1	33 26.854	4.502	47 12 50.52	2.91	89	
88	9.2	9.2	34 5.127	4.515	47 28 15.47	2.96	90	
89	9.2	9.1	35 25.845	4.462	46 25 28.80	3.08	90	S. prec. Pair
90	9.2	9.1	35 37.300	4.473	46 39 41.40	3.10	93	S. fol.
91	9.2	8.8	35 44.528	4.444	46 3 34.13	3.11	94	
92	7.3	7.5	37 5.428	4.529	47 48 31.78	3.22	94	S. fol. P.M. (?)
93	9.5	9.3	37 14.215	4.481	46 51 13.13	3.24	89	
94	8.0	8.2	38 44.175	4.525	47 45 46.30	3.37	90	
95	8.6	8.6	39 46.026	4.482	46 56 26.14	3.46	90	
96	8.4	8.3	40 43.362	4.503	47 22 10.85	3.54	93	N. fol.
97	9.0	8.9	41 34.812	4.444	46 11 21.90	3.61	89	N. fol. S. prec.
98	9.2	9.2	41 34.339	4.444	46 28 0.58	3.61	94	
99	8.0	8.0	42 22.247	4.537	48 5 8.66	3.68	94	N. fol.
100	9.0	9.0	43 52.506	4.498	47 21 3.02	3.81	90	
101	8.0	8.1	44 26.763	4.486	47 7 39.01	3.86	90	
102	7.3	7.0	44 32.568	4.445	46 16 10.47	3.87	94	-2° Bonn A.G.
103	7.6	7.5	46 18.458	4.458	46 35 14.79	4.02	89	S. fol.
104	8.0	8.0	47 16.517	4.491	47 17 41.58	4.10	94	
105	8.8	8.5	48 25.378	4.501	47 31 2.57	4.20	90	N. fol.
106	8.2	8.0	48 29.830	4.442	46 18 34.05	4.21	90	
107	9.2	9.1	48 33.922	4.537	48 13 52.49	4.21	93	N. pr.
108	9.0	9.2	50 2.512	4.517	47 52 58.92	4.34	94	N. fol.
109	7.9	7.9	51 46.786	4.489	47 22 28.71	4.49	89	
110	9.5	9.1	52 30.886	4.487	46 41 33.30	4.55	90	

OBSERVATIONS OF EROS STARS. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
111	9.0	9.1	6 ^h 55 ^m 13 ^s .084	4.518	48° 3' 18".12	-4.78	25.90	
112	9.0	9.1	55 44.924	4.503	47 46 43.26	4.82	89	
113	8.2	8.1	56 25.470	4.474	47 12 50.15	4.88	93	S. prec.
114	8.0	8.1	57 58.509	4.512	48 1 18.84	5.01	90	
115	8.8	8.9	58 9.366	4.446	46 41 21.58	5.03	94	
116	7.6	7.6	58 15.150	4.479	47 22 0.33	5.04	94	N. prec.
117	9.0	8.6	59 54.083	4.441	46 38 28.61	5.18	89	
118	8.9	9.0	7 1 54.681	4.465	47 12 46.65	5.35	90	
119	8.7	8.6	1 54.921	4.457	46 50 20.73	5.35	93	
120	8.2	8.4	2 3.141	4.499	47 54 24.20	5.36	90	
121	8.4	8.1	4 38.019	4.450	46 59 59.89	5.57	89	
122	8.8	8.9	4 53.159	4.459	47 10 44.96	5.60	94	
123	8.0	8.0	5 34.772	4.500	48 2 7.50	5.65	94	P.M. (?)
124	8.5	8.5	8 3.638	4.474	47 37 14.84	5.86	90	
125	7.1	7.5	8 7.258	4.463	47 23 30.38	5.87	90	
126	9.2	9.4	9 2.399	4.490	47 58 53.73	5.94	89	
127	8.6	8.6	10 47.530	4.505	48 20 42.87	6.09	93	
128	7.8	7.5	11 44.297	4.516	48 35 46.36	6.17	94	N. prec.
129	9.0	9.0	12 12.184	4.438	47 1 26.89	6.21	90	N. prec.
130	9.2	9.2	12 41.019	4.465	47 36 19.49	6.25	90	
131	8.0	8.5	13 7.674	4.496	48 15 25.47	6.28	89	
132	7.6	8.0	14 27.446	4.437	47 6 40.36	6.40	94	Var. (?) P.M. (?)
133	9.2	9.2	17 6.131	4.479	48 4 34.36	6.61	90	
134	9.2	8.9	17 59.204	4.432	47 9 14.04	6.69	89	P.M. (?)
135	8.6	8.6	18 20.496	4.459	47 44 1.53	6.72	90	
136	9.0	8.9	19 9.260	4.450	47 35 16.24	6.78	93	
137	7.8	7.8	20 2.012	4.419	46 57 24.64	6.86	94	P.M. (?)
138	7.8	7.9	21 39.092	4.455	47 47 39.17	6.99	90	
139	8.7	8.7	22 30.551	4.408	46 50 24.89	7.06	89	
140	7.9	8.4	22 51.214	4.473	48 12 38.92	7.09	90	
141	9.0	9.0	23 10.380	4.488	48 31 45.79	7.11	93	
142	7.8	7.8	23 58.963	4.433	47 26 34.21	7.18	94	
143	9.1	8.9	25 35.936	4.482	48 31 48.10	7.31	90	
144	8.6	8.9	27 10.022	4.420	47 18 58.40	7.44	89	
145	9.0	9.1	27 20.757	4.401	46 55 37.25	7.45	90	
146	8.5	8.5	29 28.334	4.427	47 35 13.40	7.63	93	
147	8.9	8.6	30 15.184	4.472	48 33 30.57	7.69	98	
148	9.0	9.3	31 33.637	4.392	46 57 12.30	7.79	97	
149	9.0	8.9	31 42.423	4.451	48 12 20.18	7.81	98	
150	7.6	7.6	32 49.286	4.434	47 55 7.74	7.90	94	P.M. (?)
151	8.3	8.3	33 28.261	4.401	47 14 30.18	7.95	97	
152	8.4	8.0	34 0.985	4.443	48 8 31.83	7.99	94	Pr. Mo. -1' Bonn A.G.
153	8.7	8.5	34 59.458	4.419	47 43 18.91	8.07	98	
154	8.5	8.8	35 34.772	4.377	46 50 53.94	8.12	97	N. fol.
155	8.7	8.6	35 39.538	4.461	48 37 7.94	8.12	98	
156	8.5	8.5	37 27.641	4.406	47 34 2.59	8.27	97	
157	9.2	9.4	39 46.057	4.393	47 25 10.53	8.45	98	
158	9.0	9.0	41 9.903	4.385	47 19 35.92	8.56	97	
159	7.6	7.7	41 56.814	4.391	47 31 11.52	8.62	98	
160	8.1	8.0	43 6.068	4.408	47 57 12.04	8.71	97	Pr. Mo.
161	8.9	8.8	43 27.951	4.355	46 47 55.79	8.74	26.00	N. fol. P.M. (?)
162	8.0	8.2	44 4.031	4.422	48 17 35.22	8.79	25.98	P.M. (?)
163	8.4	8.3	45 43.691	4.372	47 19 19.10	8.92	97	
164	9.2	9.2	46 55.444	4.342	46 43 37.98	9.01	98	S. prec.
165	9.2	9.4	47 8.482	4.378	47 32 35.99	9.03	97	N. prec.

OBSERVATIONS OF EROS STARS. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
166	7.8	8.0	7 ^h 49 ^m 48.752	4.407	48° 20' 33".49	-9".24	25.98	
167	8.8	8.8	50 4.273	4.375	47 40 0.74	9.26	97	N. fol. S. fol.
168	8.5	8.5	52 39.556	4.350	47 17 20.18	9.46	98	
169	8.4	8.3	52 43.063	4.321	46 38 11.40	9.46	97	S. prec.
170	8.0	8.3	54 35.841	4.376	47 59 16.20	9.61	97	
171	9.2	8.9	55 58.584	4.326	46 58 14.77	9.72	98	S. fol. S. fol.
172	9.1	9.1	56 50.397	4.360	47 47 34.60	9.78	98	S. fol.
173	9.2	9.4	56 51.623	4.380	48 14 16.12	9.78	97	
174	9.1	9.2	57 57.236	4.307	46 40 8.24	9.86	26.00	
175	9.2	9.5	58 38.751	4.332	47 18 18.68	9.92	25.98	
176	8.1	8.2	59 43.899	4.366	48 6 56.69	10.00	98	
177	9.0	9.0	8 1 15.648	4.284	46 21 50.38	10.12	98	
178	8.1	8.0	1 52.434	4.311	47 2 6.20	10.16	97	
179	9.0	8.5	2 0.930	4.350	47 55 46.95	10.17	26.00	
180	9.2	9.2	3 26.353	4.315	47 14 18.50	10.28	04	
Anon		9.8	3 48.013	4.315	47 11 7.31	10.30	25.94	
181	9.0	9.2	5 50.569	4.281	46 38 16.97	10.46	99	N. fol. S. fol.
182	7.7	7.7	6 47.981	4.300	47 9 45.35	10.53	98	
183	8.8	8.8	7 28.609	4.308	47 23 53.34	10.58	95	
184	8.7	8.5	7 44.938	4.263	46 21 46.19	10.60	97	
185	8.6	9.0	10 10.249	4.254	46 20 4.30	10.78	98	
186	8.7	8.3	10 44.538	4.314	47 47 13.25	10.82	26.00	N. prec.
187	7.0	7.0	10 54.232	4.281	47 1 44.00	10.84	25.98	
188	9.2	9.0	12 23.424	4.252	46 27 20.80	10.95	95	
189	9.1	9.2	12 27.888	4.291	47 23 21.40	10.95	98	P.M. (?)
190	8.8	8.4	13 41.686	4.308	47 54 14.67	11.04	26.00	
191	9.1	8.9	14 52.074	4.254	46 42 4.98	11.12	25.98	
192	8.0	8.1	15 43.648	4.290	47 39 9.80	11.19	98	N. prec. N. prec.
193	8.1	8.0	18 2.332	4.265	47 14 15.81	11.36	97	
194	9.1	9.2	18 46.560	4.212	46 0 45.87	11.41	94	
195	8.6	8.9	19 26.094	4.235	46 37 46.84	11.46	98	
196	7.9	7.8	19 31.125	4.267	47 24 42.88	11.46	26.00	
197	8.9	9.2	21 57.380	4.194	45 50 28.85	11.64	25.98	S. prec.
198	8.0	7.8	22 54.997	4.260	47 32 50.75	11.71	97	
199	7.0	7.4	22 56.485	4.217	46 29 37.14	11.71	94	
200	9.1	9.5	24 20.996	4.184	45 46 56.21	11.80	98	
201	8.5	8.5	25 17.588	4.193	46 6 29.94	11.87	26.00	
202	8.2	8.0	25 25.592	4.209	46 32 3.40	11.88	00	N. fol.
203	8.8	9.0	26 23.659	4.218	46 51 7.10	11.95	25.98	
204	9.3	9.4	28 26.540	4.184	46 10 19.79	12.09	98	
205	7.8	7.5	29 9.787	4.154	45 27 1.38	12.14	98	
206	8.9	8.6	30 40.175	4.207	46 57 31.30	12.25	98	
207	8.8	8.8	31 38.956	4.139	45 17 57.01	12.32	94	
208	9.0	8.9	31 56.035	4.180	46 23 19.62	12.34	26.00	S. fol.
209	9.0	9.1	32 0.579	4.160	45 53 57.58	12.34	00	
210	8.0	7.5	32 19.179	4.217	47 22 45.58	12.36	25.97	P.M. (?)
211	9.0	8.9	33 3.729	4.164	46 6 3.88	12.41	98	S. fol.
212	8.6	8.4	35 48.033	4.177	46 41 31.91	12.60	98	
213	7.8	8.5	35 54.249	4.128	45 24 43.51	12.61	95	
214	7.7	8.0	35 54.959	4.128	45 9 22.74	12.61	26.00	S. fol. N. prec.
215	9.0	8.9	36 28.720	4.195	47 13 50.41	12.65	00	P.M. (?)
216	8.9	9.2	37 21.757	4.113	45 8 51.33	12.71	25.98	N. fol.
217	8.1	8.1	38 50.548	4.128	45 43 16.75	12.81	98	P.M. (?)
218	8.2	7.9	40 17.980	4.149	46 26 28.76	12.90	98	
219	9.0	9.2	40 45.130	4.105	45 16 49.33	12.94	97	
220	9.0	8.5	41 36.604	4.086	44 50 59.36	12.99	26.02	

OBSERVATIONS OF EROS STARS. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
221	9.1	9.0	8 ^h 43 ^m 58.676	4.142	46° 39' 6".81	—13".15	25.98	
222	8.9	8.5	44 32.899	4.106	45 42 52.66	13.19	98	
223	7.6	7.9	44 43.614	4.088	45 15 10.32	13.20	94	N. prec. P.M. (?)
224	7.8	7.5	46 19.803	4.117	46 12 38.35	13.31	97	N. fol.
225	8.0	7.5	47 21.530	4.076	45 10 35.00	13.37	26.00	P.M. (?)
226	8.9	9.1	47 25.633	4.057	44 38 43.86	13.38	00	
227	9.1	9.0	48 57.823	4.083	45 33 58.85	13.48	25.98	P.M. (?)
228	8.8	8.5	50 34.656	4.090	45 56 41.29	13.58	98	
229	8.0	7.9	50 49.992	4.054	44 56 45.81	13.60	98	
230	8.6	7.9	51 31.576	4.037	44 32 4.95	13.64	97	
231	9.5	9.5	53 3.732	4.069	45 38 26.12	13.74	98	
232	8.0	7.8	54 0.801	4.033	44 42 44.94	13.80	26.00	N. prec. P.M. (?)
233	8.6	8.0	56 38.929	3.999	43 58 18.00	13.97	25.97	
234	8.9	8.6	56 57.942	4.046	45 25 41.00	13.99	98	
235	8.8	8.9	58 42.973	4.051	45 48 47.18	14.10	98	
236	9.0	9.3	59 4.331	4.026	45 6 50.27	14.12	94	
237	8.1	7.9	59 57.285	3.979	43 45 7.20	14.17	26.00	N. fol. P.M. (?)
238	8.8	8.4	9 0 31.183	3.995	44 20 50.04	14.21	00	P.M. (?)
239	9.1	9.1	2 16.840	4.009	45 0 6.34	14.31	25.98	N. fol.
240	8.6	8.5	2 23.274	4.024	45 28 28.14	14.32	97	P.M. (?)
241	9.5	10	3 50.938	3.968	43 55 10.05	14.41	98	
242	8.9	9.1	4 7.353	3.981	44 21 8.83	14.43	95	P.M. (?)
243	9.0	8.6	5 4.050	3.991	44 48 21.97	14.49	26.00	
244	8.3	8.3	5 16.477	3.942	43 14 29.21	14.50	00	
245	7.5	7.0	6 52.301	3.994	45 7 38.97	14.59	25.97	P.M. (?)
246	9.1	8.9	7 19.838	3.948	43 42 29.98	14.62	98	
247	8.3	8.0	8 29.860	3.971	44 36 57.18	14.69	98	
248	8.8	8.4	8 52.087	3.954	44 7 59.40	14.71	95	
249	9.4	9.0	10 54.410	3.959	44 33 51.99	14.83	95	
250	7.8	7.4	11 7.545	3.904	42 44 28.00	14.85	98	
251	8.7	8.5	11 15.435	3.936	43 52 3.61	14.85	26.00	
252	8.7	7.8	12 15.461	3.918	43 23 41.17	14.91	00	
253	8.3	8.4	13 18.612	3.884	42 22 35.79	14.97	25.98	N. prec.
254	9.2	9.5	13 40.171	3.953	44 44 50.26	14.99	95	
255	8.8	8.5	14 20.607	3.893	42 49 42.88	15.03	94	
256	9.6	9.3	14 38.539	3.921	43 50 29.84	15.05	97	
257	8.1	8.4	18 3.527	3.869	42 31 9.24	15.25	98	P.M. (?)
258	9.5	9.1	18 22.480	3.910	44 0 1.50	15.27	98	N. fol.
259	8.4	8.6	18 38.628	3.858	42 11 16.60	15.28	94	
260	8.2	8.0	19 30.953	3.885	43 17 53.02	15.33	97	
261	8.5	8.9	21 7.634	3.830	41 31 36.16	15.42	26.00	N. fol.
262	8.1	8.5	21 14.991	3.849	42 15 44.70	15.43	00	P.M. (?)
263	8.8	8.5	21 54.745	3.876	43 20 1.96	15.46	25.98	
264	8.7	9.0	24 24.287	3.825	41 49 22.91	15.60	97	
265	8.6	8.5	24 29.035	3.815	41 27 9.39	15.61	98	P.M. (?)
266	9.0	8.9	24 49.008	3.864	43 21 25.75	15.63	95	P.M. (?)
267	7.8	7.6	25 18.934	3.841	42 35 20.12	15.65	26.00	
268	8.9	8.7	26 19.041	3.795	40 58 7.05	15.71	25.98	P.M. (?)
269	9.5	10.5	27 14.313	3.823	42 13 38.62	15.76	26.06	
Anon		10.5	27 15.863	3.823	42 13 44.20	15.76	05	
270	8.5	8.6	27 28.180	3.834	42 39 33.52	15.77	25.94	
271	9.6	9.1	29 4.411	3.842	43 11 49.88	15.86	98	
272	8.5	8.6	29 57.651	3.799	41 43 9.42	15.90	95	
273	8.7	8.7	30 39.174	3.761	40 19 10.64	15.94	94	N. prec.
Anon		8.2	30 45.257	3.761	40 17 35.29	15.94	97	S. fol.
274	9.3	9.5	31 9.858	3.776	41 0 14.07	15.96	98	
275	9.0	9.2	31 13.088	3.809	42 19 32.83	15.97	26.00	

OBSERVATIONS OF EROS STARS. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
276	8.5	8.4	9 ^h 33 ^m 9.946	3.771	41° 7' 28".25	-16".07	25.98	
277	8.8	8.8	33 37.797	3.740	39 53 19.64	16.10	26.00	P.M. (?)
278	9.0	9.3	35 8.791	3.783	41 55 4.04	16.18	25.98	
279	8.8	8.9	35 55.014	3.767	41 24 41.96	16.21	98	
280	9.0	8.9	36 13.066	3.741	40 24 18.59	16.23	95	
281	9.5	10	36 14.814	3.732	40 0 5.50	16.23	26.00	S. prec.
282	9.2	9.3	36 46.954	3.749	40 50 0.06	16.26	08	
283	8.7	8.8	37 32.741	3.717	39 35 5.25	16.30	09	
284	8.8	9.2	37 57.150	3.753	41 11 27.20	16.32	08	S. fol. S. fol.
285	9.0	9.2	39 2.276	3.738	40 44 32.25	16.37	10	
286	8.7	9.1	39 21.820	3.714	39 45 50.81	16.39	11	P.M. (?)
287	8.7	8.7	39 44.615	3.691	38 47 53.53	16.41	11	
288	8.8	8.8	39 52.512	3.723	40 15 54.66	16.42	12	
289	8.9	9.2	40 36.670	3.749	41 29 44.33	16.45	12	S. prec. N. prec.
290	8.7	8.9	42 9.342	3.711	40 7 16.34	16.53	08	S. fol. P.M. (?)
291	9.0	9.0	42 56.636	3.693	39 26 42.67	16.57	09	
292	9.1	9.1	43 16.882	3.717	40 36 11.39	16.59	10	
293	9.3	9.9	43 45.695	3.664	38 15 43.78	16.61	09	
294	8.4	8.6	43 45.837	3.721	40 52 9.15	16.61	11	P.M. (?)
295	9.2	9.1	44 55.663	3.686	39 30 24.49	16.67	11	
296	6.8	6.9	45 41.552	3.693	39 58 37.28	16.70	12	
297	8.4	8.6	46 45.815	3.701	40 32 39.87	16.75	12	
298	9.0	9.1	46 50.337	3.646	37 57 3.95	16.76	08	
299	8.8	8.9	47 5.188	3.662	38 47 4.50	16.76	09	
300	8.8	9.1	47 10.815	3.672	39 16 14.97	16.77	10	
301	9.0	9.3	48 23.233	3.683	40 0 37.61	16.83	10	
302	8.6	8.6	49 51.525	3.661	39 15 46.43	16.90	11	
303	8.9	9.1	50 0.303	3.631	37 49 48.98	16.91	11	N. prec. P.M. (?)
304	8.7	8.7	50 13.078	3.664	39 28 10.91	16.92	12	
305	9.0	9.0	50 51.578	3.637	38 17 40.84	16.95	12	
306	8.7	8.7	51 19.018	3.620	37 32 47.96	16.97	26.08	P.M. (?)
307	8.7	8.7	52 21.507	3.608	37 7 47.63	17.02	09	
308	9.5	9.0	52 36.567	3.633	38 24 23.40	17.03	10	
309	9.2	9.1	52 52.336	3.597	36 35 21.56	17.04	10	
310	8.2	8.0	53 39.376	3.646	39 17 21.52	17.08	11	
311	9.1	8.8	54 31.705	3.607	37 29 37.13	17.12	11	P.M. (?)
312	9.2	9.1	54 51.529	3.618	38 7 36.98	17.13	12	
313	9.0	8.7	55 15.828	3.633	38 57 56.39	17.15	12	
314	9.2	9.2	56 12.811	3.576	36 9 59.19	17.20	08	
315	9.2	9.2	56 54.784	3.598	37 28 45.02	17.23	09	
316	9.2	9.3	56 59.962	3.567	35 51 17.45	17.23	10	
317	7.9	8.3	57 8.597	3.585	36 52 5.25	17.24	10	
318	8.7	8.6	57 30.001	3.607	38 6 1.43	17.25	11	
319	8.7	8.6	58 41.538	3.586	37 12 34.47	17.31	11	
320	9.3	9.3	59 23.020	3.595	37 51 47.50	17.34	12	
321	9.1	9.0	59 38.755	3.567	36 22 28.34	17.35	12	N. fol. S. fol.
322	8.7	8.6	59 58.281	3.539	34 51 24.14	17.36	05	P.M. (?)
323	8.3	8.5	10 0 12.571	3.549	35 27 45.37	17.37	11	
324	9.3	9.5	0 51.104	3.556	36 0 12.21	17.40	07	P.M. (?)
325	8.4	8.5	1 32.308	3.528	34 33 43.23	17.43	10	S. fol.
326	9.3	9.0	1 41.558	3.581	37 33 14.87	17.44	11	N. fol. N. fol.
327	9.0	8.9	1 59.318	3.540	35 18 26.44	17.45	11	S. fol.
328	9.0	9.0	2 11.424	3.515	33 53 34.43	17.46	11	S. prec.
329	9.0	9.1	3 55.825	3.539	35 41 55.41	17.53	12	
330	9.3	8.8	4 1.245	3.556	36 38 19.78	17.54	12	

OBSERVATIONS OF EROS STARS. 1926.0—*Continued*

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
331	7.6	7.5	10 ^h 4 ^m 23.868	3.519	34° 36' 22".17	—17".55	26.11	
332	8.7	8.6	4 50.824	3.501	33 33 52.10	17.57	06	P.M. (?)
333	8.4	8.4	5 22.996	3.492	33 9 13.82	17.59	09	
334	9.1	8.9	5 42.034	3.541	36 10 24.77	17.60	10	
335	8.4	8.5	6 3.871	3.511	34 29 41.87	17.62	10	
336	9.1	9.1	6 45.240	3.521	35 12 40.78	17.65	11	
337	8.0	8.0	7 6.054	3.477	32 33 29.49	17.67	11	P.M. (?)
338	8.4	8.5	7 31.948	3.490	33 29 24.23	17.68	12	
339	9.3	9.1	8 14.400	3.522	35 35 17.64	17.71	12	
340	8.5	8.7	8 23.218	3.499	34 14 28.77	17.72	11	
341	8.0	8.1	9 0.146	3.458	31 44 40.63	17.74	05	N. fol.
342	8.3	8.5	9 51.430	3.458	31 55 49.85	17.78	11	N. fol.
343	9.1	9.2	9 58.454	3.477	33 11 31.67	17.78	07	
344	8.6	8.7	10 35.569	3.489	34 6 36.33	17.81	08	
345	8.8	8.5	10 39.211	3.447	31 21 33.23	17.81	08	
346	9.0	8.5	10 44.608	3.459	32 9 26.50	17.81	11	
347	8.6	8.6	11 52.296	3.486	34 9 47.39	17.86	11	
348	8.9	8.9	11 54.488	3.462	32 38 6.52	17.86	12	
349	8.5	8.6	11 54.859	3.495	34 46 11.86	17.86	12	
350	8.4	8.3	12 3.542	3.475	33 31 25.38	17.87	11	
351	8.9	8.4	12 44.927	3.422	30 3 15.03	17.89	05	
352	9.3	9.0	13 1.169	3.462	32 53 56.01	17.90	11	
353	7.9	7.8	13 21.739	3.437	31 15 25.35	17.92	12	
354	8.9	8.8	13 47.108	3.430	30 47 57.17	17.93	07	
355	9.3	9.0	14 30.800	3.412	29 43 29.86	17.96	08	
356	8.5	9.0	14 50.424	3.464	33 24 5.65	17.98	08	
357	9.1	9.4	15 0.445	3.419	30 20 10.01	17.98	09	
358	9.1	8.8	15 28.249	3.447	32 23 13.73	18.00	11	
359	8.9	8.4	15 51.794	3.399	28 48 43.25	18.02	12	
360	9.1	8.5	15 55.880	3.407	29 37 53.90	18.02	12	S. fol.
361	8.8	8.9	16 27.293	3.435	31 48 2.43	18.04	11	
362	9.0	9.0	16 35.938	3.411	30 3 16.99	18.04	11	N. fol.
363	7.3	7.5	16 38.712	3.424	31 2 8.61	18.05	05	
364	8.7	8.7	16 55.801	3.416	30 32 16.72	18.06	11	
365	9.3	9.4	17 2.123	3.443	32 31 29.38	18.06	12	
366	9.3	8.9	17 16.521	3.386	28 22 35.03	18.07	12	
367	8.1	8.0	17 18.689	3.393	28 54 35.53	18.07	07	
368	8.9	8.3	18 6.476	3.431	31 55 3.69	18.10	08	
369	9.0	8.6	18 26.266	3.397	29 25 11.19	18.12	08	
370	7.6	7.3	18 49.280	3.419	31 12 41.49	18.13	09	
371	8.5	8.4	19 17.844	3.405	30 13 25.50	18.15	11	
372	9.1	9.0	19 49.416	3.367	27 25 55.72	18.16	12	
373	8.7	8.5	19 51.580	3.407	30 34 39.12	18.17	12	
374	9.6	8.6	19 58.392	3.358	26 43 32.85	18.17	11	N. fol.
375	9.4	9.4	20 20.558	3.385	28 58 10.28	18.18	11	
376	9.5	9.9	20 45.556	3.375	28 14 56.13	18.20	09	
377	7.8	7.9	20 49.692	3.407	30 44 58.67	18.20	05	P.M. (?)
378	8.8	8.6	21 35.207	3.340	25 35 21.33	18.23	07	
379	7.3	7.2	22 9.258	3.332	25 5 30.93	18.25	08	
380	8.6	8.5	22 14.765	3.344	26 7 15.65	18.25	08	
381	8.8	8.3	22 32.238	3.353	26 54 7.25	18.26	09	P.M. (?)
382	9.3	10	22 37.037	3.384	29 26 38.69	18.26	09	
383	7.8	7.5	22 49.592	3.391	30 3 9.65	18.27	09	S. fol. P.M. (?)
384	8.9	8.6	23 8.464	3.368	28 14 56.07	18.28	12	
385	9.0	9.1	23 14.614	3.318	24 4 41.25	18.29	13	

OBSERVATIONS OF EROS STARS. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
386	9.0	8.8	10 ^h 23 ^m 35.886	3.309	23° 19' 53".39	—18".30	26.09	
387	8.4	8.1	23 47.855	3.358	27 35 23.83	18.31	10	P.M. (?)
388	8.9	8.5	24 5.474	3.372	28 45 57.88	18.32	11	P.M. (?)
389	8.8	8.6	24 13.109	3.329	25 16 4.56	18.32	11	Pr. Mo.
390	9.6	8.5	24 59.698	3.339	26 16 37.70	18.35	05	
391	9.1	9.2	25 5.411	3.320	24 38 37.84	18.35	07	
392	9.0	8.8	25 13.666	3.362	28 14 38.57	18.36	14	N. prec.
393	9.3	9.1	25 17.509	3.306	23 26 26.96	18.36	14	
394	9.1	9.2	25 23.321	3.292	22 11 29.83	18.37	11	
395	8.2	8.0	25 31.148	3.316	24 21 0.41	18.37	12	
396	9.0	9.0	26 5.571	3.276	20 50 26.97	18.39	08	
397	9.5	10.4	26 7.464	3.342	26 44 46.07	18.39	05	
398	8.2	8.1	26 14.040	3.353	27 42 56.30	18.39	09	
399	9.2	8.5	26 14.714	3.282	21 27 40.89	18.39	12	P.M. (?)
400	9.5	10.2	26	3.328	25 36 59.77	18.40	07	2 obs. Decl.
Anon.		10	27 27.87	3.33	25 34 26.3	18.4	07	1 obs. S. fol.
401	8.7	8.5	27 5.430	3.293	22 36 15.03	18.42	13	
402	7.9	7.9	27 40.941	3.312	24 27 48.57	18.44	14	N. fol. S. fol.
403	7.9	7.5	27 52.792	3.318	25 6 14.15	18.45	14	
404	8.8	8.9	28 8.371	3.295	23 2 33.29	18.46	09	
405	8.2	8.0	28 17.918	3.302	23 41 58.27	18.47	10	
406	8.4	8.4	28 37.877	3.334	26 40 10.49	18.48	11	P.M. (?)
407	8.9	8.9	28 39.158	3.270	20 44 13.68	18.48	11	
408	9.6	9.0	28 50.406	3.265	20 22 10.51	18.48	05	
409	8.8	8.4	29 4.608	3.261	19 56 51.43	18.49	11	
410	7.7	7.6	29 11.815	3.319	25 30 23.39	18.50	07	
411	8.9	8.5	29 38.131	3.272	21 7 53.83	18.51	08	
412	9.0	9.0	29 40.518	3.280	21 57 0.83	18.51	08	
413	8.9	8.5	29 52.350	3.293	23 12 16.60	18.52	09	S. fol.
414	8.9	8.5	30 49.685	3.279	22 5 34.57	18.55	09	
415	8.9	9.0	31 10.429	3.297	23 55 38.73	18.56	12	
416	9.7	8.8	31 28.617	3.255	19 52 50.70	18.57	13	
417	8.8	8.5	32 40.561	3.274	21 59 21.15	18.61	11	P.M. (?)
418	8.9	8.8	32 57.056	3.267	21 19 55.61	18.62	08	
419	8.4	8.2	10 33 48.053	3.255	20 21 40.05	18.65	09	

PALO ALTO, July, 1927.

Issued September 6, 1927.

LICK OBSERVATORY BULLETIN

NUMBER 392

MERIDIAN CIRCLE OBSERVATIONS OF EROS STARS

SECOND LIST

BY

R. H. TUCKER

These observations of comparison stars for positions of the asteroid *Eros* were made on eighteen nights, beginning March 23, and ending May 2, 1926. The second list contains 402 stars, and 260 of these have been observed once, clamp west, or about sixty-five per cent of the list. Ten stars were observed twice. All the stars of the list could have been observed once, in about twenty nights, if the distribution in right ascension had been uniform. Actually more than double that many nights would be required to observe every star in the most condensed portion.

All of the stars of the first list had been observed twice before the observations of the second list were begun. The observations of this list number 530, and for both lists the total is close to 2100 observations. The observing season was the best encountered in more than thirty years at Mount Hamilton, 77 nights having been utilized, in six months, including all of the worst months in the year.

Had the second list been received early enough to combine the observations with those of the last hour of the first list, it might have been feasible, in this exceptionally good winter season, to secure one observation of all stars in both lists, and also a second observation of a large proportion of the stars. There are 525 stars in one full hour of right ascension, in the two lists, all south of the zenith. Such a number of stars could have been observed once, in about thirty nights, if uniformly distributed. Fully double that number of nights would have been required, with the actual distribution.

The transits of the *Eros* stars have been corrected for my magnitude equation, $-0^{\circ}008$ per unit of magnitude,

for stars south of the zenith. The mean of the applied corrections is $-0^{\circ}034$, for the stars of average magnitude 8.73. My estimate of magnitude is given below, as well as that of the published list. A few stars are faint for precise observation with the modern type of meridian instrument of six-inch aperture. The precessions have been brought forward from catalogues of reference, as a rule. They will serve as checks upon the computation of precise precessions. The positions have been given with three places of decimals in right ascension, and two in declination, to ensure precision in the final significant figure, following the application of precessions and any systematic corrections that may be derived. Proper motion has been questioned, in the Notes below, when the observed difference from some early catalogue gave a residual much in excess of the average.

The probable errors of single observations of the fundamental stars in this series are $\pm 0^{\circ}029$ in right ascension, and $\pm 0^{\circ}28$ in declination, not including graduation errors. The number of pairs of observations of the *Eros* stars is not large enough for a reliable discussion of probable error, and the figures from the numerous comparisons of the first list may be accepted for this. These figures were $\pm 0^{\circ}024$ sec δ in right ascension, and $\pm 0^{\circ}29$ in declination, including graduation errors.

The Fundamental Stars.—Corrections to the places of the *Berliner Jahrbuch* have been derived from this series of observations. The original observed corrections to the ephemeris right ascensions, (bracketed below), apparently include the systematic correction to transits observed near sunset, which has been dis-

cussed in detail in our Bulletins.¹ The observed difference between the two groups, of five stars each, is $+0.064$. Part of this difference may be due to systematic errors of the *B. J.* right ascensions. One effect of such a difference is to give an excess of the computed clock rates, during periods of observation, above the daily rates of the clock, derived from the same observations. This excess is -0.030 per hour, in this series.

It is possible to evaluate the actual systematic errors in the ephemeris right ascensions of groups of clock stars, by carrying a series of observations through a full year, and deriving the errors of each group after closing the cycle of groups. Such a series was taken at the Lick Observatory during the years 1921 and 1922, and the discussion of the complete cycle is given in our recent meridian circle volume.²

It does not seem to be reasonable to account for this systematic difference by assuming larger rates in the azimuth of the instrument. Rates in azimuth were derived from three readings on the *mire*, during a period of three hours each night. The mean of the *mire* rates on eighteen nights was -0.016 per hour, all but one small rate having the minus sign. Hourly rates in azimuth were adopted on fourteen nights, the mean being -0.015 , or a mean of -0.012 for the eighteen nights. The tendency of the object-end of the telescope to swing to the west, during the observing periods, is strongly marked in this series. Since only one pair of circumpolar stars was observed each night no rates in azimuth could be computed from these transits. To account for the above systematic difference between observations of the two groups of clock stars it would be necessary to assume average rates in azimuth -0.06 per hour in excess of the rates given by the

readings on the *mire*. No such assumption would be justified by an experience of more than thirty years with this instrument.

The corrections derived by Dr. Kopff in number 5481 of the *Astronomische Nachrichten* for the *B. J.* right ascensions have been tabulated below. His difference for the two groups of clock stars is $+0.029$. If his mean corrections are accepted there would remain a difference of $+0.035$ to be accounted for in these observations. This would be equivalent to an average excess of hourly clock rates of -0.016 , in place of -0.030 above. To account for the difference by rates in the azimuth of the instrument these would still have to be increased by -0.03 above the rates given by the *mire* readings.

The first group of clock stars came to the meridian 2^h4 after sunset at the beginning of this series, and at 0^h5 before sunset at the close. The figure 0^h3 is that actually derived in the detailed discussion in our Bulletins, and this series appears to confirm the sunset effect on transits, with the same correction, applicable to the transits of the first group. Accordingly, the corrections to the *B. J.* right ascensions adopted below are the residuals from the mean of each group, and thus represent merely the accidental errors for each star.

The explanation of this effect, and the question whether it is due to a horizontal component in atmospheric refraction, near the times of sunrise and sunset, must be left to meridian observers of the future. If there is a horizontal deviation at the meridian, the determination of azimuth from transits of circumpolar stars near sunrise and sunset will also be affected, as described in our Bulletins. The effect apparently can not be ascribed to magnitude equation, in transits occurring with bright sky illumination, since this feature has been investigated here.

OBSERVATIONS OF B. J. FUNDAMENTAL STARS 1926.0 LIST D

Star	Mag.	No.	Obs. R.A.	Obs.—B.J.	Diff. Corr.	Kopff—B.J.	Obs. Decl.	Obs.—B.J.	ϕ
6 Hydr.	5.1	14	8 ^h 36 ^m 31 ^s .095	(-0.031)	-0.001		$-12^{\circ} 12' 46''.62$	$-0''.26$	25 ^h 75
δ Cam.	4.2	15	8 40 28.926	(-0.070)	-0.040		+18 25 38.56	+0.30	25.17
γ Pyx.	4.2	15	8 47 23.486	(-0.002)	+0.028	+0.005	-27 26 4.45	-0.17	25.64
83 Cam.	6.6	15	9 14 51.250	(-0.050)	-0.020	-0.046	+18 1 12.03	+0.27	25.20
α Hydr.	2.2	15	9 23 57.131	(+0.002)	+0.032	-0.002	- 8 20 13.49	-0.04	25.51
Mean	4.5		8 56	-0.030	0.000	-0.014	- 2 18	+0.02	25.45
α Leon.	3.8	4	9 37 12.178	(-0.074)	-0.033	-0.018	+10 13 47.05	-0.10	25.29
θ Ant.	5.0	4	9 40 54.145	(-0.006)	+0.035	+0.002	-27 25 48.01	+0.09	25.10
6 Sex.	6.2	4	9 47 30.336	(-0.044)	-0.003	-0.020	- 3 53 44.53	+0.69	24.50
Mean	5.0		9 2	-0.041	0.000	-0.012	- 7 1	+0.23	24.96
1 Leon.	5.3	18	10 45 22.105	(-0.026)	-0.060	-0.020	+10 56 13.53	-0.16	25.55
β Hydr.	5.1	17	11 1 45.841	(+0.086)	+0.052	+0.049	-26 53 38.16	-0.05	25.43
χ Crater.	4.5	18	11 8 0.983	(+0.047)	+0.013	+0.002	-22 25 18.28	-0.88	26.26
δ Crater.	3.8	18	11 15 38.345	(+0.022)	-0.012	+0.011	-14 22 40.04	+0.35	25.02
ϵ Leon.	4.0	18	11 20 4.077	(+0.044)	+0.010	+0.034	+10 56 13.46	+0.37	25.00
Mean	4.5		11 6	+0.034	0.000	+0.015	- 8 22	-0.05	25.45
Difference, first five and last five stars,				+0.064		+0.029			
Average correction to B. J.					± 0.026	± 0.019		± 0.29	

OBSERVATIONS OF B. J. CIRCUMPOLAR STARS

Star	Mag.	No.	R.A.	Corr.	Decl.	Corr.	φ
Br. 2777 L.C.	5.9	13	21 ^h 7 ^m 0 ^s .629	+0.013	77° 49' 35".90	-0.10	25° 80
β Ceph. L.C.	2.9	4	21 27 42.728	-0.020	70 14 8.64	+0.34	26.24
λ Draco. U.C.	4.1	18	11 27 1.838	-0.033	69 44 22.86	+0.12	25.78
$\frac{1}{2}$ L.C.				-0.003		+0.12	26.02
$\frac{1}{2}$ (U.C.+L.C.)				-0.018		+0.12	25.90

Systematic Differences.—The corrections to the *B. J.* places derived by Dr. Kopff have been applied, in order to give the places of the *Eros* stars on his system. The various mean corrections are recapitulated here, for both lists. Also the corresponding differences for the

places of the *American Ephemeris and Nautical Almanac*, and for those of the *Preliminary General Catalogue* of Lewis Boss are given, for the purpose of reducing these observations to either system, if desired to do so.

List	No.	Kopff—B.J.	$\Delta\alpha$	$\Delta\delta$	A.E.—B.J.	$\Delta\alpha$	$\Delta\delta$	Boss—B.J.	$\Delta\alpha$	$\Delta\delta$	Decl.
A	1 to 146	6 stars	-0.030	+0.10	7 stars	+0.063	+0.16	8 stars	-0.012	-0.16	+44.8
B	147 to 281	8	-0.037	+0.14	6	+0.028	+0.21	8	-0.016	-0.22	+42.9
C	282 to 419	7	-0.022	-0.03	8	+0.006	+0.05	8	-0.008	-0.40	+29.7
D	420 to 821	11	0.000	-0.06	10	+0.007	+0.02	13	-0.008	-0.23	- 5.7
	Means (4)		(-0.022)	(+0.04)		(+0.026)	(+0.11)		(-0.011)	(-0.25)	

Systematic Differences: A.E.—Kopff, (+0.048), (+0.07); Boss—Kopff, (+0.011), (-0.29). All of these mean differences are enclosed in brackets, to indicate the approximate values of the figures. The figures for each group of fundamental stars have real significance.

The Latitude.—Results for astronomical latitude have been computed from every transit of a fundamental and circumpolar star. The corrections to the Poulkova refractions, as derived from the meridian observations at this station, have been applied. The correction is $-0.20 \tan Z.D.$ during the night hours, but is close to zero at sunset and sunrise. Using the declinations of the *Berliner Jahrbuch*, the following mean values have been obtained for the stars of List D, in this series.

FUNDAMENTAL STARS

Stars	Nights	Epoch	φ	φ_0
5	15	1926.29	25.45	25.46
3	4	.33	24.96	25.00
5	18	.30	25.45	25.47
13	Mean	.30	(25.34)	25.36

CIRCUMPOLAR STARS

L. C.	18	1926.30	26.02	26.04
U. C.	18	.30	25.78	25.80
$\frac{1}{2}$			(25.90)	25.92
Half sum, Fund. and C. P. stars,				25.64

From the observations of the first list of *Eros* stars the value of the normal latitude, φ_0 , was 25.47, on 58 nights, at epoch 1926.02. The weighted mean of the two results is 37° 20' 25".51, on 77 nights, at epoch 1926.09. The results clamp west have been reduced to the mean of the two clamp positions, with the correction found in the earlier series.³ The corrections for ($\varphi - \varphi_0$) have been derived for this station from the results at the international zenith telescope stations, for the past thirty years. The adoption of the half

sum from stars north and south of the zenith is presumed to eliminate bisection error, errors of flexure, and any vagaries of atmospheric refraction. For this reason it is preferred to a weighted mean. The above result for the latitude is close to that obtained in our first meridian volume, 25.55, from observations made more than thirty years before. It is smaller than the figure, 25.65, derived from all the observations of thirty years, 1893 to 1923, recorded on page 229, volume XV, of the *Publications of the Lick Observatory*. But such differences can legitimately be ascribed to the errors of observation, and the errors of the adopted declinations of the fundamental and circumpolar stars. It is not likely that any change of position on the surface of the Earth is indicated, either sudden or progressive. The probable error of a latitude from the observations of one night is ± 0.22 , in this series. The computed probable error of the mean of 19 nights would be ± 0.05 , but certain systematic errors may be larger than this figure.

Catalogue Comparisons.—The observations of the *Eros* stars in this list, reduced with the *B. J.* places, have been compared with twenty published catalogues, and the following mean differences have been taken out in the sense observed minus catalogue. The average residuals per star are given for each comparison, when there are four or more stars included. Individual differences for which the residual exceeds four times the average have generally not been included in the means. Such residuals may be due to sensibly large proper motions, or in some cases to mistakes. Although there are 351 comparisons tabulated, some stars were found in more than one catalogue, and there are

³ *L. O. Bull.* 13, 95, 1927.

actually comparisons of 255 separate stars, 5 stars not having any earlier meridian places. The comparisons extend from 27° south to 20° north declination, and may be used for the investigation of systematic errors depending on zenith distances. The five southern *A. G.* zones, of epoch 1900, give the mean differences $-0^{\circ}030$ in right ascension, and $+0^{\circ}11$ in declination, weighting them equally. The five northern *A. G.* zones, of epoch 1875, give the mean differences $-0^{\circ}135$ in right ascension, and $+0^{\circ}11$ in declination. Seven other recent catalogues, generally of higher weight than the *A. G.* zones, give the means $-0^{\circ}040$ in right ascension, and

$-0^{\circ}12$ in declination. The effect of mean proper motion is evident in the right ascensions, while that in declination is partly masked, apparently, by the respective systematic errors of the authorities. The general effect of the right ascension differences has the mean value $+0^{\circ}002$ ($+0^{\circ}03$) per year, for stars of 8.7 magnitude, at $10^{\text{h}}0$ right ascension. The averages of the residuals are $\pm 0^{\circ}08$ and $\pm 0^{\circ}9$ per star, for the *A. G.* catalogues, and $\pm 0^{\circ}06$ and $\pm 0^{\circ}6$, for the others. Some proportion of the residuals is due to proper motion. The epochs of the catalogues are in most cases not far distant from the epochs of observation.

CATALOGUE COMPARISONS

Catalogue	Epoch	Stars	Decl.	$\Delta\alpha$	ϖ	$\Delta\delta$	ϖ
Cordoba Zone	1900	95	$-24^{\circ}3$	$-0^{\circ}066$	$\pm 0^{\circ}09$	$+0^{\circ}18$	$\pm 1^{\circ}0$
Algiers A.G.	1900	55	-21.4	-0.090	± 0.09	-0.15	± 1.0
Wash. A.G.	1900	23	-16.1	-0.051	± 0.08	$+0.79$	± 0.7
Harv. A.G.	1900	19	-12.5	$+0.044$	± 0.06	$+0.18$	± 1.0
Vienna A.G.	1900	15	-8.3	$+0.023$	± 0.06	-0.07	± 0.6
Strass. A.G.	1900	16	-4.6	-0.077	± 0.09	-0.20	± 0.9
Nicol. A.G.	1875	7	-1.0	-0.066	± 0.07	$+0.09$	± 0.9
Albany A.G.	1875	13	$+3.1$	-0.119	± 0.09	-0.02	± 0.7
Leip. A.G.	1875	5	$+6.9$	-0.240	± 0.08	$+0.68$	± 1.0
Leip. A.G.	1875	5	$+11.4$	-0.127	± 0.08	$+0.28$	± 0.8
Berlin A.G.	1875	11	$+16.5$	-0.122	± 0.08	-0.46	± 0.9
Albany Zone	1900	13	-24.9	$+0.021$	± 0.09	$+0.10$	± 0.5
Albany Zone	1900	2	-1.5	$+0.045$		$+0.35$	
Bordeaux	1890	12	-17.4	-0.073	± 0.10	$+0.88$	± 0.9
Abbadia (3)	1900	21	-7.0	-0.071	± 0.05	$+0.16$	± 0.6
Abbadia (2)	1900	21	$+1.0$	-0.047	± 0.05	$+0.12$	± 0.4
Abbadia (1)	1900	4	$+18.2$	-0.037	± 0.05	-0.87	± 0.7
Bonn VI	1855	2	$+11.6$	$+0.06$		-0.4	
Bonn X	1900	8	$+8.2$	-0.052	± 0.08	-0.11	± 0.5
Wash. IX	1900	4	$+5.6$	-0.022	± 0.04	-1.12	± 0.6

OBSERVATIONS OF EROS STARS. SECOND LIST. 1926.0

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes	
400	9.5	10.5	$10^{\text{h}}26^{\text{m}}18^{\text{s}}.534$	3.328	$+25^{\circ}37'2^{\circ}00$	$-18^{\circ}40$	26.29	From list 1.	2 observations.
420	8.9	9.2	9 30 52.213	2.671	$-26^{\circ}25'9.81$	-15.95	26.22		
421	10.2	9.5	30 52.359	2.677	$-26^{\circ}4'26.02$	-15.95	28		
422	8.2	8.5	31 21.883	2.687	$-25^{\circ}34'51.55$	-15.98	23		
423	9.4	9.0	31 47.660	2.662	$-27^{\circ}1'20.69$	-16.00	28		
424	8.1	8.0	32 51.317	2.689	$-25^{\circ}37'48.24$	-16.06	30	S. fol. S. fol.	2 observ.
425	8.1	8.1	33 12.756	2.666	$-26^{\circ}59'18.39$	-16.07	30		2 observ.
426	7.8	8.0	33 34.808	2.675	$-26^{\circ}31'31.00$	-16.09	29		
427	9.4	8.8	33 43.446	2.704	$-24^{\circ}52'58.10$	-16.10	30	N. fol.	
428	8.3	8.5	34 35.527	2.693	$-25^{\circ}36'1.77$	-16.15	31		2 observ.
429	8.3	8.5	34 52.452	2.687	$-26^{\circ}2'22.17$	-16.16	22		
430	9.1	9.2	35 30.392	2.711	$-24^{\circ}39'52.14$	-16.19	28		
431	8.6	9.3	35 46.692	2.673	$-26^{\circ}57'12.01$	-16.21	23		
432	9.1	9.0	35 51.843	2.682	$-26^{\circ}27'52.52$	-16.21	28		
433	9.2	9.5	36 7.768	2.699	$-25^{\circ}29'30.83$	-16.23	31		
434	9.4	9.0	36 30.326	2.719	$-24^{\circ}20'21.88$	-16.25	31		
435	9.1	8.9	37 13.354	2.693	$-25^{\circ}58'41.03$	-16.28	30		2 observ.
436	9.1	9.5	37 20.041	2.678	$-26^{\circ}52'50.54$	-16.29	28		
437	8.5	8.5	38 22.308	2.702	$-25^{\circ}37'37.22$	-16.34	30		2 observ.
438	9.0	9.5	38 37.356	2.689	$-26^{\circ}25'6.53$	-16.35	30	Pr. Mo. (?)	
440	9.2	9.2	38 40.944	2.708	$-25^{\circ}16'13.57$	-16.36	30		

OBSERVATIONS OF EROS STARS. SECOND LIST. 1926.0—*Continued*

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
441	8.9	9.2	9 ^h 38 ^m 44 ^s .975	2.723	−24° 23′ 36″.44	−16″.36	26.22	
442	8.9	9.2	39 11.475	2.696	−26 4 4.08	−16.38	28	
443	8.9	9.0	39 27.331	2.685	−26 46 37.10	−16.40	23	
444	8.7	9.1	39 41.292	2.738	−23 33 38.31	−16.41	29	2 observ.
445	8.9	8.8	39 47.926	2.729	−24 8 27.17	−16.41	31	
446	8.5	8.5	40 50.701	2.723	−24 38 34.90	−16.46	31	
447	7.5	8.0	41 13.097	2.750	−23 1 14.09	−16.48	28	
448	8.1	8.5	41 18.654	2.693	−26 32 53.20	−16.49	29	Pr. Mo. (?)
449	8.5	8.5	41 39.687	2.699	−26 13 17.38	−16.50	32	
453	8.9	9.0	42 4.215	2.713	−25 27 18.68	−16.53	29	S. pr. N. pr.
454	8.1	8.0	42 40.174	2.707	−25 55 5.26	−16.55	30	
455	9.2	9.0	43 3.365	2.764	−22 21 17.98	−16.57	30	Error 0.02 Algiers precess.
456	9.0	8.0	43 7.589	2.717	−25 19 49.11	−16.58	22	S. pr.
457	9.1	9.0	43 10.806	2.758	−22 44 23.52	−16.58	28	
458	9.0	9.0	43 26.549	2.749	−23 21 51.93	−16.59	23	
459	9.1	8.8	43 39.700	2.743	−23 47 52.22	−16.60	28	
460	9.2	8.8	44 56.290	2.767	−22 23 39.25	−16.67	31	
461	9.1	8.5	44 58.186	2.737	−24 22 8.56	−16.67	31	
462	8.9	8.8	45 11.066	2.724	−25 11 50.44	−16.68	31	
463	9.0	9.0	45 38.747	2.776	−21 51 56.69	−16.70	28	
464	8.9	8.5	45 47.876	2.731	−24 52 54.92	−16.71	29	
465	9.2	8.8	45 56.226	2.759	−23 2 14.73	−16.72	31	S. fol.
466	9.2	8.5	45 57.496	2.788	−21 8 13.88	−16.72	32	S. fol.
468	8.9	9.0	46 14.380	2.756	−23 18 40.96	−16.73	30	
470	8.3	8.3	46 34.428	2.746	−24 1 11.68	−16.75	30	
471	9.4	8.8	46 56.528	2.741	−24 24 23.60	−16.76	22	
472	9.3	9.5	47 12.240	2.734	−24 52 25.39	−16.78	28	N. pr.
Anon.		9.5	47 16.779	2.734	−24 54 38.10	−16.78	28	S. fol.
473	8.6	8.5	47 13.637	2.796	−20 43 53.50	−16.78	23	N. fol. Pr. Mo. (?)
474	8.1	8.5	47 22.239	2.791	−21 6 57.59	−16.78	28	
479	9.3	9.5	48 35.424	2.746	−24 16 38.86	−16.84	31	
480	8.7	8.5	48 49.887	2.757	−23 34 44.61	−16.85	31	
481	8.5	8.0	49 10.024	2.761	−23 22 38.05	−16.87	31	N. fol.
482	9.3	9.3	49 31.312	2.788	−21 35 30.62	−16.89	29	
483	8.9	8.8	49 47.519	2.805	−20 27 4.28	−16.90	28	
484	9.0	8.8	49 58.649	2.802	−20 40 57.96	−16.91	30	
485	8.3	8.3	49 59.372	2.726	−25 48 39.26	−16.91	31	S. pr.
486	8.1	8.0	50 7.258	2.757	−23 45 33.46	−16.91	30	
487	9.0	9.0	50 24.059	2.781	−22 10 50.50	−16.93	32	
488	9.0	8.8	50 28.275	2.820	−19 28 43.42	−16.93	32	
489	8.3	8.5	50 44.088	2.738	−25 7 10.48	−16.94	28	
490	8.5	8.3	50 52.318	2.814	−19 56 30.13	−16.95	22	
491	8.6	8.5	51 0.449	2.771	−22 57 45.09	−16.96	28	Pr. Mo. (?)
492	9.4	9.3	51 26.612	2.748	−24 34 55.21	−16.98	23	
493	9.2	9.2	51 57.862	2.818	−19 48 27.70	−17.00	28	
494	9.2	9.0	52 24.727	2.795	−21 29 23.75	−17.02	31	
495	8.8	8.8	52 26.689	2.800	−21 8 32.87	−17.02	31	
496	9.1	9.0	52 29.322	2.781	−22 31 13.06	−17.02	33	
497	8.0	8.2	52 44.531	2.826	−19 21 37.87	−17.04	31	S. fol.
499	7.9	8.0	53 6.102	2.764	−23 44 54.00	−17.05	33	Pr. Mo. (?)
500	9.2	9.0	53 6.691	2.739	−25 26 56.13	−17.05	29	
501	9.2	9.2	53 14.059	2.811	−20 29 30.07	−17.06	31	
502	8.7	8.7	53 17.565	2.746	−24 59 0.26	−17.06	33	
504	9.3	9.9	53 36.793	2.834	−18 53 12.70	−17.07	29	
506	9.2	9.2	54 6.877	2.754	−24 34 38.70	−17.10	30	
507	9.1	8.8	54 43.987	2.771	−23 29 35.37	−17.13	32	
508	9.2	9.0	54 54.296	2.804	−21 12 36.53	−17.14	22	
509	9.2	9.0	54 56.817	2.822	−19 56 17.34	−17.14	28	
510	9.0	9.0	55 4.250	2.809	−20 52 17.54	−17.15	32	
512	9.2	8.8	56 2.534	2.778	−23 14 57.39	−17.19	23	

OBSERVATIONS OF EROS STARS. SECOND LIST. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
513	9.5	10	9 ^h 56 ^m 13 ^s .138	2.840	-18° 46' 39".13	-17° 20'	26.28	
514	9.2	8.8	56 15.636	2.807	-21 12 27.71	-17.20	31	
515	9.1	9.0	56 20.102	2.826	-19 49 8.39	-17.20	31	
516	9.1	9.2	56 30.146	2.757	-24 45 57.23	-17.21	31	
517	8.9	8.8	56 39.641	2.766	-24 8 52.07	-17.21	29	
518	8.6	8.5	56 50.834	2.858	-17 28 26.92	-17.22	31	
519	9.0	9.2	56 52.308	2.847	-18 16 20.48	-17.22	33	
520	9.5	9.5	57 32.062	2.772	-23 50 10.15	-17.25	33	
521	8.7	8.5	57 50.605	2.835	-19 19 2.36	-17.27	28	
522	9.0	9.0	57 55.323	2.866	-17 0 34.40	-17.27	30	S. pr.
523	8.8	8.5	58 0.662	2.850	-18 10 57.87	-17.27	30	Pr. Mo. (?)
525	8.8	8.8	58 13.234	2.818	-20 36 32.17	-17.28	32	
526	8.6	8.7	58 18.185	2.783	-23 14 2.38	-17.29	33	
528	8.0	8.2	58 28.892	2.858	-17 38 6.31	-17.30	32	
532	9.0	9.0	59 3.494	2.878	-16 12 37.34	-17.32	22	S. fol.
533	8.8	8.8	59 6.968	2.778	-23 42 38.76	-17.32	28	
535	9.0	9.0	59 50.611	2.764	-24 46 29.96	-17.36	31	N. fol.
536	9.2	9.0	10 0 24.696	2.850	-18 31 33.69	-17.38	29	N. pr.
537	9.2	8.8	0 25.078	2.855	-18 8 8.81	-17.38	31	S. fol. S. pr.
538	9.3	9.3	0 25.214	2.871	-16 55 3.92	-17.38	31	
539	9.0	9.0	0 30.226	2.837	-19 32 40.78	-17.38	32	Pr. Mo. (?)
540	8.8	8.5	0 30.340	2.891	-15 17 23.04	-17.38	23	
541	8.7	8.7	0 33.279	2.896	-14 51 53.57	-17.39	28	
542	9.1	9.0	0 40.157	2.883	-15 55 19.62	-17.39	33	
543	9.0	9.0	0 48.201	2.792	-22 55 9.34	-17.40	33	
545	9.3	9.3	1 14.918	2.783	-23 39 41.47	-17.42	28	
547	8.8	9.0	1 16.564	2.776	-24 10 5.44	-17.42	33	
548	8.3	8.0	1 38.278	2.876	-16 35 34.59	-17.43	28	
549	9.4	9.3	1 41.296	2.894	-15 6 1.84	-17.43	32	
550	9.3	9.5	1 52.624	2.797	-22 47 7.40	-17.44	30	Wrong star (?). Only one star in D.M.
551	9.0	8.5	2 18.022	2.904	-14 26 59.87	-17.46	30	
552	9.2	9.5	2 55.412	2.786	-23 44 46.51	-17.49	22	Pr. Mo. (?)
553	9.3	8.8	2 57.850	2.777	-24 23 51.14	-17.49	28	S. pr.
554	8.1	8.2	2 59.529	2.886	-15 58 37.85	-17.49	32	
557	8.7	8.7	3 29.714	2.794	-23 10 57.45	-17.51	31	
559	9.0	8.8	3 43.410	2.910	-14 4 47.06	-17.52	33	
560	7.9	8.2	3 45.517	2.898	-15 5 37.45	-17.52	31	
561	8.6	8.3	3 59.214	2.777	-24 34 23.73	-17.53	29	N. pr.
562	9.0	9.0	4 3.198	2.874	-17 6 49.82	-17.54	31	
563	9.1	9.0	4 41.587	2.904	-14 45 0.28	-17.56	32	N. fol.
564	9.2	9.2	4 41.744	2.924	-13 3 5.47	-17.56	33	
565	9.0	9.0	4 55.394	2.916	-13 42 40.20	-17.57	32	
566	7.9		5 25.722	2.927	-12 49 52.62	-17.59	30	Pr. Mo. (?)
567	9.0	8.8	5 27.509	2.786	-24 6 28.78	-17.60	23	
568	7.5	7.5	5 34.056	2.900	-15 7 37.76	-17.60	28	Pr. Mo. (?)
570	8.0	8.5	5 40.617	2.799	-23 12 10.09	-17.61	28	S. pr.
571	7.9	7.8	6 7.998	2.910	-14 23 54.81	-17.62	29	
572	8.4	8.7	6 14.367	2.796	-23 31 26.25	-17.61	30	
573	9.1	9.5	6 48.593	2.790	-24 6 46.72	-17.65	32	Decl. (?) Pr. Mo. (?)
574	8.9	8.5	6 56.659	2.900	-15 15 37.28	-17.66	33	
575	9.3	9.3	6 58.976	28.87	-16 25 13.38	-17.66	22	
576	8.9	9.0	7 4.624	2.922	-13 27 57.24	-17.66	28	
577	8.6	8.5	7 10.126	2.924	-12 29 50.25	-17.67	31	Pr. Mo. (?)
578	9.3	9.0	7 14.370	2.809	-22 41 44.98	-17.67	31	N. fol.
579	8.1	8.2	7 58.800	2.918	-13 54 32.59	-17.70	29	
581	8.7	8.5	8 22.143	2.927	-13 11 48.80	-17.72	31	
582	8.1	8.2	8 22.788	2.817	-22 15 58.09	-17.72	32	
583	8.1	8.0	8 39.413	2.896	-15 48 44.80	-17.73	33	S. pr.
584	9.1	9.5	9 3.676	2.806	-23 14 53.72	-17.75	31	
585	8.7	8.7	9 4.499	2.903	-15 20 24.83	-17.75	33	

OBSERVATIONS OF EROS STARS. SECOND LIST. 1926.0—*Continued*

No.	Mag.	Obs.	Obs. R.A.		Prec.	Obs. Decl.		Prec.	Epoch	Notes
590	9.1	8.5	10 ^h	9 ^m 16.631	2.912	-14° 34'	6.86	-17.76	26.23	
591	9.3	9.5		9 28.816	2.797	-23 59 24.43		-17.76	28	
592	9.1	9.2		9 59.514	2.950	-11 18 25.16		-17.78	28	
593	9.3	9.0		10 22.649	2.801	-23 52 37.26		-17.80	29	
594	9.0	9.0		10 29.471	2.969	- 9 38 58.02		-17.80	30	
595	8.8	8.8		11 5.201	2.922	-13 54 33.79		-17.83	31	N. fol.
596	9.0	9.0		11 9.655	2.817	-22 45 26.74		-17.83	31	
597	9.1	8.5		11 12.966	2.821	-22 27 53.07		-17.83	32	Pr. Mo. (?)
598	9.1	9.0		11 13.395	2.946	-11 44 6.19		-17.83	33	
600	8.8	8.5		11 24.524	2.928	-13 25 42.70		-17.84	22	
601	9.0	9.2		11 26.457	2.941	-12 17 3.98		-17.84	28	
603	8.1	8.0		11 59.542	2.957	-10 49 6.94		-17.86	31	
604	9.1	9.0		12 16.632	2.812	-23 21 29.99		-17.87	32	
605	9.0	9.0		12 29.487	2.974	- 9 21 58.17		-17.88	30	
606	9.1	9.2		12 47.631	2.982	- 8 33 24.15		-17.90	31	
610	7.2	7.5		13 9.472	2.937	-12 43 46.73		-17.91	33	
614	8.6	9.2		13 45.609	2.809	-23 49 31.55		-17.93	23	
615	9.0	9.0		13 53.492	2.975	- 9 20 15.01		-17.94	28	
616	7.3	7.5		14 5.357	2.814	-23 30 22.32		-17.95	28	
617	9.0	9.2		14 14.207	3.005	- 6 32 31.49		-17.95	29	
618	9.2	9.0		14 18.072	2.995	- 7 30 38.61		-17.95	30	
619	9.6	9.0		14 20.815	2.967	-10 5 30.23		-17.96	32	
620	8.5	8.5		14 54.155	2.829	-22 23 10.44		-17.98	31	
621	9.1	9.0		14 54.800	2.999	- 7 7 50.37		-17.98	29	
622	8.9	8.5		15 1.253	2.987	- 8 17 18.93		-18.98	31	
623	9.0	8.8		15 12.476	3.011	- 5 59 35.07		-17.99	33	
624	8.7	8.5		15 24.632	2.955	-11 21 5.39		-18.00	33	S. fol. S. fol. S. fol.
625	8.7	8.7		15 29.124	2.949	-11 54 10.77		-18.00	31	
626	8.1	8.0		15 29.967	2.980	- 8 57 50.80		-18.00	22	
627	8.8	8.8		15 33.844	2.965	-10 22 40.11		-18.00	28	
630	8.7	8.5		15 40.760	2.963	-10 36 58.48		-18.01	32	S. pr.
632	7.9	7.5		16 18.903	2.835	-22 9 5.14		-18.03	30	
633	9.1	9.0		16 22.469	3.025	- 4 44 9.34		-18.03	31	
635	8.7	8.7		16 46.047	2.835	-22 14 53.44		-18.05	33	Fol. star, pair.
639	7.0	7.0		17 23.552	3.022	- 5 2 33.88		-18.07	32	S. fol.
642	8.3	8.0		17 28.492	2.988	- 8 20 9.47		-18.08	23	
643	9.1	8.8		17 39.355	3.031	- 4 13 31.56		-18.08	28	
644	9.2	9.0		17 42.624	2.845	-21 34 43.03		-18.09	28	
645	9.0	9.0		18 2.216	3.014	- 5 51 11.18		-18.10	29	
646	9.2	9.0		18 33.349	2.831	-22 57 51.69		-18.12	29	
647	9.0	8.8		18 43.924	2.987	- 8 32 44.36		-18.12	30	
648	8.9	8.5		18 55.146	3.040	- 3 22 3.30		-18.13	31	
649	9.0	8.8		18 55.875	3.023	- 5 0 51.78		-18.13	31	
650	8.6	8.5		19 4.168	3.009	- 6 22 45.84		-18.14	31	
651	9.2	8.8		19 5.569	3.012	- 6 9 5.89		-18.14	32	
652	9.1	9.0		19 7.907	3.003	- 7 2 50.91		-18.14	33	
653	8.8	8.8		19 12.309	2.826	-23 27 28.57		-18.14	22	
654	9.2	9.2		19 17.791	2.995	- 7 48 21.37		-18.14	28	
655	9.0	8.5		19 29.639	2.980	- 9 18 36.94		-18.15	33	
657	9.2	8.5		19 49.121	3.047	- 2 40 14.17		-18.16	30	Pr. Mo. (?)
658	7.7	7.0		20 3.124	3.054	- 1 54 4.83		-18.17	31	
659	8.9	8.5		20 11.984	3.005	- 6 52 59.95		-18.18	33	
669	9.0	9.0		21 40.548	3.018	- 5 42 23.74		-18.23	23	
670	9.1	9.0		21 43.324	3.031	- 4 18 19.23		-18.23	28	S. fol.
672	7.9	7.5		22 3.708	2.854	-21 34 36.75		-18.25	28	
673	9.7	9.0		22 25.308	3.062	- 1 10 6.97		-18.26	33	
674	8.9	8.5		22 30.145	3.038	- 3 40 31.48		-18.26	29	
675	9.5	8.8		22 40.844	3.054	- 1 57 37.31		-18.27	31	N. fol. S. pr.
676	9.3	9.0		22 50.665	2.839	-23 1 39.26		-18.28	31	
677	9.3	9.0		22 58.154	3.014	- 6 8 10.00		-18.28	32	Mistake Decl. (?) Δδ Strass. 20'

OBSERVATIONS OF EROS STARS. SECOND LIST. 1926.0—Continued

No.	Mag.	Obs.	Obs. R.A.	Prec.	Obs. Decl.	Prec.	Epoch	Notes
678	8.1	7.8	10 ^h 23 ^m 6 ^s .367	3.075	+ 0° 13' 45".87	-18°28'	26.31	
679	9.0	8.8	23 7.246	3.067	- 0 39 47.03	-18.28	33	S. fol.
680	8.7	8.5	23 15.580	3.048	- 2 40 27.11	-18.29	32	S. pr.
681	8.8	8.5	23 15.749	3.022	- 5 17 52.90	-18.29	22	
682	8.9	8.8	23 15.371	3.084	+ 1 10 6.78	-18.29	28	S. pr.
683	8.3	8.3	23 27.516	3.092	+ 1 53 32.14	-18.30	30	Pr. Mo. (?)
685	8.8	8.8	23 58.682	2.844	-22 49 21.59	-18.32	33	Pr. Mo. (?)
695	9.1	9.2	25 23.410	3.120	+ 4 59 57.02	-18.37	32	Pr. Mo. (?)
696	8.8	8.0	25 30.577	2.858	-21 52 27.29	-18.37	28	N. pr.
697	8.9	8.5	25 30.284	3.044	- 3 8 53.00	-18.37	30	
698	9.0	9.2	25 31.586	2.846	-22 56 30.67	-18.37	33	Pr. Mo. (?)
699	9.3	9.0	25 39.696	3.066	- 0 42 41.50	-18.38	23	Pr. Mo. (?)
700	9.3	9.0	25 41.140	3.113	+ 4 16 34.66	-18.38	28	
701	8.9	8.8	26 3.196	3.089	+ 1 44 11.00	-18.39	33	
702	8.7	8.5	26 8.140	3.105	+ 3 26 49.11	-18.39	32	
703	8.5	8.3	26 16.020	3.056	- 1 49 26.95	-18.40	31	
704	8.4	8.3	26 26.928	3.110	+ 3 55 59.29	-18.40	31	
705	8.5	8.5	26 31.023	3.130	+ 6 6 23.78	-18.40	31	Pr. Mo. (?)
708	7.1	7.0	26 41.634	3.097	+ 2 31 50.96	-18.41	32	
710	9.0	9.0	26 51.885	3.113	+ 4 17 0.60	-18.42	22	
711	9.3	9.3	27 6.311	3.084	+ 1 10 53.15	-18.42	28	
712	8.0	8.0	27 9.817	3.126	+ 5 43 17.98	-18.43	33	
735	9.3	8.8	28 51.696	3.212	+15 0 6.95	-18.49	32	
736	8.8	8.8	28 54.940	3.171	+10 40 22.74	-18.49	33	Pr. Mo. (?)
737	8.9	8.8	28 56.538	3.069	- 0 29 4.25	-18.49	30	
740	9.5	10	29 18.361	3.079	+ 0 36 15.74	-18.50	30	R.A. List+10 ^s
741	9.2	9.2	29 9.720	3.186	+12 23 15.15	-18.50	28	
744	8.9	9.0	29 26.621	3.240	+17 57 21.26	-18.51	33	Pr. Mo. (?)
749	8.7	8.7	29 41.809	2.872	-21 24 43.57	-18.51	32	
750	8.7	8.7	29 49.857	3.154	+ 8 58 7.33	-18.52	28	Pr. Mo. (?)
752	8.7	8.3	29 53.190	2.864	-22 12 10.19	-18.52	29	S. fol.
753	9.1	9.0	29 55.983	3.229	+16 52 47.34	-18.52	31	
755	8.1	8.0	30 6.894	3.100	+ 3 0 19.07	-18.53	31	
756	8.5	8.5	30 6.912	3.103	+ 3 20 52.06	-18.53	31	
758	9.0	9.0	30 10.805	3.088	+ 1 42 34.98	-18.53	33	
760	7.7	8.0	30 11.635	3.189	+12 45 24.49	-18.53	31	
761	9.0	8.5	30 28.260	3.113	+ 4 29 38.08	-18.54	22	
762	9.0	8.8	30 36.119	3.109	+ 4 0 1.79	-18.54	28	
778	7.9	8.2	31 55.481	3.240	+18 25 48.58	-18.59	33	
782	9.0	8.8	32 16.054	2.879	-21 14 57.86	-18.60	32	
786	8.1	8.0	32 24.634	3.208	+15 10 38.32	-18.60	30	
789	8.7	8.5	32 30.860	3.148	+ 8 33 28.98	-18.61	33	
790	8.5	8.5	32 35.543	3.220	+16 25 12.23	-18.61	28	
794	9.2	9.0	32 50.817	3.164	+10 23 52.35	-18.62	23	
795	8.9	8.8	32 55.169	3.211	+15 34 15.69	-18.62	28	Pr. Mo. (?)
796	9.0	8.8	33 4.101	2.876	-21 44 43.61	-18.62	32	
799	9.1	9.2	33 16.035	3.182	+12 23 41.40	-18.63	33	
800	8.5	8.5	33 18.905	3.158	+ 9 45 34.66	-18.63	29	N. fol.
801	9.2	9.0	33 28.392	3.171	+11 11 20.69	-18.64	31	Pr. Mo. (?)
804	8.3	8.5	33 37.186	3.240	+18 44 12.72	-18.64	28	
805	8.7	8.5	33 38.944	3.184	+12 44 17.54	-18.64	31	
806	7.6	7.5	33 47.977	3.230	+17 39 49.06	-18.65	31	Pr. Mo. (?)
807	8.5	8.7	33 56.613	3.208	+15 23 35.12	-18.65	22	S. fol.
808	9.1	9.0	34 11.237	3.195	+13 56 20.50	-18.66	28	
809	9.3	9.5	34 16.258	3.167	+10 51 0.69	-18.66	31	Δα 4 ^s , B VI.
811	9.1	8.8	34 24.215	3.201	+14 39 8.05	-18.67	30	
817	8.0	8.5	35 1.800	3.215	+16 22 6.16	-18.69	33	
818	9.2	8.8	35 54.984	2.876	-22 23 15.98	-18.72	31	Pr. Mo. (?)
819	7.4	7.0	36 9.237	2.879	-22 8 40.63	-18.72	30	S. fol.
820	9.1	8.9	36 57.555	2.890	-21 11 1.18	-18.75	30	Pr. Mo. (?)
821	8.9	8.8	37 16.243	2.878	-22 25 0.40	-18.76	27	

PALO ALTO, November, 1927.

Issued January 14, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 393

ON THE RADIAL VELOCITY CURVE OF *S SAGITTAE*

BY

T. S. JACOBSEN

Our existing knowledge of the radial velocity curves of Cepheid variables shows that some are subject to secular changes in shape, amplitude, and systemic velocity, and that others change quite rapidly, differences having been noted in a few adjacent cycles of variation. Since the latter evidence rests principally upon results of moderate accuracy, it is of interest to discuss the existing three-prism spectrograms of as many Cepheids as possible, with a view to finding out whether these stars as a class show semi-irregular velocity variation. As a number of the brighter Cepheids have been followed with high dispersion spectrographs during several clear seasons at Mount Hamilton, series of observations exist from which the changes occurring in the course of a star's variation may be traced more or less completely.

The observational material for *S Sagittae* discussed in this paper consists of 39 one-prism spectrograms taken in the years 1903 to 1906, and 51 three-prism Mills spectrograms taken in the years 1921 to 1927. The 1903 observations, by Curtiss¹, from which the star's velocity variation was discovered, have so far not been included in any other treatment of the velocity curve; but the series obtained by Maddrill in 1906 was subjected to a formal discussion by Hellerich², in which spectroscopic elements were derived. The variation in systemic velocity, definitely shown by Aldrich³ in 1924 to exist, and to be periodic, was adumbrated by a linear term in Hellerich's expression for this element.

The present investigation deals primarily with the three-prism plates mentioned above. Of these the 1921 series was secured by various observers, especially

Messrs. Moore, Thiele, and Trumpler. These plates were measured on a Toepfer machine by Miss A. M. Hobe. Two-thirds of the plates taken after 1921 were secured by myself, and most of the others by Messrs. Maxwell and Moore. All but one were measured by me on a Hartmann spectrocomparator against a standard spectrogram of the sky used previously in radial velocity measurements of Mills plates. In addition to the usual systematic corrections, the one due to flexure⁴ of the telescope was applied to all three-prism results.

The task of grouping the observations, drawing plausible observational curves, and comparing these, prior to computation, was gratifying in its results; since by a few trials, several striking features of all the curves were revealed; the well marked variation of the systemic velocity was recognized; and, by taking this into account, the general maze of exceedingly scattered observational material was reduced to a probable mean curve with greatly diminished residuals. The groups were formed according to time, and if the observations in any one indicated a noticeable change in systemic velocity toward the end, that group was shortened. In this way fairly homogeneous curves were obtained for comparison. Table I shows the change in velocity of system determined by superposition of observational curves upon the final computed curve.

Several attempts to represent the values in Table I by elliptic elements, using Aldrich's³ and other periods, were unsatisfactory. It was therefore decided to wait for further observations before computing an orbit for the long period system, and to eliminate the effect of

¹ *L.O.B.*, 3, 40, 1904.

² *A.N.*, 210, 65, 1919.

³ *Pop. Ast.*, 32, 218, 1924.

⁴ *L.O.B.*, 12, 142, 1926.

TABLE I. DATA FOR VARIABILITY OF SYSTEMIC VELOCITY

Group	Julian day	Velocity of System km./sec.	Remarks
1	2416352	-18.4	1903, 6 one-prism observations
2	7371	- 1.3	1906, 10 one-prism observations
3	7428	- 4.4	1906, 10 one-prism observations
4	7473	-20.9	1906, 12 one-prism observations
5	22917	-22.2	1921, 20 three-prism observations
6	3981	+ 2.8	1924, 10 three-prism observations
7	4725	+ 2.0	1926, 10 three-prism observations
8	4891	- 5.7	1926, 9 three-prism observations
9	5073	-19.5	1927, 2 three-prism observations

The Julian day refers to the mean of the times of observation.

the variability of systemic velocity by an approximate but more direct method; namely, by applying to each group of observations the correction, obtained from Table I, which will refer it to the arbitrary constant value of -10 km./sec.;—the provisional value given by the most satisfactory preliminary curve being -9.76 km./sec.

It is recognized that these data are uncertain in the cases where the groups cover a long period of time or such a short one that the curves are materially influenced by the accidental distribution of observations. But the three-prism curves in 1921 and 1924 are well determined and very similar in shape. They form also a good mean curve for all the observations. Although

it is improbable that Cepheid variables are true spectroscopic binary stars, elliptic elements are still convenient as a mathematical means of defining the radial velocity curve, particularly as such elements have already been derived for earlier observations.

The observed values were collected into normal places using the phase differences as criteria for combination. A normal place formed from two plates was considered of unit weight. Preliminary elements were derived by the method of Lehmann-Filhés.⁵ With these as a starting point, and using Schlesinger's formulae and tables,⁶ the observations were adjusted by the method of least squares, two solutions being necessary. The final elements with their probable errors, derived from the plate residuals for 1921-4, are as follows:

$$\begin{aligned}
 U &= 8^{\text{h}}38^{\text{m}}20^{\text{s}} \text{ (assumed)} \\
 T &= \text{J. D. } 2422920.588 \pm 0^{\text{d}}133 \\
 e &= .484 \pm .017 \\
 K &= 20.54 \text{ km./sec.} \pm .53 \text{ km./sec.} \\
 \omega &= 43^{\circ}63' \pm 2^{\circ}94' \\
 \gamma &= -10.26 \text{ km./sec.} \\
 r &= \pm 1.49 \text{ km./sec.}
 \end{aligned}$$

The last quantity, r , is the probable error of a single three-prism observation of unit weight used in the solution.

⁵ *A.N.*, 136, 17, 1894.

⁶ *Publ. Alleghany Obs.*, 1, 33, 1908.

TABLE II. RADIAL VELOCITIES OF *S Sagittae*.

Julian day	Heliocentric phase in days	Observed velocity km./sec.	Reduced velocity km./sec.	O-C km./sec.	Remarks
2416343.9	0.0	-30.2	-22.6	- 1.1	Lick one-prism observations, 1903-4, group 1, Curtiss. ¹
6336.9	1.4	-20.3	-12.7	+ 3.2	
6353.9	1.7	-20.2	-12.6	+ 2.0	
6371.8	2.8	-17.3	- 9.7	- 2.3	
6682.9	3.8	- 3.9	+ 3.7	+ 0.5	
6364.9	4.3	+ 3.9	+11.5	+ 0.4	
6341.9	6.4	-32.2	-24.6	- 3.0	
7375.906	1.052	- 6.5	-11.3	+ 6.5	Lick one-prism observations, 1906, group 2, Mad-drill. ²
7375.984	1.130	-14.0	-18.8	- 1.6	
7384.915	1.679	- 6.8	-11.6	+ 3.0	
7352.959	3.251	- 1.5	- 6.3	- 3.0	
7369.986	3.514	+ 7.8	+ 3.0	+ 3.4	
7361.941	3.851	+18.5	+13.7	+ 9.8	
7379.890	5.036	+13.7	+ 8.9	- 7.0	
7379.978	5.124	+13.9	+ 9.1	- 4.6	
7363.905	5.815	-20.7	-25.5	-12.7	
7363.994	5.940	- 3.7	- 8.5	+ 6.2	
7425.788	0.642	-17.4	-14.5	+ 4.5	Lick one-prism observations, 1906, group 3, Mad-drill. ²
7384.985	1.749	-12.6	-10.0	+ 4.3	
7445.732	3.822	-10.7	- 8.1	-11.4	
7437.974	4.446	+ 1.0	+ 3.6	- 9.8	
7446.919	5.009	+ 1.8	+ 4.4	-11.7	
7448.873	6.963	-30.6	-28.0	- 4.4	
7390.813	7.577	-22.4	-19.8	+ 3.6	
7390.884	7.648	-15.1	-12.5	+10.7	

TABLE II. RADIAL VELOCITIES OF *S Sagittae*.

Julian day	Heliocentric phase in days	Observed velocity km./sec.	Reduced velocity km./sec.	O-C km./sec.	Remarks
2417390.974	7.738	-14.3	-11.7	+11.4	
7441.856	8.328	-21.1	-18.5	+ 3.0	
7475.665	0.226	-18.8	-11.7	+ 9.1	Lick one-prism observations, 1906, group 4, Mad-drill. ²
7475.701	0.262	-27.6	-20.5	+ 0.1	
7485.827	2.006	-19.8	-12.7	+ 0.1	
7452.717	2.424	-14.4	- 7.3	+ 2.9	
7469.776	2.719	-18.9	-11.8	- 3.9	
7486.690	2.869	-17.0	- 9.9	- 3.1	
7479.684	4.245	- 0.3	+ 6.8	- 3.2	
7454.681	4.388	+ 2.5	+ 9.6	- 2.8	
7480.815	5.376	+ 0.7	+ 7.8	+ 4.6	
7481.660	6.221	-28.0	-17.1	- 1.0	
7481.699	6.260	-39.3	-28.4	-11.9	
7448.907	6.997	-21.9	-14.8	+ 8.8	
2423979.945	0.004	- 7.47	-20.22	+ 1.23	Mills three-prism observations, 1921-4, groups 5 and 6, various observers.
2940.692	0.131	-33.89	-21.66	- 0.59	
3988.747	0.424	- 4.80	-17.55	+ 2.55	
2932.700	0.521	-30.13	-17.90	+ 1.85	
2924.705	0.908	-28.67	-16.40	+ 1.80	
2916.853	1.438	-28.47	-17.22	- 1.35	
3989.939	1.616	- 3.89	-16.64	- 1.67	
2925.708	1.911	-29.60	-17.37	- 4.00	
3973.849	2.290	- 9.90	-13.65	- 2.59	
2917.711	2.296	-24.06	-11.83	- 0.81	
2926.685	2.888	-17.85	- 5.62	+ 1.01	
3974.860	3.301	+11.60	- 1.15	+ 1.62	
2918.846	3.431	-14.74	- 2.51	- 1.13	
2910.736	3.703	- 7.71	+ 4.52	+ 2.64	
2902.774	4.123	- 1.40	+10.83	+ 3.89	
3975.844	4.285	+21.83	+ 9.08	- 1.51	
2911.725	4.692	+ 1.91	+14.14	- 2.50	
2903.770	5.119	+ 4.37	+16.60	+ 3.06	
3976.772	5.209	+19.66	+ 6.91	- 3.61	
2920.711	5.296	- 5.01	+ 7.22	+ 0.27	
2937.682	5.503	-13.37	- 1.14	+ 0.95	
2954.668	5.724	-25.13	-12.90	- 3.18	
3985.821	5.880	+ 1.80	-10.95	+ 3.36	
2921.696	6.281	-35.07	-22.84	- 2.39	
2913.739	6.706	-36.34	-24.11	- 1.15	
3978.780	7.221	-11.80	-24.55	- 0.95	
2905.883	7.232	-33.35	-21.12	+ 2.48	
2939.689	7.510	-36.80	-24.57	- 1.18	
3987.767	7.826	- 8.17	-20.92	+ 1.91	
2923.707	8.292	-33.93	-21.70	+ 0.03	
4692.926	0.508	- 5.41	-15.43	+ 4.4	Mills three-prism observations, 1926, group 7, various observers.
4752.812	1.719	- 7.96	-17.98	- 3.5	
4728.904	2.957	+ 3.45	- 6.57	- 0.5	
4720.945	3.380	+12.70	+ 2.68	+ 4.6	
4738.839	4.510	+19.55	+ 9.53	- 4.8	
4755.747	4.654	+18.89	+ 8.87	- 7.5	
4714.895	5.713	+ 5.97	- 4.05	+ 6.0	
4707.944	7.144	-12.99	-23.01	+ 0.7	
4749.879	7.168	-16.04	-26.06	- 2.3	
4683.949	8.295	- 8.59	-18.61	+ 3.1	
4810.674	0.906	-10.69	-17.38	+ 0.9	Mills three-prism observations, 1926, group 8, various observers.
4768.778	0.921	- 8.01	-14.70	+ 3.5	
4803.679	2.294	- 9.44	-16.13	- 5.0	
4787.688	3.067	+ 1.49	- 5.20	- 0.2	

TABLE II. RADIAL VELOCITIES OF *S Sagittae*

Julian day	Heliocentric phase in days	Observed velocity km./sec.	Reduced velocity km./sec.	O-C km./sec.	Remarks
2424771.747	3.890	+12.57	+ 5.88	+ 1.5	
4781.800	5.561	+ 6.60	- .09	+ 4.3	
4773.758	5.901	- 4.84	-11.53	+ 3.5	
4807.646	6.261	-15.26	-21.95	- 1.7	
4816.666	6.898	-23.75	-30.44	- 6.9	Measurement by Miss D. Applegate.
5078.897	0.903	-24.72	-15.50	+ 2.76	Mills three-prism observations, 1927, group 9,
5067.958	6.728	-35.00	-25.78	- 2.75	Jacobsen.

The observations with their residuals are collected in Table II, in which all those available to me are included for the sake of completeness. Using Luizet's⁷ period and Hellerich's⁸ epoch, J. D. 2417400.00, the heliocentric phase is as given in the second column. While the third column contains the observed velocities corrected for systematic errors, the fourth contains these values reduced to constant systemic velocity by application of a correction obtained from Table I. The values of the residuals in column five are with

for the one- and three-prism results, respectively. By the least squares solutions the sum of the three-prism residuals used was reduced from 148.80 to 142.42. In the final solution the greatest difference between residuals computed from the observation equations and from elements is 0.08 km./sec., so that the effect of second order terms is very small.

A few attempts to improve the representation by superposing a secondary oscillation on the primary gave the result that while a sine curve of one-half, or

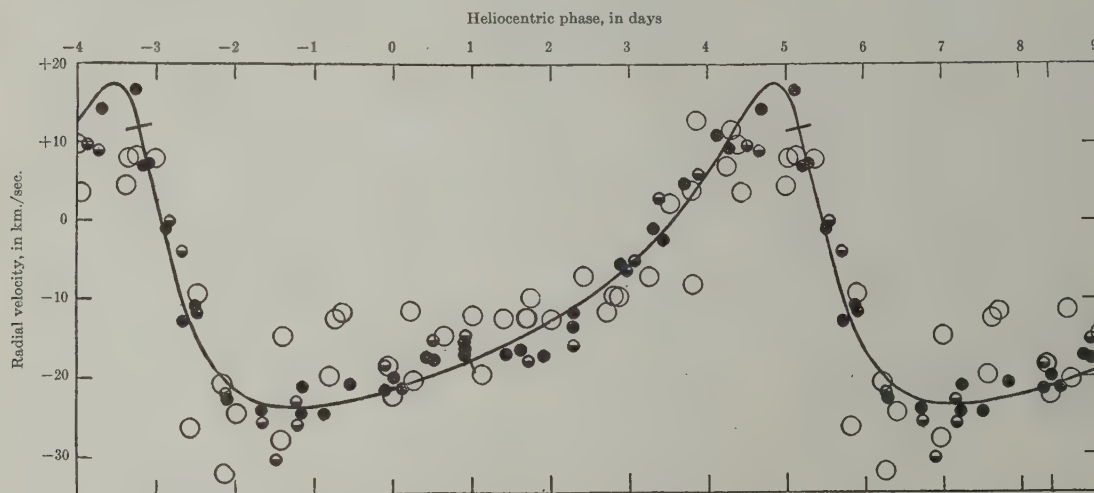


FIG. I. Radial velocity curve of *S Sagittae*. The plotted points are reduced velocities. Filled and half filled circles are Mills three-prism observations in 1921-4 and 1926-7, respectively. Open circles are Lick one-prism observations. The most outstanding Mills observations have relatively small weight.

respect to the final curve, shown in the figure. For one-prism and 1926 observations they were obtained graphically by reading directly from the plot; for the remaining plates they were computed by the final elements. The probable error of a single estimate by the graphical process was found to be ± 0.2 km./sec. from 30 points for which the residuals were both scaled off and computed rigorously.

The probable errors of single observations obtained from the plate residuals are ± 4.3 and ± 1.85 km./sec.

⁷ A.N., 168, 341, 1905.

⁸ A.N., 210, 66, 1919.

better, one-third the period, may perhaps be so chosen as to effect a slight reduction of the sum of the squares of the residuals, neither one represents the observational curve in all its parts. The oscillation is evidently not due to blend effect in the ordinary sense, for the nodes are near those of the principal orbit and not at the points of intersection of the curve with the velocity of the system. The three-prism plates are for the most part well exposed and of very good quality. No general shading of spectral lines is noticeable at any phase. But at maximum the spectral class is earlier than that of the Hartmann standard by approximately 0.6 divi-

sion.⁹ In order to guard against systematic errors three fine plates at phases 0.508, 1.719, and 2.294 days, where the observed curve differs the most from the mean curve, were remeasured on a Toepfer machine. The measurements of 39 to 43 star lines on each plate gave differences, Hartmann *minus* Toepfer, of +0.10, -0.63, and +0.04 km./sec., respectively. It is thus unlikely that a careful remeasurement of all the plates, by whatever method, would result in a materially smoother curve. An investigation of a supposed source of systematic error; namely, the effect upon the measurement of smallness of the relative displacement between Hartmann standard and plate, gave the result that there is no correlation between displacement and "humps." I therefore consider the systematic deviations real and not instrumental.

The close connection between the velocity curve and the light curves of several observers; e.g., Hertzprung¹⁰ and Luizet¹¹, should not be overlooked. A resemblance was first noted by Aldrich³. If the three-prism curve be inverted, its similarity to the light curves is striking. The secondary minimum and maximum are reflected in the velocity curve, with phase lags of the order of one day. The "secondary oscillation" thus seems to be more than a spectroscopic peculiarity, but its physical significance remains obscure. As has already been mentioned, grave doubt attaches to the reality of the elliptic elements. In view of this and of the newer theories of Cepheid variation, it seems undesirable to carry the binary analysis farther, especially as the data are complicated by variation in systemic velocity. What has already been put down is sufficient for comparison with other orbits.

³ *Pop. Ast.*, **32**, 218, 1924.

⁹ Shapley, *Ap. J.*, **44**, 290, 1916.

¹⁰ *A.N.*, **182**, 299, 1909.

¹¹ *A.N.*, **168**, 350, 1905.

The large computed probable error of even a three-prism observation is not surprising, but the dispersion of the velocities about a mean curve is considerably larger than in other Cepheids,¹² as, for example, δ Cephei, η Aquilae, or ζ Geminorum. Part of this is due to the difficulty of eliminating the variation of systemic velocity. The rest can be accounted for satisfactorily by instrumental causes, principally temperature changes during the exposures, which lasted three hours or more in most cases. The 1921-24 series suffered very little from this source of error, but in 1926 some difficulty was experienced with the heating control. Although there is no simple relation between sign or size of residuals and temperature changes during exposures with the new *Mills* spectrograph, it is a well established fact, that observations taken under conditions of large temperature changes deserve only small weight. In the present case those in which the change was greater than 0°1 C. are at phases 0.004, 1.719, 2.294, 3.380, 3.890, 4.510, 4.654, 5.561, 5.713, 5.880, 6.261, and 6.898 days—nearly all outstanding ones. In view of present evidence, it is thus unnecessary to suppose that the short period curve does not repeat itself closely at different epochs.

Although the one-prism observations are systematically too high or too low in certain parts with respect to the three-prism curve, they are so discordant that the evidence they afford for changes in spectroscopic elements between 1903 and 1927 need not be seriously considered here.

¹² *L.O.B.*, **12**, 144, *et seq.*, 1926.

LICK OBSERVATORY, December, 1927.

Issued January 21, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 394

ELEMENTS AND EPHEMERIS OF COMET *j* 1927 (SCHWASSMANN-WACHMANN)

BY

L. BERMAN AND F. L. WHIPPLE

This comet was discovered photographically by Schwassmann and Wachmann with the Lippert astrograph at Bergedorf, Germany, on November 15, 1927. The integrated magnitude was given as 13-14 by the Bergedorf observers, who described the comet as having a well defined nucleus with a circular coma about 2' in diameter. Van Biesbroeck¹ remarks that he found it difficult to locate the object on some of the 24" reflector

plates, November 26-December 1, presumably on account of an unexpected change in the brightness. Because of the extreme faintness, 16^m-17^m, subsequent observations have been secured under great difficulty and at the present time the object can only be followed photographically. To date the only sources of observations have been with one exception the Bergedorf, Yerkes, and Lick observatories. The astrographic or corresponding visual mean places, referred to the beginning of the year, so far received are:

¹ *Pop. Astron.* 36, 1, p. 69 (Comet Notes), 1928.

1927-1928, U. T.		α	δ	Observer	Authority
(1)	Nov. 15.8983	1 ^h 32 ^m 14 ^s .2	+20° 54' 42"	Schwassmann, Wachmann, Bergedorf—Lippert, phot.	Beob. Zirk. No. 39, 1927
(2)	Nov. 18.8506	1 31 10.7	+20 46 47	Schwassmann, Wachmann, Bergedorf—Lippert, phot.	Beob. Zirk. No. 39, 1927
(3)	Nov. 22.7171	1 29 54.0	+20 36 36	Baade, Bergedorf	Beob. Zirk. No. 40, 1927
(4)	Nov. 22.7693	1 29 53.5	+20 36 25	Schwassmann, Bergedorf—Lippert, phot.	Beob. Zirk. No. 40, 1927
(5)	Nov. 23.28819	1 29 43.24	+20 35 02.1	Jeffers, Lick—36", visual	Letter
(6)	Nov. 25.26013	1 29 07.39	+20 29 56.7	Jeffers, Lick—Crossley, phot.	Letter
(7)	Nov. 26.09020	1 28 53.19	+20 27 49.3	Van Biesbroeck, Yerkes—24", phot.	Letter
(8)	Nov. 28.13181	1 28 19.80	+20 22 38.7	Van Biesbroeck, Yerkes—24", phot.	Letter
(9)	Dec. 1.19676*	1 27 33.92	+20 15 02.8	Van Biesbroeck, Yerkes—24", phot.	Letter
(10)	Dec. 4.03443	1 26 56.95	+20 08 22.5	Baade, Bergedorf	Beob. Zirk. No. 43, 1927
(11)	Dec. 14.10362	1 25 27.56	+19 47 08.6	Van Biesbroeck, Yerkes—40", visual	H.C.O. Card No. 46
(12)	Dec. 14.7696	1 25 24.0	+19 46 00	Baade, Bergedorf	Beob. Zirk. No. 43, 1927
(13)	Dec. 15.21767*	1 25 21.86	+19 45 10.2	Jeffers, Lick—Crossley, phot.	Letter
(14)	Dec. 19.18740	1 25 08.72	+19 38 17.2	Van Biesbroeck, Yerkes—40", visual	Letter
(15)	Dec. 20.86860	1 25 05.72	+19 35 43.2	Wolf, Heidelberg—28", phot.	Beob. Zirk. No. 44, 1927
(16)	Dec. 21.13447	1 25 06.22	+19 35 35.7	Van Biesbroeck, Yerkes—24", phot.	Letter
(17)	Dec. 22.19289	1 25 04.88	+19 33 50.0	Van Biesbroeck, Yerkes—40", visual	Letter
(18)	Dec. 24.19799	1 25 04.93	+19 30 41.6	Van Biesbroeck, Yerkes—40", visual	Letter
(19)	Dec. 28.83815	1 25 20.41	+19 25 30.8	Baade, Bergedorf	Beob. Zirk. No. 1, 1928
(20)	Dec. 29.19183*	1 25 21.84	+19 25 09.0	Van Biesbroeck, Yerkes—24", phot.	Letter
(21)	Jan. 11.08273	1 27 21.44	+19 17 27.8	Van Biesbroeck, Yerkes—24", phot.	Letter
(22)	Jan. 12.09379	1 27 35.42	+19 17 24.9	Van Biesbroeck, Yerkes—24", phot.	Letter
(23)	Jan. 13.19166	1 27 50.49	+19 17 08.1	Crawford, Lick—Crossley, phot.	H.C.O. Card No. 51
(24)	Jan. 18.17708	1 29 10.49	+19 17 29.9	Crawford, Lick—Crossley, phot.	H.C.O. Card No. 51
(25)	Jan. 20.15695	1 29 45.99	+19 18 01.0	Crawford, Lick—Crossley, phot.	H.C.O. Card No. 55
(26)	Jan. 21.12042	1 30 04.21	+19 18 25.2	Van Biesbroeck, Yerkes—24", phot.	Letter
(27)	Jan. 21.19583	1 30 05.65	+19 18 26.2	Crawford, Lick—Crossley, phot.	H.C.O. Card No. 55
(28)	Jan. 22.06098	1 30 22.42	+19 18 47.0	Van Biesbroeck, Yerkes—24", phot.	Letter
(29)	Jan. 23.04020	1 30 41.98	+19 19 25.6	Van Biesbroeck, Yerkes—24", phot.	Letter
(30)	Feb. 10.17708	1 38 17.16	+19 38 17.8	Crawford, Lick—Crossley, phot.	Letter
(31)	Feb. 11.20556	1 38 47.59	+19 39 56.4	Crawford, Lick—Crossley, phot.	Letter
(32)	Feb. 12.05126	1 39 13.27	+19 41 15.8	Van Biesbroeck, Yerkes—24", phot.	Letter
(33)	Feb. 13.04803	1 39 43.77	+19 42 57.9	Van Biesbroeck, Yerkes—24", phot.	Letter

* Mean of 2 exposures.

The problem of determining the orbit has presented several points of theoretical interest and there are at the present time eleven orbits extant. In computing a preliminary parabolic orbit using Leuschner's method it was found expedient to dispense with the accelerations in α and δ because their real values appeared to be masked by the errors of observation. This step shortened the computation to such an extent that a direct solution based upon the observations (1), (4), (5) of November 15, 22, 23, was readily obtained but produced larger residuals than ordinarily would be expected. These residuals were chiefly due to an error in the observed α , November 22, and also to the inadvertent combination of the average velocities with the observation of November 22, near the end of the arc, instead of with an observation near the middle of the arc, to which the adopted velocities actually corresponded for the constant but indeterminate small accelerations. The inaccuracies thus introduced were rectified by shifting to (2) Nov. 18, as middle date, and making a differential correction of the first preliminary parabola by removing the residuals for (1) and (6), Nov. 15 and 25, which latter in the meantime had come to hand.

Because of the slow motion and small inclination which suggested ellipticity an elliptic orbit was also computed with the data contained in the differential correction for the parabola. The elements and short ephemerides were distributed in a private circular to the Yerkes, Lick, and Mt. Wilson observatories and aided in further observations. The parabolic and elliptic elements are:

I. Parabola, 1927.0				II. Ellipse, 1927.0					
$T=1929 \text{ Mar. } 18.5624 \text{ U. T.}$				$T=1924 \text{ July } 29.3302 \text{ U. T.}$					
$\omega=147^{\circ} 34' 16''$				$\omega=326^{\circ} 24' 17''$					
$\Omega=316 \quad 02 \quad 39$				$\Omega=322 \quad 20 \quad 27$					
$i= \quad 8 \quad 59 \quad 09$				$i= \quad 9 \quad 30 \quad 46$					
$\log q=0.57322$				$e=0.263319$					
				$\log q=0.67165$					
				$P=16.09 \text{ years}$					
O-C	(1)	(6)		(1)	(2)	(4)	(5)	(6)	
$\Delta\alpha$	$-2''$	$0''$		$\Delta\alpha$	$0''$	$0''$	$+10''$	$+2''$	$0''$
$\Delta\delta$	$+5$	0		$\Delta\delta$	0	0	0	-1	0

Although the representations are excellent considering that for the parabola the outstanding errors are concentrated in the first position and that there is an evident slight error in α , November 22, reliable preliminary elements were not to be expected from such a slow moving object. The greatest uncertainty was attached to the time of perihelion passage with a range of five years, according to orbits deduced by different investigators.² In the list of elements referred to we find that the elliptic orbits by Smiley, Berman,

² *Publ. A.S.P.*, 40, 232, 1928.

and Cunningham all place the perihelion in 1925 and 1926, while the elliptic orbit by C. Vick is the only elliptic orbit which, like our original parabola, puts the perihelion in 1929. The explanation for this wide divergence of results lies in the fact that the early observations were secured near the extremity of the semi-minor axis of the ellipse. In this position even minute inaccuracies of the observations or variation in the choice of the basic observations or both, coupled with the fact that the orbit is nearly circular, would change the perihelion from one to the other of the two opposite positions. This was made evident when in differentially correcting orbits we found the velocity along the radius vector fluctuating between small negative and small positive values, depending upon the observations used. This would indicate that an initial circular orbit would have satisfied the observations.

On receipt of later positions from the Yerkes and Lick Observatories a second elliptic orbit was computed by Berman by differentially correcting Elements II on the basis of observations (1), (6), November 15, 25, and a normal place formed from (11), (13), for December 14.66064. The residuals to be removed were computed from the observed α and δ for the middle date and from the geocentric distance and the heliocentric velocities for the same date corresponding to Elements II. These residuals were:

O-C	Nov. 15	Dec. 14
$\Delta\alpha$	+1"3	+78"7
$\Delta\delta$	+0.3	- 3.6

The resulting elements are:

Elements III, 1927.0			
$T=1926 \text{ Oct. } 16.123 \text{ U. T.}$			
$\omega=43^\circ 28' 56''$			
$\Omega=321 \quad 53 \quad 0.9$			
$i= \quad 09 \quad 22 \quad 59.4$			
$e=0.078681$			
$\log a=0.810074$			
$\log q=0.774484$			
$P=16.41 \text{ years}$			

O-C	Nov. 15	Dec. 14
$\Delta\alpha$	0"	-0"6
$\Delta\delta$	0	+1.4

These elements and an ephemeris were distributed on *Harvard College Observatory Announcement Card No. 48*.

Subsequent observations made by Crawford at the Lick Observatory a month later gave the following residuals:

O-C	$\Delta\alpha$	$\Delta\delta$
(23) Jan. 13.19	-26"7	+11"0
(24) Jan. 18.18	-34.7	+11.8
(25) Jan. 20.16	-45.1	+ 8.7
(27) Jan. 21.20	-48.9	+10.1

After shifting to Dec. 15 as new middle date, a normal place for Nov. 18 by combining (1), (2), (3) and the position (27), Jan. 21, were selected as first and third dates for a further differential correction of the orbit with the following results:

Elements IV, 1928.0

$T=1925$ May 15.040 U. T. $a=6.42840$
 $\omega=0^\circ 23' 42''.9$ $\mu=217^\circ 69' 75$
 $\Omega=322 \ 40 \ 45.4$ $q=5.51423$
 $i=9 \ 25 \ 47.4$ $P=16.299$ years
 $e=0.142208$

Constants for the Equator 1928.0

$x=r[9.997846]\sin(53^\circ 26' 59''.0+v)$
 $y=r[9.933960]\sin(320 \ 02 \ 15.8+v)$
 $z=r[9.717361]\sin(332 \ 50 \ 47.9+v)$

These results and an ephemeris extending from Jan. 15, 1927 to March 11, 1928, have been distributed in *Harvard College Observatory Announcement Card* No. 53. An extension of the ephemeris is being deferred until the comet will have moved into a more favorable position for observation.

This orbit represents the observations to date as follows:

1927-1928, U. T.	$\Delta\alpha$	$\Delta\delta$
(1) Nov. 15.90	- 5"	- 2"
(2) Nov. 18.85	- 1	- 1
(3) Nov. 22.72	+ 3	+ 3
(4) Nov. 22.77	[+11]	0
(5) Nov. 23.29	+ 3.6	- 1.3
(6) Nov. 25.26	- 0.5	+ 0.4
(7) Nov. 26.09	+ 0.6	+ 1.1
(8) Nov. 28.13	+ 2.8	+ 0.5
(9) Dec. 1.20	- 0.7	- 3.2
(10) Dec. 4.03	+ 0.5	- 0.9
(11) Dec. 14.10	- 1.0	- 4.6
(12) Dec. 14.77	- 2	+ 1
(13) Dec. 15.22	0.0	+ 0.1
(14) Dec. 19.19	+ 7.2	- 4.5
(15) Dec. 20.87	- 0.4	- 1.0
(16) Dec. 21.13	[+10.4]	[+15.3]
(17) Dec. 22.19	- 3.3	+ 3.0
(18) Dec. 24.20	[-23.1]	[-19.2]
(19) Dec. 28.84	+ 3.1	+ 0.2
(20) Dec. 29.19	+ 0.3	+ 0.5
(21) Jan. 11.08	+ 0.2	+ 1.2
(22) Jan. 12.09	+ 5.6	+ 9.7
(23) Jan. 13.19	- 0.8	+ 0.6
(24) Jan. 18.18	+ 3.7	+ 1.7
(25) Jan. 20.16	- 0.3	- 1.1
(26) Jan. 21.12	- 0.4	+ 1.5

1927-1928, U. T.	$\Delta\alpha$	$\Delta\delta$
(27) Jan. 21.20	- 0.4	+ 0.2
(28) Jan. 22.06	- 2.4	- 1.3
(29) Jan. 23.04	- 3.3	+ 8.2
(30) Feb. 10.18	- 2.0	- 1.4
(31) Feb. 11.21	- 4.6	+ 0.3
(32) Feb. 12.05	- 2.6	- 2.3
(33) Feb. 13.05	- 2.4	- 0.2

The small average residuals $\cos \delta \Delta\alpha = -2''.7$, $\Delta\delta = -0''.9$, for the four consistent observations Feb. 10-Feb. 13 renders unnecessary the further improvement of the orbit at this time.

Elements derived by Crommelin³ from observations made at Bergedorf Nov. 15, Dec. 4, 28, are in good agreement with this orbit.

With the arc of two months the period of 16 years is now confirmed. The comet passed perihelion 2.5 years before discovery. It appears very probable that it will remain under observation a long time if not for the duration of its entire path. According to our orbit the comet on Dec. 15 was approximately 3 astronomical units from Jupiter with no past approach in several years but indications of an approach before 1932. The orbit lies completely outside of Jupiter's orbit with a minimum distance of approach of 0.3 astronomical units. This comet has the largest perihelion distance and smallest eccentricity of any comet heretofore discovered. We have mentioned elsewhere⁴ the peculiar place this object occupies along with others among the asteroids and comets. A possible explanation for such an apparent incongruity was given by Prof. Leuschner several years ago and again more recently⁵. He concludes that since we have comets with planetary orbits and minor planets with cometary orbits, it is perfectly safe to consider them as belonging to the same class on the supposition that the physical distinction between these objects disappears when a comet loses its nebulous appendage. Moreover, it is now believed that comets are not inherently luminous but only become so as they approach near to the Sun. Confirmation of this view is derived from a spectroscopic study of both types of objects.

³ *Bureau Central Astronomique*, Circulaire No. 183, 1928.

⁴ *Publ. A.S.P.* 40, 232, 1928.

⁵ *Publ. A.S.P.* 39, 231, 1927.

BERKELEY ASTRONOMICAL DEPARTMENT,

February 15, 1928.

Issued February 27, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 395

ELEMENTS AND EPHEMERIS OF COMET *k* 1927 (SKJELLERUP)

BY

N. U. MAYALL, H. G. MILES, and F. L. WHIPPLE

This comet was discovered independently by a number of observers in Australia, New Zealand, South America, and South Africa. The only approximate discovery positions announced by cable were by Skjellerup on December 3, Melbourne, Australia, and by Maristany on December 6, La Plata, Argentina. For this reason the comet hitherto has been designated as Skjellerup-Maristany. As a number of other dis-

coverers have anticipated Maristany and possibly Skjellerup and as the discoveries were known throughout the countries referred to above, it seems appropriate to attach only the name of Skjellerup to the comet, in view of his definite priority of announcement, according to present information, by cable to a central station or to an observatory. The complete list of observations received so far is given below.

1927, U. T.	α	δ	Observer	Authority
(1) Dec. 3.7292	16 ^h 12 ^m	12°	Skjellerup, Melbourne—Cir. Rdg.	H.C.O. Card No. 40, 1927
(2) Dec. 5.6460†	16 27 05	—51 10	McIntosh, Auckland	Circ. B.C.A. No. 185, 1928
(3) Dec. 6.025	16 27	—50 00	Maristany, La Plata—Cir. Rdg.	H.C.O. Card No. 41, 1927
(4) Dec. 6.106	16 30 30	—50 22	Wood, Johannesburg—Cir. Rdg.	Circ. B.C.A. No. 185, 1928
(5) Dec. 7.8586	16 41 17	—47 48 10"	Castro, Santiago	H.C.O. Card No. 42, 1927
(6) Dec. 8.3279	16 48 28	—45 37.2	Hartmann, La Plata—Cir. Rdg.	Beob. Zirk. No. 43, 1927
(7) Dec. 9.121	16 55 48	—43 44	Wood, Johannesburg—Cir. Rdg.	A.N. 231, 415, 1928
(8) Dec. 17.4866*	17 40.1	—16 35	Schwassmann, Wachmann, Bergedorf	Circ. B.C.A. No. 185, 1928
(9) Dec. 17.5096*	17 40.2	—16 32	Schwassmann, Wachmann, Bergedorf	Beob. Zirk. No. 43, 1927
(10) Dec. 17.6264	17 40 22	—16 15.1	Graff, Bergedorf—Cir. Rdg.	Beob. Zirk. No. 43, 1927
(11) Dec. 18.6174	17 43 08	—14 05.1	Graff, Bergedorf—Cir. Rdg.	Beob. Zirk. No. 43, 1927
(12) Dec. 19.5631	17 45 11	—12 40.7	Van Biesbroeck, Yerkes—40", Cir. Rdg.	H.C.O. Card No. 48, 1927
(13) Dec. 19.6726	17 45 32	—12 35	Hoffmeister, Sonneburg—Cir. Rdg.	Beob. Zirk. No. 44, 1927
(14) Dec. 20.55115	17 46 58	—11 49.5	Van Biesbroeck, Yerkes—40", Cir. Rdg.	H.C.O. Cards Nos. 48, 49, 1927, 1928
(15) Dec. 20.6496	17 47 11	—11 45 00	G. Struve, Berlin-Babelsberg	Beob. Zirk. No. 43, 1927
(16) Dec. 20.94917	17 47 41	—11 37.7	Van Biesbroeck, Yerkes—40", Cir. Rdg.	H.C.O. Cards Nos. 48, 49, 1927, 1928
(17) Dec. 21.54744	17 48 43	—11 26.8	Van Biesbroeck, Yerkes—40", Cir. Rdg.	H.C.O. Cards Nos. 48, 49, 1927, 1928
(18) Dec. 21.6118	17 48 52.2	—11 25 54	Vaisala, Turku—Sextant	A.N. 232, 42, 1928
(19) Dec. 21.6353	17 49 55.0	—11 25 30	G. Struve, Berlin-Babelsberg	H.C.O. Card No. 47, 1927
(20) Dec. 22.5594	17 50 24	—11 23.3	Van Biesbroeck, Yerkes—40", Cir. Rdg.	H.C.O. Card No. 48, 1927
(21) Dec. 23.5549	17 51 57	—11 31.7	Van Biesbroeck, Yerkes—40", Cir. Rdg.	H.C.O. Card No. 48, 1927
(22) Dec. 24.53244	17 53 50	—11 48.0	Van Biesbroeck, Yerkes—40", Cir. Rdg.	H.C.O. Card No. 49, 1928

† Derived by Crommelin from sketch by McIntosh.

* Apparent positions.

The majority are approximate circle readings and may be considered as mean places for the beginning of the year unless they are designated as apparent places.

A general account of the work on the orbit of the

comet undertaken at the Students' Observatory under the direction of Professor Leuschner has been printed in the *Publications of the Astronomical Society of the Pacific*, 40, 233, February, 1928, pages 38-46. In addi-

tion to orbit computations this work included attempts to identify the comet with comet 1846 IV (DeVico) by Messrs. N. U. Mayall and H. G. Miles and independently by Mrs. M. Makemson, and resulted in demonstrating that the comets are not identical. The orbit computations were commenced by Mr. Mayall and Mr. Miles as soon as three observations became known. These were the observations (1) of December 3.73 by Skjellerup, (5) of December 7.86 by Castro, and (12) of December 19.56 by Van Biesbroeck. The failure to derive an orbit which represented the later observations by Van Biesbroeck is accounted for by the approximate character of the observations of December 3 and 7, which could not be suspected from the cabled information. The latter position had been used as second place and also as first place for the determination of orbits. After the excellent series of observations by Van Biesbroeck became available an orbit was computed by Mayall and Miles from Van Biesbroeck's observations (12) of December 19.56, (17) of December 21.55, and (21) of December 23.55. On account of the rapid and irregular motion of the comet the velocities and accelerations derived from these three observations were corrected for the effect of third differences obtained by the use of (16) of December 20.95 and (20) of December 22.56, by means of the expressions

$$\Delta a' = -\frac{\theta, \theta'''}{6} a'''; \Delta a'' = (\theta''', -\theta'') \frac{a'''}{3}$$

and similar expressions for $\Delta \delta'$, $\Delta \delta''$, where a''' and δ''' represent the third derivatives of the coordinates expressed in terms of $\frac{1}{k}$ mean solar days obtained from the observed third differences of the coordinates. As a check the resulting velocities and accelerations were computed also from the five observations by the rigorous formulae.² This check, which was also verified by a method of interpolation by Dr. H. Thiele, proved that the observed fourth differences in the coordinates were practically negligible. The empirical correction of the velocities and accelerations deduced from three observations by means of observed third differences, if additional observations are available, is simple and effective and greatly increases the accuracy of the solution when the third differences, as in this case, are not negligible. The heliocentric coordinates and velocities for the middle date derived on the basis of these velocities and accelerations on the parabolic hypothesis did not represent the subsequent observation (22) of December 24.53 as satisfactorily as had been expected. due to the discrepancy of (21) of December 23.55 in right ascension from the remainder of the excellent

series by Van Biesbroeck as shown by residual (21) in the table of the residuals given below. These successive attempts at deriving an orbit produced the following unsatisfactory representations:

FROM (1), (5) AND (12), AFTER AN APPROXIMATE DIFFERENTIAL CORRECTION

(O-C)	Δa	$\Delta \delta$
(1) Dec. 3	-56'20"	-19'40"
(12) Dec. 19	+13 25	-04 00

Observation (1) was then dropped and (5) of December 7.86 was chosen as first date, with (12) of December 19.58 as middle date and (20) of December 22.56 as last date, with residuals for (5) and (20) computed in the usual manner from the observed a and δ for the middle date and from the geocentric distance and the heliocentric velocities for the same date corresponding to the previous solution. These residuals clearly showed that one or more of the original three observations (1), (5), (12), were too defective to obtain satisfactory results. An approximate differential correction to remove these residuals entirely proved unsatisfactory on account of the error in the observation (5) of December 7, which later was found to be approximately a degree in each coordinate.

When the parabolic solution based on the velocities and accelerations derived from five of Van Biesbroeck's observations failed to represent the observation (22) of December 24.5 on account of the discrepancy in a of (21), the work was discontinued by Messrs. Mayall and Miles.

An approximate value of unity for the geocentric distance for the new middle date, (17) of December 21, was then derived with the preceding erroneous value of $\frac{1}{m}$ by Mayall and Whipple from Leuschner's table³ for a parabolic hypothesis with the following results:

(O-C)	Δa	$\Delta \delta$
(12) Dec. 19	-4'1	-12'0
(21) Dec. 23	0.0	+22.0

To extend the differential correction over a longer arc which in the meantime had become available, the observations (11) of December 18.62 and (22) of December 24.53 were represented with the following results on which the differential correction for the first successful choice of three consistent observations was based:

(O-C)	Δa	$\Delta \delta$
(11) Dec. 18	-7'58"	-22'51"
(22) Dec. 24	+3 32	+39 12

Since the heliocentric distance was small, the closed expressions⁴ for δf and δg were used.

¹ *Publ. Lick Obs.*, 7, 267, 1913.

² *Ibid.*, pp. 229-232.

³ *Pub. Lick. Obs.*, 7, 1913, part 7.

⁴ *Ibid.*, p. 332.

PARABOLIC ELEMENTS, 1928.0

$T=1927 \text{ Dec. } 18.1162 \text{ U. T.}$
 $\omega=46^{\circ}40'46''$
 $\Omega=77 \text{ } 13 \text{ } 32$
 $i=85 \text{ } 12 \text{ } 41$
 $q=0.17524$

CONSTANTS FOR THE EQUATOR 1927.0

$x=r [9.37256] \sin (156^{\circ}52'24''+v)$
 $y=r [9.98756] \sin (159 \text{ } 40 \text{ } 38 +v)$
 $z=r [9.99997] \sin (\text{ } 69 \text{ } 30 \text{ } 40 +v)$

CONSTANTS FOR THE EQUATOR 1928.0

$x=r [9.37223] \sin (156^{\circ}53'35''+v)$
 $y=r [9.98763] \sin (159 \text{ } 40 \text{ } 10 +v)$
 $z=r [9.99997] \sin (\text{ } 69 \text{ } 30 \text{ } 58 +v)$

An ephemeris based on this orbit extending from January 31.0 to March 11.0, has been published in *Harvard College Observatory Announcement Card No. 52*.

EPHEMERIS, 1928.0, CONTINUED

1928 U. T.	α	δ	$\log \rho$	$\log r$	Mag.
Mar. 19.0	$19^{\text{h}}17^{\text{m}}4$	$-34^{\circ}49'$	0.333	0.318	7.3
27.0	19 16.7	-36 46	0.330	0.344	
Apr. 4.0	19 13.6	-38 49	0.326	0.368	
12.0	19 07.8	-40 57	0.323	0.390	
20.0	18 58.8	-43 08	0.320	0.410	
28.0	18 46.5	-45 16	0.319	0.429	7.8

The orbit is in close agreement with those derived by Van Biesbroeck and by Mrs. Makemson.⁵ An inspection of the residuals given below shows that a very satisfactory representation of the earlier observations has been obtained if the observations which have been bracketed are disregarded. The residuals of other

observations secured at different observatories are in good agreement. The correction of the orbit on the basis of normal places, however, may be deferred, as the ephemeris derived from the foregoing elements should be sufficient to locate the comet from the southern hemisphere.

The orbit represents the observations published to date as follows:

1927 U. T.	$\Delta\alpha$	$\Delta\delta$	Averages	
(1) Dec. 3.73	[+19']	[+ 9']		
(2) Dec. 5.65	- 1.4	- 6		
(3) Dec. 6.02	[-50]	[+23]	-4'.60	-5'.67
(4) Dec. 6.11	- 7.4	- 8		
(5) Dec. 7.86	[-58]	[-69]		
(6) Dec. 8.33	- 5	- 3		
(7) Dec. 9.12	[+14]	[- 7]		
(8) Dec. 17.49	- 1.2	- 3	-2.03	-3.97
(9) Dec. 17.51	- 0.7	- 4		
(10) Dec. 17.63	- 4.2	- 4.9		
(11) Dec. 18.62*	+ 0.6	- 3.6		
(12) Dec. 19.56	+ 1.4	- 1.7	-1.80	-2.77
(13) Dec. 19.67	+ 3.4	- 3		
(14) Dec. 20.55	+ 0.3	- 0.5		
(15) Dec. 20.65	+ 0.9	+ 0.7		
(16) Dec. 20.95	+ 0.3	- 0.4	+0.50	-0.07
(17) Dec. 21.55*	+ 0.0	- 0.2		
(18) Dec. 21.61	+ 0.6	0.0		
(19) Dec. 21.64	+ 0.7	+ 0.2		
(20) Dec. 22.56	- 1.0	- 0.2	+0.43	-0.00
(21) Dec. 23.55	- 3.4	+ 0.1		
(22) Dec. 24.53*	- 0.1	- 0.2		
			-1.50	-0.10

The starred observations are those on which the solution has been based.

BERKELEY ASTRONOMICAL DEPARTMENT,

March 10, 1928.

Issued March 16, 1928.

⁵ *Publ. A.S.P.*, 40, 233, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 396

THE SPECTRA OF THE CARBON STARS

BY

C. D. SHANE

INTRODUCTION

The spectra of the carbon stars are particularly difficult to photograph. The stars themselves are faint, and in those of later Class R and of Class N the spectral intensity decreases very rapidly toward the violet. Nevertheless, it is in this region that some of the most significant features of their spectra are to be found. Moderate or high dispersion spectrograms of normal exposure record only the longer wave-lengths. Low dispersion for the most part suffices for a study of the most prominent characteristics and permits the examination of many faint stars in the more refrangible region.

The present paper contains the observations of 50 stars of Classes R and N made with very low dispersion. Higher dispersion plates are included wherever possible. The behavior of the prominent spectral features is described and on this basis certain adjustments in the Harvard Classification are made. Finally the significance of this classification is briefly discussed.

INSTRUMENTS AND OBSERVATIONS

Three spectrographic arrangements were used in securing these plates.

Spectrograph *a* is a small slitless instrument with one quartz prism and quartz lenses and is used with the Crossley reflector.¹ The average dispersion between 5100Å and 3900Å is 650Å per mm.

Spectrograph *b* is the large 2-prism quartz spectrograph designed by Mr. Wright.² It was used in its slitless adaptation in conjunction with the Crossley reflector. The average dispersion between $H\beta$ and $H\epsilon$ is 120Å per mm.

Spectrograph *c* is a slit spectrograph having one light flint 60° prism and a 16-inch camera. It was attached to the 36-inch refractor. The dispersion at 4500Å is 64Å per mm.

Plates of 50 stars were secured with Spectrograph *a*, the exposure times varying from a few seconds to 5 hours. In most cases several exposures of different lengths were impressed on one plate in order to obtain properly exposed images over as long a stretch of spectrum as possible. In this way records of the Class R spectra extending from about 5200Å to 3400Å were ordinarily secured. In Class N they ranged from about 5400Å to 3800Å although among the later Class N stars, the spectrum usually extended only as far toward the violet as 4400Å.

Higher dispersion plates of a few typical stars of the spectral sub-classes were secured with Spectrograph *b*. These were used when a more detailed examination was desirable.

The plates with Spectrograph *c* were taken by Dr. Moore for radial velocity and were kindly placed at my disposal for this study.

The results are summarized in Tables I and II. Table I includes all the observations made with Spectrograph *a*. The first four columns are self explanatory. The fifth column tabulates the readjustment in the Harvard Classification which the present observations indicated. The significance of the remaining columns is described in the discussion of the prominent spectral features. Table II includes a few additional stars of which no observations were secured with Spectrograph *a*. Material homogeneous with that of Table I was therefore not available, and the stars are tabulated separately with their reclassification and such comments on their spectra as seem important.

¹ *Lick Obs. Bull.*, 2, 47, 1902.

² *Ibid.*, 9, 52, 1917.

TABLE I

$\alpha 1900$	$\beta 1900$	Mag.	Harvard Class.	Reclassifi- cation	Int. Cy. Bands	Int. 4737A Bands	Faintness in short wave-length	Int. 4600A Int. 4800A	Designation
19 ^b 17 ^m 7	-10° 53'	7.0	R0	R0	4	3	1	.8	HD182040
23 57.0	- 3 23	9.9	R0	R0	5	1	1	.9	HD224959
17 10.4	+42 15	7.7	R0	R1	6	3	2	.8	HD156074
19 43.8	+85 9	9.6	R3	R2	6	5	3	.9	HD187216
2 30.2	- 9 53	8.3	R3	R3	9	5	2	.8	HD 16115
7 20.1	+22 7	10.0	R5	R3	8	6	2	.8	HD 58337
7 20.2	+22 5	9.8	R5	R3	8	6	2	.8	HD 58364
15 21.9	-24 49	7.4	R0	R3	7	3	3	.8	HD137613
21 59.7	+20 34	8.8	R3	R3	8	2	3	.9	HD209621
23 44.0	+ 5 50	8.8	R3	R3	8	3	3	.8	HD223392
9 8.3	+14 37	8.9	R5	R4	8	9	3	.7	HD 79319
19 3.0	-17 26	10.6	R0	R4	8	5	3	.7	HD178316
0 19.1	+53 44	9.7	R5	R5	9	9	3	.6	HD 1994
3 3.7	+57 31	8.1	R5	R5	8	10	4	.6	HD 19557
8 56.2	+50 29	9.5	R5	R5	10	10	4	.6	HD 77234
18 4.0	+ 9 26	10.0	R5	R5	10	9	4	.6	HD166097
12 54.7	+38 20	9.2-9.6	Na	R6p	5	8	3	.8	HD112869
8 42.4	-29 21	7.6	R8	R7	6	7	5	.6	HD 75021
2 9.6	+11 47	8.3-9.0	Nb	R8	2	9	5	.7	V <i>Arietis</i>
3 57.2	+61 32	7.9	R8	R8	7	6	5	.6	HD 25408
19 52.5	- 0 2	9.8	R3	R8	6	3	5	.8	HD188934
7 25.8	+24 44	8.2	R8	R9	3	4	5	.8	HD 59643
1 10.6	+25 14	7.4-8.1	Na	N0	2	4	6	.8	Z <i>Piscium</i>
5 4.9	- 5 38	8.6	Nb	N0	1	5	6	.7	HD 33404
5 41.7	+30 36	9.0-9.3	Na	N0	2	6	6	.8	HD 38572
19 25.1	+76 23		Nb	N0	1	6	6	.7	UX <i>Draconis</i>
23 41.3	+ 2 56	5.3	Na	N0	1	4	6	.8	HD223075
21 37.8	+35 3	6.4	Nb	N1	0	6	7	.7	HD206570
23 56.2	+59 48	7.6	Na	N1p	0	2	7	.9	HD224855
5 39.1	+24 23	8.7-9.5	Nb	N2 ³		6		.6	TU <i>Tauri</i>
5 39.7	+20 39	6.9-8.9	Nb	N2	0	7	7	.5	Y <i>Tauri</i>
4 45.2	+28 21	8.1-8.8	Nb	N3	0	8	8	.5	TT <i>Tauri</i>
6 4.7	+26 03	7.4-8.3	Na	N3		6	8	.4	TU <i>Geminorum</i>
6 29.7	+38 31	6.2-6.7	Na	N3		6	8	.5	UU <i>Aurigae</i>
12 40.4	+45 58	4.8-6.0	Nb	N3	0	7	9	.5	Y <i>Canum Venaticorum</i>
18 44.9	- 8 1	6.4-7.3	Nb	N3		5	9	.5	S <i>Scuti</i>
18 50.0	- 8 19	8.8-9.3	Nb	N3		6	9	.5	T <i>Scuti</i>
20 6.4	+47 33	8.5-9.4	Nb	N3		6	9	.5	SV <i>Cygni</i>
21 51.7	+22 24	7.7-8.6	Nb	N3	0	10	8	.5	RX <i>Pegasi</i>
12 52.5	+66 32	6.1-7.1	Np	N4		7	9	.4	RY <i>Draconis</i>
18 54.0	+14 14		Nb	N4		7	9	.4	UV <i>Aquilae</i>
5 0.2	+ 1 2	5.9-7.7	Nb	N5				.3	W <i>Orionis</i>
6 39.4	+ 3 25	9.6	Nc	N5				.3	HD 48664
21 39.1	+37 34	7.1-9.3	N	N5				.3	RV <i>Cygni</i>
4 55.0	-14 57	6.1-9.7	Nc	N6				.3	R <i>Leporis</i>
18 59.1	- 5 50	6.5-8.0	Np	N6				.3	V <i>Aquilae</i>
0 14.6	+44 09	8.1-9.5	Nc	N7				.2	VX <i>Andromedae</i>
5 30.2	+68 45	7.8-10.8	R8	Np ⁴					S <i>Camelopardalis</i>
12 20.1	+ 1 20	7.2-9.0	Pec.	Np ⁵					SS <i>Virginis</i>
20 16.5	+47 35	6.1-11.8	R8	Np ⁶					U <i>Cygni</i>

³ The spectrum is composite indicating a Class A0 companion of eleventh magnitude. *Publ. A.S.P.*, 37, 35, 1925.

⁴ S *Camelopardalis* Var. 7.8-10.8. One plate was secured with spectrograph *a* on March 21, 1925, when the star's magnitude was about 10. This spectrum is exactly like that of U *Cygni* at minimum light.

⁵ SS *Virginis* Var. 7.2-9.0. The Draper Catalogue describes the spectrum as similar to VX *Andromedae* which would make it late Class N. A plate taken with spectrograph *a* on March 21, 1925, when the magnitude was 8.1 records a spectrum closely resembling U *Cygni* at minimum.

⁶ U *Cygni* Var. 6.1-11.8. The spectrum is variable. A discussion of the variations based on numerous observations covering a light range mag. 6.8 to mag. 9.5 is given in *Lick Obs. Bull.* 10, 89, 1920. A plate obtained with spectrograph *a* on July 20, 1925, when the star's magnitude was 10.4 records a spectrum somewhat resembling Class N6. The Swan band at 4737A is not quite so strong as in Class N6 but to the violet of this band the spectrum fails to increase appreciably in strength, a feature peculiar to stars of classes later than N4. There is some slight evidence of the unidentified bands peculiar to the late Class N stars, but owing to the small dispersion used this is very uncertain.

TABLE II

$\alpha 1900$	$\delta 1900$	Mag.	Harvard Class.	Reclassification	Instruments	Designation
3 ^h 33 ^m 2	+62°19'	6.9-9.0	Nb	N5	b	U <i>Camelopardalis</i>
4 40.8	+67 59	7.0-8.3	Nb	N5	c	ST <i>Camelopardalis</i>
6 19.8	+14 48	6.6	Nb	N0	c	HD44984
8 49.7	+17 37	6.1-6.6	Nb	N3	bc	X <i>Cancri</i>
9 46.4	-22 32	6.5-8.0	Np	N3	c	Y <i>Hydrae</i>
10 32.6	-12 52	4.5-6.3	Nb	N2	bc	U <i>Hydrae</i>
10 38.1	+67 56	6.3	Na	N0	c	HD92839
18 39.4	+36 51	8.1	Nb	N4	c	HD173291
19 13.4	-16 5	7.2	Na	N2	bc	HD180953
19 28.6	-16 35	(8.5-9.7) ⁷	Nb	N3	c	AQ <i>Sagittarii</i>
20 9.8	+38 26	7.5-8.7	Pec.	Nop ⁸	bc	RS <i>Cygni</i>

DESCRIPTION OF PROMINENT SPECTRAL FEATURES

Cyanogen bands.—The cyanogen band groups with heads at 4216A, 3883A and 3590A are found in varying strength in stars of Classes G and K. An investigation by Lindblad⁹ has shown that in the giant stars, the bands reach a maximum intensity between Classes G5 and K5, after which they rapidly decline.

The cyanogen bands exhibit a similar behavior among the carbon stars but on a much intensified scale. They are very prominent features of the Class R spectra but sink into insignificance early in Class N. Among the earliest Class R stars they exert several times the absorbing effect observed in the giant stars of Classes G and K. In Class R0 the 3883A group absorbs roughly 20% of the star's light between 3883A and 3590A, while in Class R5 the amount absorbed rises to 50% or more. In the latter case the absorption appears to be complete for a distance of at least one hundred angstroms from the head of the group. The absorption decreases rapidly beyond Class R5 and disappears entirely before Class N3 is reached.

Numbers on an arbitrary scale indicating the intensities of these bands are given in the sixth column of Table I. In the case of the earlier type stars the estimates are based principally on the 3883A group, but after R8 the spectra were frequently so weak in the ultra-violet that it was necessary to rely on estimates of the 4216A group, allowance being made for its smaller intensity. This involves a slight uncertainty because there is some evidence that as the spectral type advances the less refrangible group increases in intensity relatively to the other. This source of error is not great, however, and can not affect the qualitative value of the results.

Figure 1 illustrates graphically the variations of intensity of the cyanogen bands with spectral class. Already very strong in the Class R0 stars, they increase

rapidly as the spectral type advances reaching a maximum at R5. Beyond R5 the decrease is even more rapid and by the time N3 is reached they have entirely disappeared.

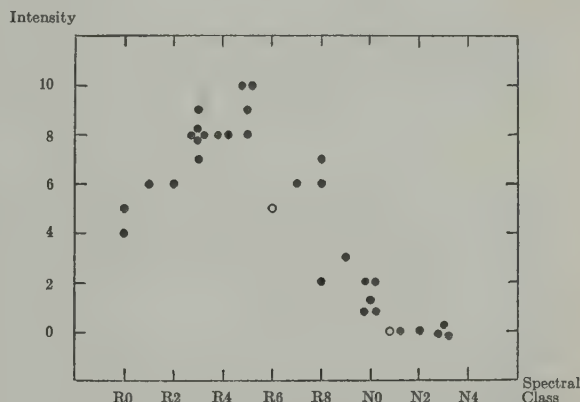


Fig. 1. Intensity of the cyanogen bands. (The open circles indicate stars with peculiar spectra.)

The above discussion relates only to the three groups at 4216A, 3883A and 3590A. Another group of cyanogen bands at 4606A behaves in an entirely different manner. Its separate bands have very little extension toward the violet, being scarcely distinguishable under low dispersion from broad lines. The intensities of these bands do not vary rapidly from one spectral subdivision to another. An examination of 17 plates taken with Spectrograph *b* reveals a noticeable increase in their strength as far as Class R8, beyond which they are nearly constant through Classes N0, N3 and N5. This behavior differs markedly from that of the other cyanogen groups.¹⁰

Carbon bands.—The spectra of the carbon stars are readily distinguished from all other stellar spectra by the presence of the "Swan Bands." If, as is generally

⁷ These magnitudes are photographic.

⁸ RS *Cygni* Var. 7.5-8.7. The spectrum resembles Class N0 except that the intensity to the violet of 4600A decreases much more rapidly than in other early Class N stars.

⁹ *Contrib. Mt. Wilson Obs.*, 11, 1, 1921.

¹⁰ Conclusions regarding the behavior of the cyanogen spectrum in the carbon stars have been heretofore largely drawn from studies of the group at 4606A. It is now clear that this group is not typical of the entire cyanogen spectrum. For references see *Publ. Yerkes Obs.*, 2, 379, 1904. *Publ. Detroit Obs.*, 2, 139, 1916. *Lick Obs. Bull.*, 10, 85, 1920.

believed, the carbon stars form a secondary branch of the spectral sequence, departing from the main stem in Class G or Class K, we might expect to observe a progressive increase in the carbon absorption with advancing spectral type. Rufus,¹¹ however, has indicated that, instead of such a progressive increase in carbon absorption running through Classes R and N, there exist two parallel sequences, one for Class R and one for Class N.

The seventh column of Table I contains estimates of the intensity of the 4737A group of the Swan spectrum. These intensities are plotted against spectral class in Figure 2. It is evident that if the badly scattered points at R7 and R8 are omitted, the conclusions

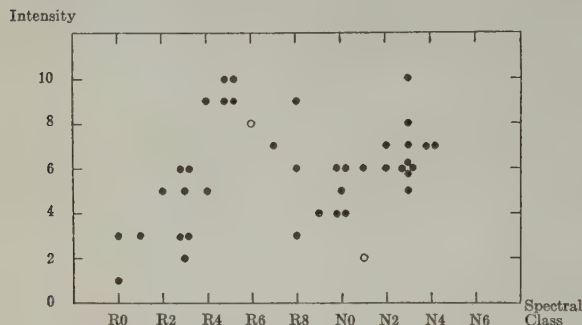


Fig. 2. Intensities of the Swan band at 4737A.

of Rufus are verified. If we include them, however, there is a clearly defined maximum at R5 followed by a minimum at N0 with a subsequent rise. Reliable estimates of the intensities in Classes N5–N7 could not be made in the case of the 4737A band because of rapidly increasing absorption toward the violet in this region.

The Swan groups at 5636A, 5165A and 4383A are represented in all the carbon stars. Their intensities appear to vary with spectral class in the same manner as that of 4737A. This is certainly true of 4383A and a rather limited number of spectrograms in the visual region supports the same conclusion in the case of the other bands.

The group with heads at 6191A, 6122A and 6060A is visible on spectrograms of the visual region. The bands, however, are very narrow, being little more than broadened lines in marked contrast to the bands of other groups which are frequently so broad as to be entirely run together. No plates are available for their study in Classes R3–R8, but spectrograms of other Class R and of Class N stars indicate the same variations of intensity that are exhibited by the band at 4737A.

Spectral intensity distribution.—In the main branch of the spectral sequence, it is observed that the more

¹¹ *Publ. Detroit Obs.*, 2, 140, 1916.

refrangible portions of the spectrum decrease in relative intensity with advancing spectral class. This same effect is present to a much greater extent in the carbon stars. Among the Class R0 stars, the intensity distribution, as shown on plates taken with the small slitless spectrograph, does not differ greatly from that of the Class G stars. As we proceed beyond Class R0, there is an increasing faintness in the ultra-violet which continues at an accelerated rate until in Class N3 it is practically impossible to photograph the spectrum beyond 3900A. The most striking illustration is afforded by *Y Canum Venaticorum* (Class N3). This star is brighter than the sixth magnitude and with the small slitless spectrograph can be photographed near 4800A in one second. An exposure of five hours, however, failed to show any light of wave-length shorter than 3900A. No doubt the stars of Classes N4–N7 are even fainter in the ultra-violet, but in the case of the brightest of these, exposures of the order of fifteen or twenty hours would probably be required to settle the question.

While it is true that among the Class N stars the decrease in spectral intensity toward the shorter wave-lengths is continuous throughout the entire region photographed, the most rapid decrease occurs between 4380A and 4200A. This sudden drop is not observable among the Class R stars, except slightly in Class R8. In Class N0 it is more pronounced and in Class N3 there is a very sudden decrease of intensity in this region. Nearly all of the later Class N stars are too faint here to be easily photographed.

Numbers on an arbitrary scale from 1 to 10, indicating the increasing faintness of the spectrum in the more refrangible region, are recorded in column 8 of Table I. These numbers are plotted in Figure 3.

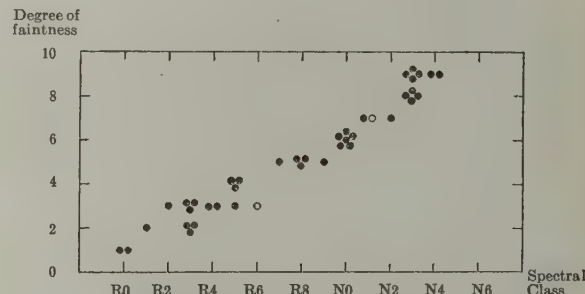


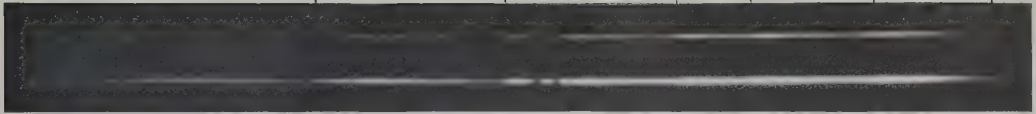
Fig. 3. Faintness in the more refrangible region.

As in the main spectral sequence, it is evident that the relative weakening of the violet spectrum parallels the increasing redness of the stars.

Relative intensities on either side of the 4737A band.—In the Harvard Classification the subdivisions of Class N depend largely on the ratio of the spectral intensities on either side of the 4737A band. This ratio was also

3590A 3883A 4216A 4382A 4737A 5165A

HD182040
Class R0



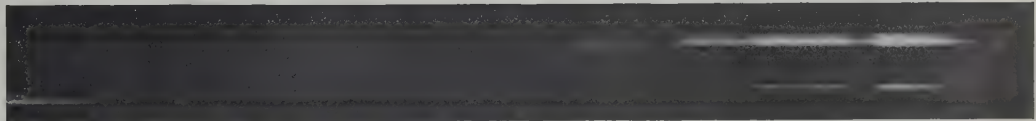
HD16115
Class R3



HD19557
Class R5



HD25408
Class R8



HD223075
Class N0



Y Canum
Venaticorum
Class N3



V Aquilae
Class N6



Cyanogen Bands

Swan Bands

determined in the present study, the intensities at approximately 4600Å and 4800Å being compared. The estimated values are tabulated in column 9 of Table I and are plotted against spectral class in Figure 4.

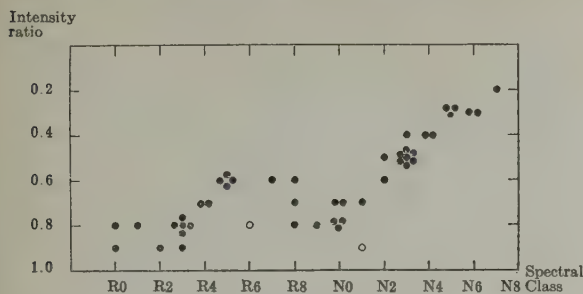


Fig. 4. $\frac{\text{Intensity at 4600}\text{\AA}}{\text{Intensity at 4800}\text{\AA}}$

The ratio is of course influenced by the general weakening of the spectrum toward the violet, and so might be considered a measure of the redness of the stars. An examination of Figure 4, however, reveals a definite maximum at Class R5. The similarity of this curve to that of Figure 2 suggests that we are dealing with a property depending on the intensity of the 4737Å band in addition to the general weakening of the spectrum toward the violet. If such a relation actually exists, there should be a tendency for stars departing from the mean positions for their spectral classes in Figure 2 to depart in the same sense in Figure 4. This was tested in the following manner: Mean values of the intensity of the 4737Å band and of the ratio Int. 4600Å/Int. 4800Å were calculated for each spectral subdivision, and these values compared with corresponding results for individual stars. Among 23 cases in which departures from the mean existed, 16 were favorable to the hypothesis and 7 unfavorable. It therefore appears very probable that the ratio Int. 4600Å/Int. 4800Å depends on the strength of the carbon absorption as well as on the general faintness in the short wave-lengths.

The G band.—The hydrocarbon band designated as G by Fraunhofer is present in most of the carbon stars. The small dispersion of Spectrograph *a* did not record this band satisfactorily, so that observations were limited to 12 plates secured with Spectrograph *b*. A study of these revealed an increase in the intensity of G from R0 to R3 where it appears with unusual strength. Beyond R3 its intensity decreases slowly at first and then more rapidly until in N3 it is nearly or quite absent.

There are, however, some curious irregularities in intensity. In the star HD82040 (Class R0) it is scarcely visible, while in HD156074 (Class R1), it is very strong. The existence of such irregularities combined with the small number of stars examined causes

some uncertainty in the position of maximum development but there is not much doubt that it lies somewhere in the range R2–R5.

Unidentified bands.—Sanford¹² has directed attention to five bands of unknown origin which occur in a number of the Class N stars. The two most prominent bands, of wave-lengths 4979Å and 4868Å, appear to be degraded toward the red. The remaining three bands with the more refrangible edges at 5035Å, 4932Å and 4905Å are of uncertain character and are not definitely degraded in either direction.

On plates of the late Class N stars secured with Spectrographs *b* and *c* in the present study, the bands measured by Sanford at 5035Å and 4932Å are faintly recorded while the remaining three are conspicuous features of the spectrum. Their wave-lengths derived from plates of *W Orionis* and *V Aquilae* are 4976Å, 4905Å and 4866Å. In addition to these, three new bands were measured at 4642Å, 4572Å and 4540Å.

An examination of the plates at once suggests that these six bands fall into two very similar groups. In the two groups the separations of corresponding bands

Group I	Group II	Description of bands
4642Å	4976Å	Wide, strongly degraded toward red
4572Å	4905Å	Narrow
4540Å	4866Å	Moderate width, slightly degraded toward red

are nearly the same. Furthermore the relative intensities within each group appear to remain constant. These facts strongly suggest a common origin for the six bands.

TABLE III
INTENSITIES OF UNIDENTIFIED BANDS

Star's α	Class	Int.	4868Å	4979Å	Designation
6 ^b 19 ^m 8	N0	0			HD44984
10 38.1	N0	0			HD92839
19 25.1	N0	0			UX <i>Draconis</i>
23 41.3	N0	0			HD223075
21 37.8	N1	0			HD206570
23 56.2	N1p	0			HD224855
5 39.7	N2	1			Y <i>Tauri</i>
10 32.6	N2	2	V. Weak	Weak	U <i>Hydrae</i>
19 13.4	N2	0			HD180953
6 29.7	N3	0	V. Weak	Moderate	UU <i>Aurigae</i>
8 49.7	N3	1	?	Weak	X <i>Cancris</i>
9 46.4	N3	1			Y <i>Hydrae</i>
12 40.4	N3	1	?	V. Weak	Y <i>Canum Venaticorum</i>
18 44.9	N3	0			S <i>Scuti</i>
19 28.6	N3	0			AQ <i>Sagittarii</i>
18 39.4	N4	2			HD173291
3 33.2	N5	5			U <i>Camelopardalis</i>
4 40.8	N5	3			ST <i>Camelopardalis</i>
5 00.2	N5	4	Moderate	Strong	W <i>Orionis</i>
21 39.1	N5	5	Strong	V. Strong	RV <i>Cygni</i>
4 55.0	N6	5	Weak	Moderate	R <i>Leporis</i>
18 59.1	N6	5			V <i>Aquilae</i>

¹² Publ. A.S.P., 38, 177, 1926.

Although the relative intensities of the bands remain essentially the same in all stars, the absolute intensities vary greatly. Estimates were made on an arbitrary scale of the mean strength of 4976A and 4866A as an index of the group intensity and are included in Table III with Sanford's remarks. There is clearly a rapid increase in the intensity of the bands at Class N5. This is so marked that their prominent occurrence in Classes N5 and N6 gives these spectra a very characteristic appearance and is valuable in classifying the later Class N stars.

CLASSIFICATION

In classifying the spectra of the carbon stars, it is desirable to consider the region of short wave-length. Among the Class R stars the very characteristic cyanogen bands occur in this part of the spectrum, while in Class N the weakness or absence of the cyanogen absorption is equally important. Moreover the rapid decrease in the spectral intensity toward the violet is of prime significance in classification. If the more refrangible region is not observed, it is possible to fall into serious error. An example is afforded by Classes R5 and N3. If through underexposure the recorded spectrum of a Class R5 star does not extend beyond 4400A, there is little to distinguish it from a Class N3 star, unless high dispersion is employed. This is also exemplified in Classes N3 to N6. The visual spectra of these classes exhibit no appreciable variation throughout the sequence and are therefore not suitable for use in classification.

With these considerations in mind the stars were reclassified according to the following criteria:

Class R0: Cyanogen bands strong.¹³ Spectral intensity distribution as in Class G or K. Swan bands weak.

Class R3: Intermediate between R0 and R5.

Class R5: Cyanogen bands very strong. Considerably weaker in short wave-lengths than Class R0. Swan bands very strong.

Class R8: Cyanogen bands much weaker than in Class R5. Swan bands show a great range of intensity in this class. Spectral intensity distribution intermediate between Class R5 and Class N0.

Class N0: Cyanogen bands very weak. Spectrum can be photographed as far as H and K only with difficulty. Swan bands weak or moderate.

Class N3: Cyanogen bands missing. Spectrum extremely weak to the violet of 4383A. Swan bands strong.

Class N5: Spectrum very faint to the violet of the Swan band at 4737A. Unidentified bands at 4979A and 4686A strong.

¹³ The cyanogen group at 4606A was not used in the classification.

Classes N6 and N7: Continue in greater strength the characteristics of Class N5.

The spectral class assigned on the basis of these criteria is tabulated in the fifth column of Tables I and II. The fourth column indicates the Harvard class. Since the new criteria were chosen to follow the Harvard system, the two classifications are for the most part in agreement, discrepancies in individual stars being due to accidental error or perhaps in some cases to actual variations in spectral class.

Spectrograms of typical stars belonging to various subclasses are reproduced in Plate XI. These were all secured with the quartz 2-prism spectrograph, referred to here as Spectrograph b.

DISCUSSION OF CLASSIFICATION

The problem of spectral classification would be much simplified if stellar spectra could be accurately arranged in a one dimensional system. In such a case, a smooth curve would be obtained if the intensity of any spectral feature were plotted against spectral class. From this we could infer the complete dependence of stellar spectra on a single variable.

Among the stars of the main spectral sequence, however, we find that in general such a diagram represents a band rather than a line. This indicates that not less than two independent variables are required for a complete description of stellar spectra. Although some progress has been made toward such a classification, a sufficiently comprehensive system has not been devised.

A similar situation probably exists among the stars of the R-N branch. An examination of Figures 1 to 4 shows that by plotting the different spectral features against spectral class, a band instead of a smooth curve is obtained. The notable scattering of points, particularly where the intensities of the carbon bands are concerned, is a very definite indication that a one-dimensional system is valuable only as a rough means of describing the spectra. The small amount of observational material available for these stars would render an attempt at a more elaborate classification unprofitable.

It is of interest to inquire into the physical significance of the R-N spectra. Along the main branch of the spectral sequence, temperature is undoubtedly a controlling factor, while pressure, density and chemical composition probably exert some influence. Despite the progressive redness of the carbon stars, there is considerable doubt that temperature plays as important a role in this branch of the sequence. In the first place, let us consider the black body temperature of the carbon stars. Making the conservative estimate that the spectrum of *Y Canum Venaticorum* at 4800A is five

hundred times as intense as the spectrum at 3900A (see page 126), we find the black body temperature to be 953°K. This is unreasonably small and leads to the conclusion that the redness of the star is not entirely due to low temperature. In the second place the behavior of the Swan bands is puzzling. It is difficult to see how a progressive decrease in temperature or in fact a steady variation in any single physical condition, could lead to the peculiar forms of the curves in Figures 2 and 4. A maximum of a spectral feature followed by a decrease in its prominence is readily understandable, but it is not evident how this can be followed by another increase unless rather complicated changes in a number of physical conditions are involved. It appears from these considerations that temperature may not be of prime importance in determining the spectrum of the carbon stars.

The physical interpretation of the R-N spectrum is evidently a problem of some complexity. In the main spectral sequence, the variation of the spectrum from

class to class unquestionably depends in great part upon temperature. Among the stars of Classes R-N, however, neither the behavior of the carbon bands nor the redness of the stars, two most prominent spectral characteristics, can be satisfactorily related to temperature. Lack of observational material contributes further to the difficulty of the problem. In the present state of our knowledge, it is advisable to regard the classification in this spectral sequence as purely descriptive and not to insist on any particular physical interpretation.

I wish to express my deep obligation to Associate Director Aitken for the privilege of using the equipment of the Lick Observatory, to Dr. Moore for placing the 1-prism spectrograms at my disposal and to Dr. Bobrovnikoff for securing spectrograms of *W Orionis* which were used in the foregoing discussion.

LICK OBSERVATORY, February, 1928.

Issued March 28, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 397

OBSERVATIONS MADE WITH A PAIR OF FIVE-FOOT CAMERAS ON THE
LIGHT-DEFLECTIONS IN THE SUN'S GRAVITATIONAL FIELD
AT THE TOTAL SOLAR ECLIPSE OF SEPTEMBER 21, 1922

BY

W. W. CAMPBELL* AND R. J. TRUMPLER

The Wm. H. Crocker Eclipse Expedition of the Lick Observatory, observing the total solar eclipse of Sept. 21, 1922 at Wallal, Western Australia, employed two pairs of instruments of different type for photographing the star field surrounding the eclipsed Sun. The first of these, a twin camera of 15 feet focal length, with 5-inch doublet objectives, furnished four successful plates, and the results derived from these concerning the existence of light deflections in the Sun's gravitational field were published in 1923¹. The present paper is concerned with the observations made with the second instrument, a twin camera of 5 feet focal length, with two wide-angle quadruplet objectives of four

inches aperture, designed by Dr. F. E. Ross. The two pairs of instruments used plates of the same size (17 x 17 inches). The 15-foot cameras thus served to photograph the immediate surroundings of the Sun ($5^{\circ} \times 5^{\circ}$) on a large scale (45" to the mm) and were best suited for observing the stars nearest to the Sun's limb. The 5-foot cameras, on the other hand, covered a large field ($15^{\circ} \times 15^{\circ}$) on a smaller scale (135" to the mm) and were planned principally to determine the law according to which the light deflections diminish with increasing angular distance from the Sun's center.

The 5-foot cameras and their mounting, in design quite similar to that of the larger instrument, were constructed in the workshop of the Lick Observatory. To the common tube of the cameras, built as a framework of steel covered with black cloth, a guiding telescope was fastened, consisting of a square wooden tube with a 6-inch objective of $6\frac{1}{2}$ feet focal length.

1. THE OBSERVATIONS

The comparison photographs giving the normal star positions in the field of the sky surrounding the eclipsed Sun were obtained on the island of Tahiti in May, 1922, four months before the eclipse, when they could be taken at night. As the telescope had to be dismounted subsequently, transported to the eclipse station in Australia and reassembled there, it was anticipated that some of the finer adjustments would not be the same for the eclipse photographs as for the night comparison plates. In order to eliminate any errors that might arise from this source, an auxiliary star field (check field) was selected with center at R. A.

* It is desirable that the individual relations of Director Campbell and Dr. Trumpler to this paper be stated, in outline.

Director Campbell organized the expedition to observe the total solar eclipse of September, 1922, at Wallal, Western Australia, and the subsidiary expedition to Tahiti, the latter to secure the reference photographs needed in connection with the Einstein problem; he made the general plans for the equipment to be used at both stations; he outlined the general design of the 15-foot and the 5-foot pairs of cameras for the detection and measurement of the Einstein effect, and he supervised their construction at the Lick Observatory; at the Wallal station he directed the work of the expedition, and with the assistance of Chief Warrant Officer Rhoades and Petty Officer Kenny of the Royal Australian Navy he individually secured the observations of the eclipsed sun and its surroundings, with the twin 5-foot cameras, upon which the present paper is based.

Dr. Trumpler devised the auxiliary apparatus for placing the lenses and plateholders of the two pairs of Einstein cameras in precise and refined adjustment; he was in charge of the subsidiary station at Tahiti, and there he secured the comparison plates with the 15-foot and 5-foot pairs of cameras; he mounted and adjusted these cameras at Wallal; the laborious measurements of the plates obtained with the 5-foot cameras, the elaborate computations based upon these measurements, and the extensive discussion of the final and general results were made at Mount Hamilton in the past two years exclusively by him; and he alone has prepared the manuscript and the illustrations for the printer.—W. W. CAMPBELL.

¹ *L.O.B.*, 11, 41, 1923.

18^h5^m5, Decl. +1° 1' (following the eclipse star field by 6^h14^m9). At the date of the eclipse as well as at the time of the Tahiti photographs this check field was more than 90° distant from the Sun and therefore unaffected by any light deflections; in both cases moreover it could be observed at night. This check field was to be photographed on the same plates as the eclipse star field, at the same hour-angle, during the same day, and no alterations were to be made at the telescope between the two exposures except opening and closing the plateholder slides.

The similarity of the two instruments made it possible at Tahiti to mount the tube of the 5-foot cameras on the polar axis of the 15-foot cameras, after the observations with the latter had been completed and the cameras dismounted. At the eclipse station (Wallal), of course, two separate mountings and observing houses had to be provided for, since the observations there had to be made simultaneously. The methods described in the former publication were followed at both observing stations for the fine adjustments of the cameras, which consisted in centering the objectives, setting the plate perpendicular to the optical axis, determining the focus, and adjusting the guiding telescope so that when the guiding star (β *Virginis* for the eclipse field, 70 *Ophiuchi* for the check field) was set on the cross wires, the center of the eclipse or check field would fall in the middle of the plate. The focal surface of the five-foot camera objectives is so nearly plane² that no compromise in the focus setting was necessary, and the mean focus estimate of 20–30 stars evenly distributed over the field was adopted. The telescope was not equipped with any slow motion in declination, and after being set it remained clamped during the observing period. The declination difference between the two guiding stars β *Virginis* and 70 *Ophiuchi* was taken care of by different settings of the declination micrometer of the guiding telescope.

Mr. Trumpler was in charge of the work at Tahiti, where the observations with the 5-foot cameras were made between May 24 and May 30. All photographs were taken as nearly as possible at the same zenith-distance which the eclipsed Sun had at Wallal. The exposures on the eclipse star field had to be made in the evening between 9^h and 9^h30^m. Telescope and plateholders were then left undisturbed (except for closing the slides) until 3^h15^m to 3^h45^m in the morning, when the same plates were exposed to the check star field. In one case, however, (plates M7 and N9) the check field was photographed first and the eclipse field in the evening of the following day. Plate glass plates 17 x 17 inches in size, 1/4 inch thick, coated with Seed 23 emulsion were used for all observations with the 5-foot

cameras. The fine grain of this emulsion seemed important on account of the small scale of the photographs. Exposures of 3 minutes were made at Tahiti on both star fields, and the plates were developed at night for 10^m–15^m with Rodinal developer diluted at the rate 1:50 at a temperature of 70°–75°.

At Wallal an exposure on the check star field of 1 minute duration was made with both cameras on Sept. 20 at 7^h45^m in the evening. The plateholders were then left undisturbed in the telescope until the next afternoon when the eclipse took place. An exposure of 1 minute, started 10 seconds after the beginning of totality, was then made on the same plates, showing the eclipsed Sun and the surrounding star field. Changing of plateholders by two helpers took about 40 seconds, and a second pair of plates was exposed for 102 seconds. The plateholders were then changed again, and a third exposure of 62 seconds was completed a few seconds before the end of totality. The last pair of plates was again left undisturbed in the telescope until 7^h45^m in the evening, when an exposure of 1 minute on the check field was made on them. Six eclipse plates were thus available; four of these had an exposure of about 1 minute on the eclipsed Sun and an exposure of equal duration on the check field; two plates had an eclipse exposure of 1^m42^s, but no check field exposure. The first four plates were developed at Wallal with Rodinal developer 1:50, with varying additions of potassium bromide, at a temperature of 70°F for 3–17 minutes. The last two plates were developed at Broome where ice for cooling was obtainable.

The data of the eclipse plates and of the three best pairs of Tahiti plates, selected for measurement, are found in Table 1.

Despite of the shorter exposure and the slower emulsion the 5-foot camera plates are considerably more blackened by the sky-background than those of the 15-foot cameras. The 1^m exposures of the 5-foot cameras (ratio of aperture 1:15) are, outside of the corona, about as strongly fogged as is permissible without serious loss of fainter star images. As a consequence the 102 second exposures do not show any fainter stars than the shorter ones; evidently the gain in exposure time is offset by the increase in fogging.

On the 5-foot camera plates the corona is very strong and extends as a general, nearly circular halo to about 2° from the Sun's center. These photographs, therefore, do not contain as many measurable star images in the immediate surroundings of the Sun as do those of the larger instrument. The star images, however, as a rule are quite sharp and well defined. For objective N they are uniformly good all over the field, a trace of elongation or distortion being noticeable only near the edges and in the corners of the plate.

² For the optical qualities of these objectives see *Publ. A.S.P.*, 35, 152, 1923.

TABLE I
DATA OF PHOTOGRAPHS

Plate	Station	Date 1922	Local M. T.	Exposure	Star field	Hour angle, W	Temperature, F	Barometer, inches
M15	Wallal.....	Sept. 20	7 ^h 48 ^m 0	60 ^s	Check	1 ^h 36 ^m 9	+72°5	30.28
		Sept. 21	1 30.7	60	Eclipse	1 37.3	79.2	30.26
M7	Tahiti.....	May 25	15 39.7	180	Check	1 47.5	71	30.34
		May 26	9 21.8	180	Eclipse	1 47.3	73	30.34
M16	Wallal.....	Sept. 21	1 32.6	102	Eclipse	1 39.4	79.2	30.26
M11	Tahiti.....	May 30	9 8.1	180	Eclipse	1 49.4	76	30.34
M17	Wallal.....	Sept. 21	1 34.7	62	Eclipse	1 41.3	79.2	30.26
		Sept. 21	7 48.0	60	Check	1 40.7	70.5	30.31
M9	Tahiti.....	May 29	9 10.0	180	Eclipse	1 47.4	73	30.31
		May 29	15 27.7	180	Check	1 51.1	72	30.24
N18	Wallal.....	Sept. 20	7 48.0	60	Check	1 36.9	72.5	30.28
		Sept. 21	1 30.7	60	Eclipse	1 37.3	79.2	30.26
N11	Tahiti.....	May 29	9 10.0	180	Eclipse	1 47.4	73	30.31
		May 29	15 27.5	180	Check	1 51.1	72	30.24
N19	Wallal.....	Sept. 21	1 32.6	102	Eclipse	1 39.2	79.2	30.26
N13	Tahiti.....	May 30	9 8.1	180	Eclipse	1 49.4	76	30.34
N20	Wallal.....	Sept. 21	1 34.7	62	Eclipse	1 41.3	79.2	30.26
		Sept. 21	7 48.0	60	Check	1 40.7	70.5	30.31
N9	Tahiti.....	May 25	15 39.7	180	Check	1 47.5	71	30.34
		May 26	9 21.8	180	Eclipse	1 47.3	73	30.43

All photographs at Wallal taken by Campbell, all those at Tahiti by Trumpler.

Observing station at { Tahiti: long. 9^h58^m3 W of Gr., lat. -17°32'
Wallal: long. 8 2.7 E of Gr., lat. -19°46'

The images of objective M, while perhaps a trifle sharper at the center, are not quite so good near the edge where they become somewhat diffuse, and in the S. E. corner they are slightly distorted. Fortunately the distortion is the same as on the Tahiti comparison plates, and for differential measures these images were still usable, although, of course, they received smaller weight.

2. STAR LIST

A first survey of the 5-foot camera eclipse plates was made during the search for intramercorial planets³; about 550 stars were noted and charted. For the study of the light deflections it seemed sufficient to measure only a limited number of the best images, and a program of 147 stars was drawn up according to the following rules:

1. All measurable star images within 3° of the Sun's center were included (55 stars).
2. Drawing circles with radii of 3°, 4°, 5°, 6°, 7° around the Sun's center four concentric rings 1° wide were obtained. In each ring about 16 stars, four in every quadrant were selected (66 stars).
3. In each corner of the plate 6-7 stars (26 stars).

As far as possible the stars in groups 2 and 3 were chosen between the photographic magnitudes 7 and 9 with well exposed images. Two stars (66 and 90) of

the first group were later dropped because they were found too faint for measurement. The number was thus reduced to 145.

A list of the stars measured in the eclipse field is found in Table 4. After the number in the first column, the name of the star or its B.D. number in the 2^d col. follow the right ascension and declination for 1900. The photographic magnitudes of the 5th col. are mostly those of the *Henry Draper Catalogue*, but for the stars observed with the 15-foot cameras they were taken from Table II of the former publication⁴. For many of the stars the proper motions were available, while for others they were roughly derived by comparing the *Catalogue Abbadia* 1900 with the *A. G. Catalogues* and other catalogues of early epoch. Four stars had a measured parallax, and for 16 stars with large proper-motions a hypothetical parallax was obtained from the spectral type and proper-motion⁵. Columns 6 and 7 contain the combined proper-motion and parallax corrections to be applied to the differential measures Eclipse-Night; they are small, and exceed 0".1 only in four cases. The rectangular coordinates of columns 8 and 9 are referred to the Sun's center at the middle of the eclipse as zero point, are expressed in units of 200 mm or 27084", and oriented to the pole of 1922.0.

In the check field 75 stars uniformly distributed over the plate, mostly of magnitudes 7-8, were selected for measurement. Their rectangular coordinates in the same units are found in the 2^d and 3^d columns of

⁴ *L.O.B.*, 11, 49, 1923.

⁵ Following the method outlined *L.O.B.*, 10, 153, 1922.

³ *Publ.A.S.P.*, 35, 214, 1923.

Table 5. They are referred to an arbitrary zero point so chosen that on plate N18 it coincided exactly with the zero point of the eclipse field.

The longer exposure of the Tahiti photographs had the advantage, that all images measured on them were well exposed. On the eclipse photographs, however, it was necessary to measure many faint images the results of which did not deserve so much confidence. Different weights were therefore assigned to the measures, depending on the quality of the eclipse plate image, according to the following table:

Quality of image	Weight
Good.....	1.
Fair.....	0.7
Poor, faint.....	0.4
Very poor, very faint.....	0.25

The weights were fixed quite mechanically by four independent estimates of the image quality, one at every measurement, the mean of the four corresponding weights being adopted.

3. MEASUREMENT OF PLATES

Each eclipse plate was compared by differential measurement with one of the Tahiti plates using a new method described by Trumpler⁶. The eclipse plate was first placed into the measuring engine and the comparison plate laid on it with the same orientation, both plates with the film side towards the observer. The measuring microscope was focused on the film of the upper plate, while the lower plate was brought into focus by inserting an optically plane parallel glass plate, 10.6 mm thick, before the microscope objective. Keeping the lower plate fixed, the upper plate was shifted until corresponding star images appeared in the same part of the microscope field, while the fine adjustment had to be continued until the two corresponding images had a small offset of 0.2 to 0.3 mm in both co-ordinates and until this offset was approximately the same in all parts of the plate. The two plates were then tightly clamped together with three brass clamps and oriented in position-angle by means of four stars situated near the edge of the field, for which the rectangular coordinates had been calculated from their catalogue positions.

Only one coordinate was measured at a time, with the micrometer moving from left to right, and the measures were repeated after turning the plate 180° in position-angle. Each pair of plates thus required four series of measures:

Δx (Diff. of *R.A.*) measured "direct" (*R.A.* increasing to the right).

⁶ *Publ. A.S.P.*, 36, 9, 1924.

Δx measured "reversed" (*R.A.* increasing to the left).

Δy (Diff. of decl.) measured "direct".

Δy measured "reversed".

The plate was divided into 8 horizontal zones, each about 50 mm wide. Their measurement, intermingling stars of the eclipse and the check field, proceeded in the order 1, 7, 3, 5; 4, 6, 2, 8. The first four were completed without interruption on the same day, the last four on another, generally the following day. Six to eight of the most important stars situated close to the Sun were measured four times: at the beginning and end of each measuring session.

One differential measurement consisted of four bisections of each star image made in the following order:

Two bisections of the image of the comparison plate.

Inserting glass plate before microscope objective.

Four bisections of the image of the eclipse plate.

Removing glass plate.

Two bisections of the image of the comparison plate, or in the analogous order with the two images interchanged. The whole measuring work thus comprised about 37,000 bisections on 2,280 star images.

4. REDUCTION OF MEASURES

The first step in the reduction was to take the means of the "direct" and "reversed" measures and to convert these into seconds of arc. The following screw values used for this purpose were determined for each plate separately by measuring the distances of six wide double stars found in the eclipse or check field.

Plate	Screw value of micrometer
M15-M7.....	23.85
M16-M11.....	22.93
M17-M9.....	23.85
N18-N11.....	23.80
N19-N13.....	23.84
N20-N9.....	20.40

The measured differences Δx and Δy between the eclipse and comparison plates represent a combination of the following effects:

(1) Arbitrary offset of the two plates during measurement, constant for all stars of the same pair of exposures.

(2) Inaccurate orientation of the two plates relatively to each other during measurement.

(3) Different scale of the two plates, due to changes in the focal length of the telescope.

(4) Different inclination of the plane of the plate to the optical axis of the telescope, the latter being defined as the line joining the main Gaussian point of the objective with the adopted plate center.

(5) Optical distortions in the projection of the star field on the plane of the plate by the lens system. These distortions may be quite considerable for a field of such large dimensions as here employed. They would be eliminated from differential measures if they were the same on both plates; they must, however, to some extent, depend on the focusing and centering of the objectives, as well as on the inclination of the plate, and will on this account differ slightly for the two plates compared.

(6) Difference in the effects of differential refraction on the two photographs.

(7) Difference in the effects of differential aberration on the two photographs.

(8) Proper-motions of the stars during the four months separating the two photographs, and the effects of stellar parallax. These were neglected for the check field stars, but were applied to the stars of the eclipse field according to the data of Table 4.

(9) Apparent star displacements due to light deflection at the eclipse.

(10) Errors of observation: ν .

To determine the amount of light deflection at the eclipse, it is necessary, first to correct the measures

for all the other effects. While those of (6), (7), (8) can be calculated, the corrections for (1), (2), (3), (4), (5) are of instrumental character and have to be derived from the measures themselves.

It is convenient to develop the differential refraction and aberration as well as the instrumental corrections (1)–(5) for each coordinate into a series according to powers of x and y , the rectangular coordinates of the star. In most astronomical photographs only the first three terms of this series are used; for the wide-angle field of our photographs, however, terms of higher order become sensible. For the refraction and aberration corrections it was found sufficient to include the terms of the first, second, and third order, and the numerical values of the coefficients are given in Table 2. Of the instrumental corrections: (1) is a constant, while (2) and (3) are linear terms. With good adjustment of the cameras only the main terms of the inclination correction (4), which are of the second order, need to be considered. No information was available about the optical distortions (5). A trial reduction of the check field of plates N20–N9 with terms of the first and second order only for the instrumental corrections was therefore made. The resulting residuals, however, did not seem satisfactory, and another solution including also third order terms led to considerably smaller and better distributed residuals.

TABLE 2
CORRECTIONS FOR DIFFERENTIAL REFRACTION

$$R_x = b_r x + c_r y + d_r x^2 + e_r xy + f_r y^2 + g_r x^3 + h_r x^2 y + i_r xy^2 + k_r y^3$$

$$R_y = b'_r x + c'_r y + d'_r x^2 + e'_r xy + f'_r y^2 + g'_r x^3 + h'_r x^2 y + i'_r xy^2 + k'_r y^3$$

	M7 and N9		M9 and N11		M11, N13		M15 and N18		M17 and N20		M18, N19	
	Ecl. F.	Check F.	Ecl. F.	Check F.	Ecl. F.	Ecl. F.	Ecl. F.	Check F.	Ecl. F.	Check F.	Ecl. F.	Ecl. F.
b_r	+9".48	+9".48	+9".42	+9".58	+9".49	+8".92	+9".03	+9".07	+9".20	+9".00		
c_r	-1.50	-1.51	-1.50	-1.57	-1.53	-1.46	-1.47	-1.54	-1.55	-1.50		
d_r	-.63	-.64	-.63	-.67	-.65	-.53	-.54	-.57	-.58	-.55		
e_r	+.19	+.19	+.19	+.21	+.20	+.17	+.17	+.18	+.18	+.18		
f_r	-.58	-.58	-.57	-.58	-.58	-.52	-.52	-.54	-.55	-.53		
g_r	+.21	+.21	+.20	+.21	+.21	+.19	+.19	+.19	+.19	+.19		
h_r	-.09	-.09	-.09	-.10	-.10	-.09	-.09	-.09	-.09	-.09		
i_r	+.20	+.20	+.19	+.20	+.20	+.19	+.19	+.19	+.19	+.19		
k_r	-.03	-.03	-.03	-.03	-.03	-.03	-.03	-.03	-.03	-.03		
b'_r	-1".37	-1".37	-1".36	-1".42	-1".39	-1".34	-1".35	-1".42	-1".43	-1".38		
c'_r	+8.57	+8.57	+8.52	+8.54	+8.52	+8.67	+8.78	+8.68	+8.84	+8.68		
d'_r	+.47	+.47	+.46	+.47	+.47	+.49	+.49	+.50	+.51	+.49		
e'_r	-.14	-.14	-.14	-.15	-.14	-.15	-.15	-.16	-.16	-.16		
f'_r	+.42	+.42	+.42	+.42	+.42	+.47	+.48	+.48	+.49	+.48		
g'_r	-.03	-.03	-.03	-.03	-.03	-.03	-.03	-.03	-.03	-.03		
h'_r	+.20	+.20	+.19	+.20	+.20	+.19	+.19	+.19	+.19	+.19		
i'_r	-.08	-.08	-.08	-.09	-.09	-.09	-.09	-.09	-.09	-.09		
k'_r	+.17	+.17	+.17	+.17	+.17	+.18	+.18	+.18	+.18	+.18		

TABLE 2—(Continued)
CORRECTIONS FOR DIFFERENTIAL ABERRATION:

$$A_x = b_a x + c_a y + d_a x^2 + e_a x y + f_a y^2 + g_a x^3 + i_a x y^2$$

$$A_y = b'_a x + c'_a y + d'_a x^2 + e'_a x y + f'_a y^2 + h'_a x^2 y + k'_a y^3$$

	M7 and N9		M9 and N11		M11, N13	M15, M16, M17 N18, N19, N20 Eclipse F.	M15, N18	M17, N20
	Ecl. F.	Check F.	Ecl. F.	Check F.	Ecl. F.		Check F.	Check F.
b_a :	-2'.49	+1'.11	-2'.54	+'.96	-2'.56	°.00	-2'.43	-2'.44
c_a :	-.02	-.04	-.01	-.04	-.01	+.04	.00	.00
d_a :	+.06	+.16	+.05	+.16	+.05	-.16	+.01	+.01
e_a :	-.05	-.06	-.05	-.05	-.04	+.14	+.15	+.15
f_a :	-.06	-.16	-.05	-.16	-.05	+.16	-.01	-.01
i_a :	-.02	-.01	-.02	+.01	-.02	.00	-.02	-.02
b' :	+.02	+.04	+.01	+.04	+.01	-.04	°.00	°.00
c' :	-2.49	+1.11	-2.54	+.96	-2.56	.00	-2.43	-2.44
d' :	+.03	+.03	+.02	+.03	+.02	-.07	-.07	-.07
e' :	+.12	+.32	+.10	+.32	+.10	-.32	+.03	+.02
f' :	-.03	-.03	-.02	-.03	-.02	+.07	+.07	+.07
k' :	-.02	+.01	-.02	+.01	-.02	.00	-.02	-.02

INCLINATION CORRECTION DUE TO DIFFERENCE IN PLATE CENTER

$$I_x = d_a x^2 + e_a x y \quad \text{to be applied to check field measures.}$$

$$I_y = e'_a x y + f'_a y^2$$

p and q are the coordinates of the check field center of the Tahiti plate, measured from that of the Wallal plate, when both plates are so adjusted that the centers of the eclipse field photographs coincide.

	M15-M7	M17-M9	N18-N11	N20-N9
p :	+18"	+ 5"	+ 2"	+27"
q :	- 8"	- 7"	- 2"	- 8"
$d_i = e'_i$:	+.31	+.08	+.03	+.46
$e_i = f'_i$:	-.14	-.12	-.03	-.13

It was thus decided to reduce the measures with the following reduction formulae, which are complete in all terms of the first, second, and third order:

$$\Delta x + a + b x + c y + d x^2 + e x y + f y^2 + g x^3 + h x^2 y + i x y^2 + k y^3 = D_x + v_x$$

$$\Delta y + a' + b' x + c' y + d' x^2 + e' x y + f' y^2 + g' x^3 + h' x^2 y + i' x y^2 + k' y^3 = D_y + v_y$$

Δx and Δy are the differential measures taken in the sense Eclipse-Night, and corrected in case of the eclipse field for proper-motion and parallax. D_x and D_y are the components of the star displacements due to light deflection; they vanish, of course, for the check-field.

The important point of our reduction method is, that the coefficients of the second and third order terms, which are due only to the inclination of the plate (4) and to optical distortions (5), when corrected for refraction, aberration, etc., must be the same for the eclipse field as for the check field photographed on the same plate. They can then be determined from the latter without any assumption about the light-deflections.

For the four pairs of plates which have a check field exposure, the reduction of the latter is taken up first. As in this case no light deflections are present, the task

is simply to find the 20 constants $a, b, c, \dots k; a', b', c', \dots k'$ from the measured Δx and Δy of the 75 stars. To avoid the great labor of strict least squares solutions with so many unknowns, the following method, which is shorter and nearly as good, was employed.

The field of the plate was divided into four zones of equal width arranged parallel to the x -axis. In each of these zones (y -zones) every measured Δx and Δy was plotted as a function of x alone, and interpolated graphically by a curve of the second order with the equations $\Delta x = a + \beta x + \gamma x^2$ and $\Delta y = a' + \beta' x + \gamma' x^2$. The four zones furnished 12 values of a, β, γ and 12 values of a', β', γ' . From these, approximate data for the coefficients a, b, c, d, e, f or a', b', c', d', e', f' , in a few cases also for h and i or h' and i' , were obtained. The whole procedure was then repeated by dividing the field into four zones parallel to the y -axis (x -zones). With the mean results of the two determinations the differential measures Δx and Δy were reduced, as a first approximation.

The remaining residuals which were of the order of 1" or less were used as the basis for a second approximation, determining the coefficients of the third order

terms and corrections to the approximate values of the others. This second approximation was made in a quite similar manner, except that the graphical method was replaced by a least squares solution. In each of the y -zones the Δx and Δy were represented by formulae of the form:

$$\Delta x = a + \beta x + \gamma x^2 + \delta x^3, \text{ and } \Delta y = a' + \beta' x + \gamma' x^2 + \delta' x^3$$

After setting up the normal equations for the unknowns $a_1, \beta_1, \gamma_1, \dots, a'_1, \beta'_1, \gamma'_1, \dots$, the conditions were introduced to which they are subject:

$$\begin{array}{llll} \gamma_1 = d + hy_1 & \beta_1 = b + ey_1 + iy_1^2 & a_1 = a + cy_1 + fy_1^2 + ky_1^3 \\ \gamma_2 = d + hy_2 & \beta_2 = b + ey_2 + iy_2^2 & a_2 = a + cy_2 + fy_2^2 + ky_2^3 \\ \gamma_3 = d + hy_3 & \beta_3 = b + ey_3 + iy_3^2 & a_3 = a + cy_3 + fy_3^2 + ky_3^3 \\ \gamma_4 = d + hy_4 & \beta_4 = b + ey_4 + iy_4^2 & a_4 = a + cy_4 + fy_4^2 + ky_4^3 \end{array}$$

where the indices refer to the four zones, and y_1, y_2, y_3, y_4 are the mean y -coordinates of the zones.

A similar set of normal equations and of conditions was set up for the x -zones. The solution of the normal equations with conditions was accomplished by gradual approximation, introducing the further condition that the results of the x -zones should be identical with those of the y -zones, and giving consideration to the fact that the coefficients $b, d, g, h; b', d', g', h'$ are better determined from the y -zones, $c, f, i, k; c', f', i', k'$ better from the x -zones. The only neglect of our method as compared with a rigorous least squares solution is, that in each y -zone the stars were treated as if they had the same y coordinate (mean of the zone), the same y^2 , the same y^3 . The application of approximate corrections (first approximation), and duplication of the

solution by using the x -zones as well as the y -zones should prevent any errors from this neglect.

The results for the reduction constants of the check field are found in Table 3 under "Check F., det." These coefficients still include the effects of refraction and aberration, which have to be deducted to obtain the purely instrumental part. It is further necessary to take into account the fact that the adopted center of the check field (zero point of coordinates) does not for all plates coincide exactly with the center of the eclipse field. The optical axis of the telescope thus

differs for the two exposures taken on the same plate, and this requires a small correction for plate inclination given in Table 2 and to be deducted from the coefficients of the reduction formula of the check field. The resulting data (Table 3: "Check F., corr.") represent the instrumental part of the reduction and are, for the second and third order terms ($d \dots k, d' \dots k'$), applicable also to the eclipse field. Adding to them the refraction and aberration corrections of the eclipse field leads to the coefficients found in Table 3 under "Eclipse F.: adopt." which were used for the reduction of the eclipse field.

After correcting the eclipse field measures for second and third order terms, the coefficients of the linear terms $a, b, c; a', b', c'$ were determined by least squares solution from the residuals of 55 reference stars which

TABLE 3
COEFFICIENTS OF REDUCTION FORMULAE

	M15-M7				M17-M9				M16-M11			
	Check F. det.	corr.	Eclipse F. adopt.	corr.	Check F. det.	corr.	Eclipse F. adopt.	corr.	Eclipse F. adopt.	corr.	Eclipse F. adopt.	corr.
a :	-21°91		-39°93		-40°51		-45°41		+43°77			
b :	+ 1.14	+5°13	+ 6.31	+4°38	+ 6.68	+10°46	+11.66	+9°47	+ 4.23	+2°16		
c :	+ 1.17	+1.09	+ .02	- .08	+ 2.34	+ 2.28	+ 1.66	+1.65	- .03	- .11		
d :	- 4.46	-4.72	- 4.85		- 1.69	- 1.71	- 1.86		- 4.48	-4.37		
e :	+ 1.43	+1.38	+ 1.55		+ 2.96	+ 2.91	+ 3.09		+ 2.02	+1.86		
f :	+ .07	- .14	+ .14		+ .36	+ .18	+ .42		+ .50	+ .24		
g :	+ .11	+ .16	+ .16		- .52	- .47	- .46		+ 1.54	+1.54		
h :	+ 1.09	+1.09	+ 1.09		- .36	- .37	- .37		+ .49	+ .48		
i :	- .72	- .68	- .67		- 1.55	- 1.51	- 1.49		- .24	- .25		
k :	+ .29	+ .29	+ .29		- .31	- .31	- .31		+ .17	+ .17		
a' :	+18°30		+27°36		-56°07		-49°45		+47°37			
b' :	+ 1.26	+1°28	+ 1.24	+1°27	- 5.20	- 5°15	- 4.68	-4°57	+ 1.15	+1°19		
c' :	+ 2.51	+5.84	+ 7.19	+4.60	+ 7.13	+10.23	+11.52	+8.82	+ 5.01	+2.29		
d' :	+ .08	+ .16	+ .08		+ .10	+ .16	+ .11		- .31	- .24		
e' :	- 3.58	-3.59	- 4.03		- 2.48	- 2.23	- 2.67		- 3.89	-3.45		
f' :	+ 1.81	+1.79	+ 1.95		+ 3.12	+ 3.07	+ 3.22		+ 1.80	+1.65		
g' :	- .41	- .41	- .41		- .41	- .41	- .41		- .24	- .24		
h' :	+ 1.49	+1.53	+ 1.54		+ .21	+ .25	+ .27		- .14	- .15		
i' :	+ .59	+ .60	+ .59		+ .38	+ .38	+ .37		- .04	- .04		
k' :	- 1.16	-1.14	- 1.11		- 1.33	- 1.31	- 1.28		+ 1.78	+1.75		

	N18-N11				N20-N9				N19-N13			
	Check F.		Eclipse F.		Check F.		Eclipse F.		Check F.		Eclipse F.	
	det.	corr.	adopt.	corr.	det.	corr.	adopt.	corr.	det.	corr.	adopt.	corr.
<i>a</i> :	+41".36		+39".17		-11".86		-39".06		-38".85			
<i>b</i> :	+ .38	+4".32	+ 5.22	+3".18	- .63	+ 3".20	+ 4.12	+2".04	+ 5.91	+3".84		
<i>c</i> :	+ 2.19	+2.05	+ 1.45	+1.36	+ 1.12	+ 1.12	+ .20	+ .18	+ 1.04	+ .96		
<i>d</i> :	- 5.62	-5.64	- 5.76		- 5.33	- 5.70	- 5.86		- 7.38	-7.27		
<i>e</i> :	+ .78	+ .65	+ .83		+ 3.12	+ 3.05	+ 3.23		+ 5.81	+5.66		
<i>f</i> :	+ .17	- .04	+ .22		- .33	- .51	- .25		+ .32	+ .06		
<i>g</i> :	- .95	- .90	- .89		+ 1.47	+ 1.52	+ 1.52		- .51	- .51		
<i>h</i> :	- 2.48	-2.49	- 2.49		- 1.57	- 1.57	- 1.57		- .05	- .06		
<i>i</i> :	.00	+ .04	+ .06		- .27	- .23	- .22		- .20	- .21		
<i>k</i> :	- .67	- .67	- .67		+ .20	+ .20	+ .20		- .24	- .24		
<i>a'</i> :	-52".79		-50".39		-49".14		-39".93		-39".31			
<i>b'</i> :	- .86	- .89	+ .16	+ ".19	- 3.29	- 3".19	- 2.54	-2".23	- 1.53	-1".49		
<i>c'</i> :	+ 1.06	+4.21	+ 5.59	+2.90	+ .16	+ 3.44	+ 4.83	+2.23	+ 6.55	+3.83		
<i>d'</i> :	- .49	- .41	- .47		- .29	- .23	- .30		- .16	- .09		
<i>e'</i> :	- 5.74	-5.48	- 5.91		- 5.57	- 5.71	- 6.18		- 7.03	-6.59		
<i>f'</i> :	+ .81	+ .68	+ .83		+ 3.33	+ 3.29	+ 3.45		+ 5.24	+5.09		
<i>g'</i> :	- .64	- .64	- .64		+ .04	+ .04	+ .04		- .54	- .54		
<i>h'</i> :	- .83	- .79	- .77		+ 1.20	+ 1.24	+ 1.25		- 1.67	-1.68		
<i>i'</i> :	- 1.74	-1.74	- 1.75		- 1.04	- 1.03	- 1.04		+ .38	+ .38		
<i>k'</i> :	+ .48	+ .50	+ .53		- .53	- .51	- .48		+ .14	+ .11		

are more than 5° distant from the Sun's center. In this solution the light deflections of the reference stars, which must be small, were neglected. This does not affect the orientation constants, but leads to an erroneous scale value. The scale of the eclipse field photographs is necessarily linked together with the light deflections; the two cannot be separated unless some assumption is made concerning the law according to which the light deflections are related to the star's angular distance from the Sun's center. Our reduction method is therefore equivalent to applying an arbitrary scale value so chosen that the residuals become zero for the reference stars on the average, i. e., at a mean distance of 7°.3 from the Sun's center. Our observations do not furnish the absolute values of the light deflection, but the relative displacement of each star as compared with the mean of the reference stars situated in the outer portions of the field.

The final values of the zero corrections *a*, *a'* were not taken from the least squares solutions of the reference stars, but determined from all stars of the eclipse field (except those within 1°.5 of the Sun's center which have rather large displacements), giving equal weight to the area between 1°.5 and 4°.5 from the Sun's center and to that outside of this limit and dividing the weight equally among the four quadrants. The reduction constants derived from the eclipse field measures are given in Table 3 in italics under "Eclipse F. adopt.", and after deducting the effects of refraction and aberration under "Eclipse F. corr."

The two plates M16 and N19 having no check field exposure were reduced last. The unknown effects of light deflection were first eliminated by forming the means of the observed star displacements of the other four plates and deducting these from the differential measures of the two plates M16 and N19. The 20

reduction constants were then determined from the residuals by dividing the field into 8 *y*-zones (excluding the stars within 1° of the Sun's center) and proceeding in a similar way as for the check field. A duplication of the solution with *x*-zones, however, did not seem necessary in this case, as the zones were much narrower.

After correcting the differential measures Eclipse-Night for all known sources of error according to the formula of p. 135 (with the coefficients "Check F. det." and "Eclipse F. adopt." of Table 3) we obtain the data of Tables 4 and 5. For the eclipse field they represent the combined effect of apparent star displacements due to light deflection, error in the adopted scale of the plates, and errors of observation; for the check field they should have the character of accidental measuring errors.

On several plates evidence was found of deformations of the photographic film at the edge of the plate, especially near the southern edge, causing abnormally large residuals. In such cases all star images of the eclipse field as well as of the check field were rejected within the affected area. All measures rejected on account of large residuals are entered in Tables 4 and 5 in brackets. Only two of these belong to stars in the central part of the plate, both of them extremely faint, so that a spot may have possibly been mistaken for the star image.

5. ACCURACY OF THE MEASURES

From the residuals of the check field (mean of 4 plates) given in the three last columns of Table 5 the probable error of a differential measure of weight 1 was found to be:

Check field	Prob. error of wt. 1
	<i>x</i> : ±".23
	<i>y</i> : ±.25
	Mean ±".24

When the small scale of the plates is considered, this p.e. (in linear measure $\pm .0018\text{mm}$), which includes the imperfections of two star images, seems very satisfactory and speaks for the excellent quality of the objectives.

For the eclipse field the p.e. of unit weight was derived in two different ways. First, from the differences between the individual plate results and the mean of the six plates. For every fifth star (excluding those with less than 3 measures) the plate residuals were formed and from these the p.e. was computed taking into account the number of unknowns determined from these measures. In the second place, the observed mean star displacements (last column of Table 5) were used, resolving them into a radial component (in the direction of the radius vector of the star) and a tangential component (perpendicular to the radius vector). Assuming that the light deflections operate only in the radial direction, one value of the p.e. is obtained directly from the tangential components while another is furnished from the radial components by the least squares solution made for the determination of the light deflection at the Sun's limb (see page 150).

Eclipse field:	Prob. error of wt. 1
1. Individual plate residuals	$x : \pm .25$ $y : \pm .26$ <i>Mean</i> $\pm .26$
2. Mean obs. star displacements	tangential component : $\pm .31$ radial component : $\pm .34$ <i>Mean</i> $\pm .33$

It is not surprising to find a greater accuracy for the check field stars, as these have nearly all well exposed images and differ little in brightness, while the eclipse field contains a large number of faint and difficult images. The accidental errors of the plate constants determined from the check field and used for the reduction of the eclipse field also have the effect of increasing the residuals of the latter.

The fact that the agreement of the individual plates gives a smaller p.e. than the mean results is a condition commonly met with in astronomical observations and indicates that some sources of error must have affected all plates in a similar way. The result of the second method $\pm .33$, should therefore be adopted as a measure of the accuracy of the eclipse field measures.

The probable errors of the plate constants for each of the four plates M15-M7, M17-M9, N18-N11, N20-N9 are on the average as follows:

	Check F.	Ecl. F.
Coefficients of first order terms : b, c, b', c'	$\pm .14$	$\pm .13$
second order terms d, e, f, d', e', f'	$\pm .10$	
third order terms $g, h, i, k, g', h', i', k'$	$\pm .20$	

6. THE PLATE CONSTANTS

A discussion of the plate constants of Table 3 should not be out of place here. The coefficients b and c' representing the scale corrections of the two coordinates should agree, when corrected for refraction and aberration. This test is satisfactorily fulfilled in all but two cases, where the discrepancy amounts to about three times the probable error.

	Check field	Eclipse field	Check -Eclipse
M15-M7.....	$b : +5''.13$ $c' : +5.84$	$+4''.38$ $+4.60$	$+ .75$ $+1.24$
M16-M11.....	$b : $ $c' : $	$+2.16$ $+2.29$	
M17-M9.....	$b : +10.46$ $c' : +10.23$	$+9.47$ $+8.82$	$+ .99$ $+1.41$
N18-N11.....	$b : +4.32$ $c' : +4.21$	$+3.18$ $+2.90$	$+1.14$ $+1.31$
N19-N13.....	$b : $ $c' : $	$+3.84$ $+3.83$	
N20-N9.....	$b : +3.20$ $c' : +3.44$	$+2.04$ $+2.23$	$+1.16$ $+1.21$
		<i>Mean</i>	$+1''.15$

All the Wallal plates are on a smaller scale than the Tahiti photographs, or the adjustment of the cameras must have been such that the equivalent focal length at the eclipse station was shorter by about 0.4mm for objective M and 0.2mm for objective N. The scale differences between the three plates of the same camera are undoubtedly due to inequalities of a few tenths of a millimeter in the thickness of the plateholders.

It had never been the intention of the observers to use the check field photographs for determining the scale of the eclipse photographs. Although both were made on the same plate it was anticipated that the scale of the eclipse exposure would differ from that of the check field photographed at night; for the scale is quite sensitive to temperature changes affecting the camera tubes as well as the photographic plates. Indeed we find between the differential scale corrections of the eclipse and check fields a marked systematic difference (column "check-eclipse") which on the average amounts to $1''.15$ per unit of $7''.52$, or $0''.153$ per $1''$. When trying to account for this difference we have first to keep in mind that the light deflections of the reference stars were neglected in the determination of the scale value of the eclipse field. If these light deflections had been adopted according to Einstein's theory, the scale corrections of the eclipse field would have been larger by $0''.07$, and our difference would have been reduced by this amount ($0''.009$ per $1''$).

The temperature expansion of the telescope and photographic plate can be calculated only under the assumption that all parts of the telescope had the same temperature as the air in the observing house. With a temperature coefficient of 0.0000119 per 1°C

TABLE 4

OBSERVED DISPLACEMENTS IN x AND y FOR STARS OF ECLIPSE FIELD

No.	B. D.	R. A.	Decl.	Phtgr. magn.	P. M. and Par. corrections		x	y	M15-M7		Wt.	M16-M11		Wt.
		1900							D_x	D_y		D_x	D_y	
1	+6°2448	11 ^h 20 ^m 0	+6°17'	9.1			-.979	+.694	-.67	-.32	.4	+.1743	-.69	.5
2	+8 2498	20.3	+8 30	8.5	-.06	+.04	-.972	+.993	+1.90	-1.18	.4	+.69	-.17	.5
3	+4 2463	20.7	+4 25	6.6	+.02	+.02	-.958	+.439	+.64	-.15	1	-.44	-1.05	1
4	+1 2565	21.5	+1 36	8.2	+.02	+.01	-.933	+.064	-.28	-.15	.8	-.12	+.28	1
5	-0 2443	22.9	-0 21	7.1	-.02	+.02	-.885	-.198	-.52	-.24	1	+.07	+.37	1
6	+1 2569	23.7	+1 45	8.9	+.01	+.01	-.859	+.083	-.45	-.01	1	-.14	+.59	.9
7	-3 3128	24.1	-3 54	8.4			-.845	-.675	-.28	-.52	1	+.16	-.67	1
8	+8 2512	24.5	+8 9	7.1	-.01	+.01	-.829	+.943	-.44	-.73	.4	-.12	-.35	.9
9	e Leon	25.2	-2 27	6.1	-.02	+.01	-.810	-.481	-.16	+.44	1	+.60	-.03	1
10	+3 2513	26.2	+3 37	8.3	-.01	.00	-.772	+.331	-.32	+.32	1	.00	+.25	1
11	-5 3307	26.9	-5 55	7.9	+.01	+.03	-.754	-.947	-.06	-.59	1	-.46	-.19	1
12	+0 2801	27.2	+0 8	8.6	-.01	+.04	-.742	-.133	-.69	+.10	1	-.98	+.27	1
13	+3 2519	28.5	+3 3	7.9	-.01	.00	-.698	+.255	+.07	-.04	1	-.08	+.26	1
14	-5 3313	28.5	-5 59	9.1			-.698	-.956	+.79	-1.68	.8	-.06	-.48	1
15	-4 3099	29.7	-4 24	8.3			-.659	-.741	-.52	-.54	1	+.02	-.54	1
16	-1 2540	29.7	-1 56	8.7	+.01	.00	-.659	-.411	-.80	+.15	1	+.73	-.21	1
17	+2 2449	29.6	+2 3	9.8			-.660	+.121	-2.19	-1.29	.25	-1.02	-.81	.6
18	+0 2811	32.2	-0 1	8.9	.00	+.01	-.576	-.155	+.05	+.25	1	-.24	+.36	1
19	+7 2465	32.1	+6 49	7.1			-.574	+.761	-.65	+.05	.5	+.09	+.11	1
20	-0 2464	32.9	-1 3	8.6	+.09	-.04	-.551	-.292	+.09	+.06	1	+.11	+.05	1
21	+6 2478	33.9	+6 37	8.9	+.09	+.04	-.516	+.732	-.73	-.25	.4	-.21	-.04	.8
22	+3 2527	34.4	+3 24	9.0	+.06	-.03	-.500	+.300	-.12	-.18	1	-.41	-.22	1
23	-4 3114	34.8	-4 24	9.7			-.495	-.744	-.47	-.53	.7	-.20	-.62	1
24	+5 2525	35.3	+5 42	8.2			-.468	+.609	-.55	+.10	.7	-.28	-.14	1
25	+1 2597	35.3	+1 30	6.9	-.02	.00	-.471	+.050	-.25	+.13	1	-.25	-.30	1
26	+4 2510	35.4	+4 13	9.3	-.04	+.02	-.467	+.410	-.92	+.37	.7	-.46	+.01	1
27	+0 2821	35.8	-0 7	8.2	.00	.00	-.454	-.167	-.32	+.20	1	-.66	-.01	1
28	+3 2539	37.3	+2 55	7.4	+.03	+.02	-.403	+.238	-.30	+.08	1	-.36	-.15	1
29	+0 2826	38.2	+0 44	7.8	+.01	+.01	-.373	-.053	+.06	+.15	1	-.25	-.16	1
30	-5 3340	38.8	-6 7	7.2	-.02	+.02	-.353	-.971	+.58	-.46	1	+.04	+1.03	1
31	-0 2479	39.5	-0 50	10.4	.00	.00	-.331	-.262	-.26	+.25	.6	-.14	+.23	1
32	-3 3169	39.6	-3 57	8.6			-.328	-.680	+.05	-.56	1	+.27	-.08	1
33	-1 2563	39.8	-1 29	9.5	.00	-.01	-.323	-.349	-.11	+.38	1	+.22	+.57	1
34	+3 2544	39.9	+3 38	9.2	+.01	+.01	-.318	+.331	-.15	+.19	1	-.23	-.21	1
35	+0 2831	39.9	+0 2	8.3	+.02	+.03	-.317	-.146	+.11	+.42	1	+.03	-.03	1
36	+1 2608	40.0	+1 28	9.8	.00	+.01	-.314	+.044	-.16	+.10	.7	-.54	+.67	1
37	+3 2548	40.4	+3 12	9.2	+.03	+.03	-.300	+.275	+.41	+.04	1	+.31	-.22	1
38	+4 2522	40.4	+4 43	10.0			-.298	+.477	+.41	+.30	.6	-.08	+.63	1
39	+3 2549	40.6	+3 23	8.7	+.04	+.04	-.293	+.299	-1.05	+.91	.1	-.76	+.11	1
40	+7 2480	41.0	+7 44	8.4			-.281	+.881	+.67	+.51	.5	+.56	+.07	1
41	+2 2477	41.1	+2 6	10.0	+.02	.00	-.276	+.128	+.21	+.64	.6	-.21	+.52	.9
42	+0 2833	41.2	+0 23	9.6	+.01	.00	-.276	-.100	-.90	-.53	.5	-1.01	+.12	.9
43	-2 3410	41.2	-2 27	8.7			-.274	-.478	+.58	+.32	1	+.05	-.34	1
44	-4 3137	41.3	-4 48	8.4			-.273	-.792	-.34	-.24	1	-.51	-.02	1
45	-2 3411	41.6	-3 11	7.8			-.262	-.576	+.23	-.01	1	+.12	+.13	1
46	-0 2490	41.9	-1 4	9.3	+.01	-.01	-.252	-.294	-.17	-.77	1	-.12	-.39	1
47	+2 2480	42.0	+2 4	10.2			-.247	+.124	+.46	-1.62	.25	+.86	-.79	.3
48	+0 2836	42.1	+0 24	10.0	.00	.00	-.242	-.099	-1.17	-.13	.5	+.23	+.09	.8
49	+2 2481	42.3	+2 34	9.3	+.02	-.01	-.236	+.191	-.06	+.20	1	+.05	+.83	.8
50	+0 2843	43.9	+0 14	6.9	+.04	+.01	-.184	-.120	-.48	+.08	1	-.01	-.04	1
51	+5 2545	44.0	+5 45	7.7	+.01	+.06	-.181	+.614	-.14	-.05	1	+.05	+.27	1
52	+1 2617	44.4	+1 10	9.4	+.04	.00	-.169	+.003	-.38	+1.11	.3	+.55	-.06	.4
53	+3 2560	44.7	+2 47	8.2	.00	.00	-.158	+.220	-.35	+.07	1	+.17	+.33	1
54	+3 2561	44.7	+2 55	10.2	+.03	+.02	-.157	+.236				+.63	+.40	.6
55	-0 2498	45.3	-0 42	9.7	+.11	-.01	-.139	-.245	+.91	+.32	.5	-.20	.00	.8
56	+1 2619	45.3	+1 34	10.1			-.137	+.057						
57	-1 2576	45.3	-1 52	8.5	+.02	+.01	-.137	-.400	+.09	+.13	1	-.04	-.18	1
58	β Virgin	45.5	+2 20	4.3	-.32	+.13	-.130	+.158	-.09	+.41	1	+.22	+.31	1
59	+7 2487	45.7	+6 54	9.1			-.124	+.769	+1.24	+1.34	.5	-.11	-.20	1
60	-4 3152	45.9	-4 47	6.8	.00	.00	-.116	-.790	+.75	+.10	1	.00	-.27	1
61	+2 2490	46.0	+2 21	9.7			-.112	+.160				-.64	+.88	.4
62	+1 2621	47.1	+0 48	10.0	-.01	.00	-.076	-.045						

TABLE 4

OBSERVED DISPLACEMENTS IN x AND y FOR STARS OF ECLIPSE FIELD

M17-M9			N18-N11			N19-N13			N20-N9			Mean obs. displacement			No.
D_z	D_y	Wt.	D_z	D_y	Wt.	D_z	D_y	Wt.	D_z	D_y	Wt.	D_z	D_y	Wt.	
- .33	+1.15	.6	- .41	+1.01	.4	- .18	+ .79	.8	- .75	+ .64	.7	- .17	+ .50	3.4	1
+ .61	- .23	.4	- .35	+ .23	.4	+1.44	- .86	.8	- .32	+ .43	.5	+ .73	- .34	3.0	2
+ .45	- .45	1	+ .23	+ .04	1	- .66	- .16	1	- .50	+ .08	1	- .05	- .28	6	3
- .27	+ .05	1	+ .97	- .10	1	- .14	- .12	1	- .18	- .10	1	+ .01	- .02	5.8	4
- .40	+ .35	1	+ .33	+ .32	.9	+ .25	- .01	1	+ .50	- .51	1	+ .03	+ .04	5.9	5
- .82	+ .10	1	+ .55	- .11	.8	- .27	+ .04	1	+ .05	- .34	1	- .21	+ .04	5.7	6
- .02	- .64	1	- .28	+ .13	1	- .35	+ .40	1	- .08	- .33	1	- .14	- .27	6	7
+ .44	- .57	.7	- .56	-1.32	.7	+ .60	- .65	1	+ .54	+ .32	1	+ .16	- .48	4.7	8
+ .35	+ .41	1	+ .43	+ .65	1	+ .53	+ .31	1	+ .01	- .36	1	+ .29	+ .24	6	9
.00	+ .38	1	+ .26	- .65	1	- .24	+ .06	1	- .39	- .05	1	- .11	+ .05	6	10
(+ .69	+5.26	1)	- .52	- .08	1	- .34	-1.32	1	+ .21	- .44	1	- .23	- .52	5	11
- .37	+ .44	1	+ .70	+ .34	1	+ .28	+ .22	1	- .35	- .48	1	- .24	+ .15	6	12
- .26	+ .24	1	+ .26	- .63	1	-1.11	- .03	1	+ .13	+ .18	1	- .16	.00	6	13
(+1.58	+5.01	.7)	-1.27	- .44	.7	(-3.02	-2.11	1)	+ .22	- .05	.8	- .04	- .66	3.3	14
+ .29	- .35	1	+ .05	- .24	.9	- .16	+ .33	1	- .14	- .13	1	- .08	- .24	5.9	15
+ .22	+ .10	1	- .42	- .10	.8	- .40	+ .08	1	- .27	+ .18	1	- .15	+ .04	5.8	16
-1.40	- .44	.6				+ .53	- .89	.8	-1.05	+ .12	.4	- .75	- .65	2.65	17
+ .23	+ .31	1	+ .37	+ .61	.9	- .64	+ .28	1	+ .10	+ .32	1	- .03	+ .35	5.9	18
+ .08	-1.38	1	+ .18	- .20	.8	(-2.29	+1.06	1)	- .16	+ .16	1	- .04	- .29	4.3	19
- .41	+ .24	1	+ .17	- .44	.8	- .11	- .48	1	+ .02	- .81	1	- .03	- .22	5.8	20
+ .06	- .69	.9	+ .11	- .46	1	-1.09	+ .57	1	+ .15	- .24	1	- .25	- .17	5.1	21
- .05	+ .13	1	+ .58	- .06	.9	- .84	+ .35	1	+ .22	+ .08	1	- .12	+ .02	5.9	22
+ .44	- .85	.7	- .56	- .97	.4	+ .32	- .57	.7	+ .14	- .40	.8	- .03	- .63	4.3	23
+ .16	- .44	1	+ .20	- .05	1	- .89	- .12	1	+ .17	+ .40	1	- .18	- .05	5.7	24
+ .02	- .03	1	- .23	- .39	1	- .49	- .40	1	- .08	- .13	1	- .21	- .19	6	25
+ .50	+ .10	.9	- .33	- .58	.7	- .20	+ .13	1	+ .17	- .60	.8	- .19	- .08	5.1	26
- .56	+ .07	1	- .48	- .11	1	- .12	+ .26	1	+ .11	- .17	1	- .34	+ .04	6	27
+ .04	.00	1	- .13	+ .14	1	- .11	- .20	1	- .08	- .09	1	- .16	- .04	6	28
+ .08	+ .10	1	.00	+ .25	1	- .75	.00	1	- .10	- .31	1	- .16	.00	6	29
(+ .51	+5.43	1)	+ .77	+ .07	1	+1.37	+ .29	1	(- .12	+2.21	1)	+ .69	+ .23	4	30
- .10	+ .47	.6	- .64	- .47	.4	- .07	- .13	1	+ .74	+ .29	.4	- .10	+ .11	4	31
- .09	+ .14	1	- .22	- .18	1	- .18	- .25	1	- .22	- .07	1	- .06	- .17	6	32
+ .01	+ .54	.7	-1.62	+ .20	.4	+ .58	+ .28	1	+1.94	+ .09	.4	+ .18	+ .38	4.5	33
- .14	+1.03	1	+ .48	- .39	1	- .20	- .16	1	+ .19	+ .86	1	- .01	+ .22	6	34
- .06	+ .10	1	- .79	+ .06	1	+ .10	- .22	1	- .19	- .41	1	- .13	- .01	6	35
+ .15	+ .24	.4	- .87	+ .35	.4	- .23	+ .77	.9	- .94	+ .13	.8	- .45	+ .42	4.2	36
+ .51	- .06	1	- .30	+1.02	.9	+ .41	+ .50	1	+ .34	+1.14	.7	+ .29	+ .35	5.6	37
- .95	- .94	.8	- .09	+ .52	.4	+ .96	+ .46	.8	+ .90	+1.26	.7	+ .18	+ .35	4.3	38
- .10	- .11	1	- .49	+ .02	1	- .30	+ .32	1	+ .12	- .08	1	- .43	+ .20	6	39
+ .42	- .57	1	+ .06	+ .07	.9	+ .24	- .14	1	+ .37	- .09	.8	+ .37	- .07	5.2	40
+ .42	-1.04	.4	-1.41	+ .16	.25	- .39	- .51	.6	+ .22	+ .57	.5	- .11	+ .14	3.25	41
-1.22	+ .54	.8	- .06	+ .77	.4	- .28	+ .20	1	+ .37	- .44	1	- .49	+ .08	4.6	42
+ .06	+ .03	1	- .36	- .20	.9	- .06	- .37	1	- .08	- .32	1	+ .04	- .15	5.9	43
- .13	- .35	1	+ .66	- .30	1	- .45	- .09	1	- .12	- .18	1	- .15	- .20	6	44
+ .21	.00	1	- .60	- .05	1	- .18	+ .10	1	+ .01	.00	1	- .04	+ .03	6	45
- .03	+ .06	.8	- .52	- .07	.9	+ .92	+ .09	1	- .36	- .02	1	- .04	- .19	5.7	46
+ .45	+ .15	.3	+ .15	+1.73	.4	- .42	+ .71	.5	-1.30	+1.01	.25	+ .02	+ .35	2	47
- .97	+ .35	.5	-1.24	+ .08	.4	+ .15	- .82	.8	-1.21	- .29	.25	- .48	- .15	3.25	48
+ .24	+ .02	1	- .89	- .16	.7	- .39	+ .11	1	+ .52	+ .13	1	- .05	+ .18	5.5	49
- .02	+ .15	1	- .76	- .13	1	- .43	- .17	1	- .35	- .27	1	- .34	- .06	6	50
+ .40	- .62	1	+ .18	- .33	1	+ .30	+ .12	1	+ .39	- .10	1	+ .20	- .12	6	51
- .68	- .73	.4	-1.61	+1.16	.25	+ .85	+ .22	.4	+ .82	- .64	.5	+ .08	+ .04	2.25	52
- .13	- .14	1	+ .03	+ .35	1	- .29	+ .06	1	+ .24	+ .03	1	- .06	+ .12	6	53
-1.26	- .70	.4	- .57	- .37	.4	+ .80	+1.19	.7	+ .83	+ .49	.25	+ .18	+ .32	2.35	54
- .55	+ .41	.4	- .45	-1.20	.4	+ .32	+ .12	.9	+1.12	- .02	.4	+ .19	- .01	3.4	55
- .99	+ .35	.25				-1.43	+ .75	.3				-1.23	+ .56	.55	56
- .71	+ .41	1	- .49	- .03	1	+ .27	- .15	1	+ .10	- .49	1	- .13	- .05	6	57
+ .20	+ .03	1	- .49	+ .37	1	- .51	- .07	1	- .08	+ .21	1	- .12	+ .21	6	58
-1.48	- .44	.7	+ .34	+ .02	.9	+ .24	- .28	1	+ .35	- .75	1	+ .07	- .17	5.1	59
+ .16	- .16	1	- .21	- .27	1	- .72	- .09	1	+ .34	+ .56	1	+ .05	- .02	6	60
-1.27	- .57	.3				- .70	+2.04	.25	-1.21	+ .19	.3	- .93	+ .60	1.25	61
- .04	-1.11	.3	(-1.79	-2.16	.25)				- .67	+ .41	.25	- .32	- .42	.55	62

TABLE 4—(Continued)

OBSERVED DISPLACEMENTS IN x AND y FOR STARS OF ECLIPSE FIELD

No.	B. D.	R. A. 1900	Decl.	Phtgr. magn.	P. M. and Par. corrections	x	y	M15-M7 D_x D_y	Wt.	M16-M11 D_x D_y	Wt.
63	+3°2566	11 ^b 47 ^m 3	+3°39'	9 ^m 1	+°03 -°02	-.070	+.334	+°.61 +°.07	1	-.°11 -.°20	1
64	-0 2501	47.8	-0 43	9.7	+.02 -.01	-.055	-.246			+.35 -.90	.5
65	+0 2853	48.5	+0 15	10.2		-.032	-.118	-1.24 -.74	.25	+1.20 +.29	.25
67	-2 3433	48.8	-3 13	8.1	+.01 +.02	-.022	-.581			-.18 -.05	1
68	-0 2507	48.9	-0 29	9.4	.00 .00	-.017	-.216	-.15 +.23	.8	-.32 -.10	1
69	-0 2510	49.7	-0 53	8.4	-.07 +.11	+.010	-.270	+.14 -.17	1	+.45 -.21	1
70	-1 2587	49.8	-1 49	9.7	-.01 +.02	+.012	-.394	+.80 +.06	1	-.07 -.01	1
71	+0 2858	49.9	+0 0	8.3	+.01 +.01	+.015	-.152	+.44 -.05	.8	+.28 -.47	1
72	+4 2544	50.0	+3 47	8.1	+.01 .00	+.019	+.350	+.49 +.60	1	+.72 -.05	1
73	+2 2493	49.9	+1 52	9.4	.00 .00	+.017	+.098			-.13 +.06	.25
74	+1 2628	50.3	+1 39	7.3	+.02 +.02	+.027	+.068	+.65 +.50	1	+.55 +.66	.7
75	+3 2572	50.3	+2 50	9.1	.00 -.01	+.027	+.227	-.39 -.15	.5	+.19 +1.45	.9
76	-0 2512	50.9	-0 53	8.8	.00 +.01	+.049	-.270	+.21 -.08	.8	-.06 +.18	1
77	+4 2549	51.4	+4 37	8.3		+.065	+.464	+.29 +.76	1	+.06 -.42	1
78	-3 3207	51.5	-4 3	9.5	+.04 +.01	+.070	-.692	+.49 -.46	.8	+.19 -.02	1
79	+6 2529	51.7	+5 54	8.5		+.075	+.635	+.16 +.89	1	+.22 +.04	1
80	+3 2576	51.9	+2 58	9.4	+.03 +.01	+.081	+.243	+.40 -.26	.5	+1.01 +.86	.7
81	+1 2631	51.9	+1 11	9.6	+.01 +.01	+.083	+.005			-.19 -.80	.25
82	+7 2499	52.6	+7 32	8.9		+.107	+.853	+1.32 -.37	.4	-.11 -.67	1
83	+1 2633	53.1	+1 26	8.4	+.01 +.02	+.122	+.038	+.03 +.30	1	+.11 -.01	1
84	+4 2553	53.1	+4 2	8.2	.00 .00	+.123	+.386	+.26 .00	1	+.47 +.13	1
85	+1 2636	53.9	+1 5	7.5	+.02 -.01	+.150	-.008	+.84 -.07	1	+.93 -.56	1
86	-5 3396	53.9	-6 6	8.2		+.149	-.968	-.17 +.74	1	-1.18 +1.08	1
87	-2 3446	54.1	-2 46	9.0		+.157	-.521	+.62 +.13	1	+.01 -.26	1
88	+0 2870	54.2	+0 35	9.6	+.01 .00	+.159	-.075	+.99 -.02	.4	+.13 +.53	.4
89	+2 2499	54.3	+2 23	8.0	-.01 +.01	+.161	+.165	+.43 +.56	1	+.82 +.08	1
91	-1 2600	54.4	-1 22	8.3	+.02 +.01	+.167	-.333	+.51 +.21	1	-.20 +.55	1
92	+2 2501	54.9	+1 46	9.7	.00 +.02	+.181	+.084	+1.50 +.37	.7	+.76 -.76	.7
93	+0 2875	55.3	-0 8	9.6	+.01 +.01	+.195	-.170	+1.22 -.83	.8	+.34 +.34	.8
94	+2 2505	55.9	+2 26	9.6	+.02 -.02	+.214	+.171	+.92 +1.17	.5	-.02 -.36	.9
95	-0 2520	55.9	-1 13	7.8	.00 +.02	+.216	-.313	+.16 -.24	1	+.04 -.18	1
96	+0 2877	56.3	+0 12	9.3	.00 .00	+.230	-.126	+.24 -.99	.8	+.12 +.33	.8
97	+0 2878	56.3	+0 11	9.0	.00 .00	+.230	-.128	+.17 -.14	1	-.36 +.18	.9
98	+3 2588	56.4	+2 55	9.5	-.02 +.02	+.230	+.237	+.11 +.93	.7	-.04 +.67	1
99	+1 2640	56.7	+1 5	10.0	.00 .00	+.240	-.007	+1.02 +.24	.5	+.18 +1.16	.6
100	+0 2880	56.8	+0 40	8.2	+.01 +.03	+.246	-.064	+.43 +.48	1	-.37 -.24	1
101	+0 2881	56.8	+0 38	9.5	+.02 +.01	+.246	-.067	+1.90 +.01	.25	+.90 -.29	.6
102	+1 2641	57.2	+1 7	9.3	+.01 +.01	+.260	-.003	+1.59 -.90	.8	+1.25 +.03	1
103	+0 2883	57.4	+0 36	8.8	+.01 +.01	+.265	-.075	-.03 +.53	.8	-.26 +.04	1
104	+4 2562	57.8	+4 42	9.0	+.01 +.01	+.278	+.475	+.16 -.03	.4	-.06 -.39	.9
105	-0 2524	57.7	-0 53	10.2	+.03 .00	+.276	-.269	-.76 -.55	.6	-1.35 -.52	.7
106	-0 2526	58.1	-1 4	10.2	.00 +.01	+.290	-.295	+.15 -.35	.7	+.01 +.35	.7
107	-1 2613	58.4	-1 53	8.2	+.01 .00	+.299	-.404	+.18 +.28	1	+.46 +.21	1
108	-4 3192	58.5	-4 55	7.2	+.02 +.01	+.301	-.810	-.24 -.31	1	-.36 -.07	1
109	+6 2543	58.6	+6 7	6.9	+.03 +.04	+.305	+.664	-.02 -.19	.7	-.27 +.60	1
110	+2 2509	59.1	+2 1	7.8	+.04 +.01	+.322	+.117	+.10 +.46	1	+.36 +.01	1
111	+1 2647	59.5	+1 19	10.1	-.01 .00	+.336	+.023	+.15 +.72	.5	+.55 +.63	.8
112	+4 2569	59.6	+4 8	8.2	.00 +.01	+.338	+.399	-.54 +.01	1	+.11 -.12	1
113	-0 2531	59.7	-0 36	9.0	+.01 .00	+.340	-.233	+.26 -.54	1	-.20 +.01	1
114	+2 2512	12 ^b 0.1	+2 44	9.3	+.02 +.03	+.354	+.213	-.29 +.58	1	+.06 -.12	1
115	-0 2532	0.1	-0 57	9.2	+.12 .00	+.357	-.280	-.22 +.30	1	-.20 +.38	1
116	+2 2513	0.7	+1 52	9.4	+.02 +.01	+.377	+.096	+.16 +1.31	.7	+.25 +.25	.8
117	-2 3460	0.9	-2 34	7.5	+.01 +.01	+.381	-.496	+.45 +.26	1	+.10 +.28	1
118	+0 2894	2.1	-0 4	8.7	+.01 .00	+.421	-.162	+.60 +.31	1	-.25 -.08	1
119	+1 2656	2.9	+1 11	7.5	-.04 +.02	+.448	+.004	-.07 +.09	1	+.08 +.25	1
120	-3 3239	3.1	-3 44	8.6	.00 .00	+.456	-.651	-.37 -.21	.8	-.05 -.02	1
121	-1 2622	3.2	-1 32	9.0	+.01 .00	+.458	-.357	-.76 -.53	.7	-.39 -.15	.9
122	+3 2603	3.6	+3 40	9.6	+.01 -.01	+.471	+.336	-.98 +.23	.4	-.09 -.36	1
123	+2 2517	4.6	+2 28	7.1	-.03 +.07	+.504	+.176	+.01 +.08	1	+.25 +.06	1
124	-4 3219	4.7	-4 40	7.6		+.508	-.777	-.06 -.23	.7	-.42 -.16	.9
125	+6 2559	5.0	+6 22	6.0	+.03 -.01	+.517	+.699	-.26 +.26	.7	-.05 -.02	1
126	-5 3442	6.0	-6 0	8.6		+.554	-.956	(-.92 +2.37	.4)	-.36 +.39	.4

TABLE 4—(Continued)

OBSERVED DISPLACEMENTS IN x AND y FOR STARS OF ECLIPSE FIELD

D_x	M17-M9 D_y	Wt.	D_x	N18-N11 D_y	Wt.	D_x	N19-N13 D_y	Wt.	D_x	N20-N9 D_y	Wt.	Mean obs. displacement D_x D_y Wt.			No.
- '28	+ '64	.9	+ '26	+ '63	1	+ '28	+ '18	1	+ '16	- '06	1	+ '16	+ '20	5.9	63
(-2.62	-2.31	.25)	+1.11	+ .26	.4	+ .89	- .58	.5	+ .76	- .46	1.4	+ .76	- .46	1.4	64
- .69	-1.12	.25	-1.66	- .71	.25	- .14	-1.32	.25	- .51	- .72	1.25	- .51	- .72	1.25	65
+ .45	+ .13	1	- .55	+ .15	1	- .04	+ .55	1	- .20	+ .21	5	- .20	+ .21	5	67
-1.06	- .96	.4	- .59	- .22	.7	+ .19	+ .20	1	- .24	- .06	4.9	- .24	- .06	4.9	68
+ .40	+ .04	1	- .22	- .10	1	+ .29	- .13	1	+ .15	+ .43	1	+ .15	+ .43	1	69
+ .75	- .38	.4	+ .21	- .19	.7	+ .14	- .87	1	+ .27	- .09	5.1	+ .27	- .09	5.1	70
- .02	-1.14	.7	+ .85	+ .16	.7	+ .20	- .87	1	+ .39	- .40	5	+ .39	- .40	5	71
+ .14	+ .27	1	+ .22	+ .08	1	+ .16	+ .16	1	+ .30	+ .19	6	+ .30	+ .19	6	72
-1.23	-1.57	.25				+ .49	+ .05	.3	- .25	- .45	.8	- .25	- .45	.8	73
+ .01	- .12	.4	- .35	+ .43	.7	.00	+ .18	.4	+ .26	+ .48	4.1	+ .26	+ .48	4.1	74
- .93	- .78	.5				+ .38	+ .70	.8	- .07	+ .51	2.7	- .07	+ .51	2.7	75
+ .08	+ .46	.9	+ .19	- .13	1	- .10	- .40	1	- .03	- .03	5.7	- .03	- .03	5.7	76
+ .43	+ .12	1	+ .32	- .35	.7	+ .25	+ .60	1	+ .24	+ .14	5.7	+ .24	+ .14	5.7	77
+ .58	- .63	1	- .07	- .79	.4	- .86	+ .85	1	.00	- .17	5	.00	- .17	5	78
+ .28	- .74	.7	- .40	- .13	1	- .20	+ .12	1	+ .01	+ .05	5.7	+ .01	+ .05	5.7	79
+ .44	+1.52	.9	- .42	+1.65	.4	+ .39	+1.35	.6	+ .40	+ .78	3.8	+ .40	+ .78	3.8	80
+ .77	+ .29	.25							+ .29	- .26	.5	+ .29	- .26	.5	81
+ .25	+ .82	.5	+ .51	- .09	1	- .15	- .56	1	+ .09	- .28	4.7	+ .09	- .28	4.7	82
+ .22	+ .47	1	- .17	+ .56	.8	+ .25	- .17	1	+ .20	+ .28	5.5	+ .20	+ .28	5.5	83
+ .50	+ .24	1	+ .18	+ .37	1	+ .16	+ .39	1	+ .28	+ .29	6	+ .28	+ .29	6	84
+ .43	- .05	1	- .18	+ .43	1	+ .30	- .31	1	+ .44	- .15	6	+ .44	- .15	6	85
(- .05	+5.85	.7)	+ .77	- .17	.9	+ .50	- .95	1	- .04	+ .18	3.0	- .04	+ .18	3.0	86
-1.16	+ .34	.7	- .70	- .43	1	- .93	- .22	1	- .37	- .11	5.5	- .37	- .11	5.5	87
+ .02	-1.68	.3	- .01	- .83	.3	+ .71	+ .57	.5	+ .34	- .18	2.4	+ .34	- .18	2.4	88
+ .45	- .57	1	+ .06	+1.22	1	+ .23	+1.41	.4	+ .36	+ .36	5.4	+ .36	+ .36	5.4	89
+ .24	+ .22	1	- .14	+ .31	1	+ .13	- .68	1	+ .09	+ .11	6	+ .09	+ .11	6	91
+1.05	-2.05	.5	+ .56	+1.45	.25	+1.86	+ .42	.9	- .77	+ .07	.8	+ .86	- .13	3.85	92
+ .55	+ .26	.5	+ .88	+ .59	.3	+1.00	- .40	1	+ .71	- .08	4.1	+ .71	- .08	4.1	93
+1.38	- .52	.4	+1.23	- .36	.4	+ .92	- .38	1	+ .72	- .13	3.45	+ .72	- .13	3.45	94
+ .43	+ .47	1	- .20	- .01	1	+ .39	- .26	1	+ .11	- .03	6	+ .11	- .03	6	95
+ .46	+ .51	.6	- .28	+ .09	.7	+ .80	+ .21	1	- .02	- .08	4.6	- .02	- .08	4.6	96
- .01	+ .10	1	- .16	+ .08	.7	+ .17	- .94	1	- .06	- .10	5.4	- .06	- .10	5.4	97
- .30	+ .39	.8	+ .54	+ .72	.7	- .18	- .15	1	+ .02	+ .39	5.2	+ .02	+ .39	5.2	98
+1.33	+ .04	.3	+2.03	- .29	.25	- .01	- .17	1	+ .51	+ .16	3.35	+ .51	+ .16	3.35	99
+ .37	- .18	1	- .03	+ .26	1	- .19	- .39	1	+ .06	- .09	6	+ .06	- .09	6	100
+ .34	+1.06	.4	+ .43	+ .04	.25	- .68	+ .18	.4	+ .55	+ .38	2.7	+ .55	+ .38	2.7	101
+ .14	- .11	.5	+ .21	+ .66	.4	+ .46	+ .25	.9	+ .84	+ .15	4.3	+ .84	+ .15	4.3	102
+ .51	- .66	1	+ .78	+ .15	.7	+ .02	- .47	1	+ .26	- .05	5.5	+ .26	- .05	5.5	103
- .28	+ .95	1	+ .76	+ .53	1	+ .31	+ .23	1	+ .24	+ .13	5.3	+ .24	+ .13	5.3	104
- .46	- .23	.7	- .30	- .36	.4	+ .67	- .67	.7	- .22	- .36	3.8	- .22	- .36	3.8	105
+ .29	- .22	.7	+1.16	-1.16	.7	+ .57	- .34	1	+ .18	- .24	4.8	+ .18	- .24	4.8	106
- .03	+ .08	1	- .28	+ .08	1	+ .69	+ .06	1	+ .13	+ .17	6	+ .13	+ .17	6	107
-1.12	- .20	1	+ .08	- .03	1	- .59	+ .99	1	- .47	+ .09	6	- .47	+ .09	6	108
- .18	- .39	1	+ .26	- .39	1	+ .24	- .04	1	- .11	- .03	5.7	- .11	- .03	5.7	109
+ .10	- .03	1	- .03	+ .20	1	+ .16	- .08	1	+ .11	+ .13	6	+ .11	+ .13	6	110
- .65	+ .02	.5	+ .11	+ .49	.4	+1.41	+ .34	.7	+ .46	+ .46	3.6	+ .46	+ .46	3.6	111
+ .16	- .04	1	+ .21	+ .58	1	+ .14	+ .58	1	- .05	+ .17	6	- .05	+ .17	6	112
+ .37	+ .58	1	+ .33	- .36	1	- .40	- .60	1	- .05	- .14	6	- .05	- .14	6	113
- .07	- .12	1	+ .14	+ .39	1	+ .49	- .04	1	+ .09	+ .06	6	+ .09	+ .06	6	114
- .28	+ .62	.8	+ .23	- .23	.9	- .54	- .25	1	- .08	+ .02	5.7	- .08	+ .02	5.7	115
+ .84	+ .34	.7	+ .45	- .17	.7	- .79	+ .54	.4	+ .37	+ .35	4.1	+ .37	+ .35	4.1	116
- .53	+ .51	1	+ .12	- .19	1	- .42	+ .72	1	- .16	+ .29	6	- .16	+ .29	6	117
+ .17	+ .08	1	+ .09	- .69	1	+ .15	- .59	1	+ .15	- .22	6	+ .15	- .22	6	118
+ .05	- .24	1	- .04	+ .43	1	- .24	+ .13	1	.00	+ .11	6	.00	+ .11	6	119
- .03	- .19	1	- .09	- .61	1	- .84	+ .76	1	- .28	- .04	5.6	- .28	- .04	5.6	120
+ .09	+ .33	.9	+ .42	- .11	1	+ .09	+ .29	1	+ .02	+ .08	5.5	+ .02	+ .08	5.5	121
+ .49	+1.53	.7	- .03	- .84	.4	+ .15	+ .31	1	+ .04	+ .36	4.5	+ .04	+ .36	4.5	122
- .18	+ .09	1	- .37	+ .14	1	.00	+ .03	1	+ .01	+ .07	6	+ .01	+ .07	6	123
- .89	+ .02	.8	- .29	- .04	1	- .56	+ .89	1	- .40	+ .13	5.4	- .40	+ .13	5.4	124
- .34	- .40	1	+ .07	+ .08	1	- .44	+ .19	1	- .16	- .02	5.7	- .16	- .02	5.7	125
(- .37	+5.07	.4)	- .53	- .30	.7	+ .84	-2.06	.8	+ .08	- .89	1.9	+ .08	- .89	1.9	126

TABLE 4—(Continued)

OBSERVED DISPLACEMENTS IN x AND y FOR STARS OF ECLIPSE FIELD

No.	B. D.	R. A. 1900	Decl.	Phtgr. magn.	P. M. and Par. corrections		x	y	M15-M7		Wt.	M16-M11		Wt.
									D_x	D_y		D_x	D_y	
127	+0°2907	12 ^h 6 ^m 3	+0°45'	8 ^m 7	+°.01	°.00	+ .562	-.051	+°.48	+°.34	.7	+°.17	+°.01	.5
128	+8 2576	6.4	+7 55	8.5			+ .565	+ .909				+ .07	+1.21	.4
129	+3 2612	6.7	+3 41				+ .570	+ .465	- .56	- .20	.7	+ .54	+ .10	1
130	-1 2635	7.6	-1 55	8.6	+ .01	+ .04	+ .605	-.407	- .12	+°.20	.7	- .20	- .70	.8
131	-0 2549	8.5	-0 41	8.2	+ .03	+ .01	+ .636	- .243	- .07	+ .16	.7	+ .06	+ .02	.9
132	+4 2594	11.0	+4 7	9.0	+ .02	.00	+ .719	+ .399	- .05	- .86	.5	- .10	+ .14	.4
133	+7 2526	11.2	+6 47	8.9			+ .723	+ .756	-1.30	+ .58	.25	+1.10	- .19	.5
134	-3 3257	11.1	-3 54	9.3			+ .726	- .674	+ .26	+ .68	.4	- .12	- .78	.5
135	+8 2583	11.4	+7 55	8.0			+ .729	+ .909	(-3.54	-1.02	.4)	- .73	- .15	.4
136	-5 3465	11.2	-5 17	8.6			+ .729	- .862	- .59	- .56	.4	+ .62	.00	.5
137	-1 2639	11.6	-2 11	9.2			+ .740	- .444	+1.17	- .67	.4	+ .82	+ .11	.5
138	+1 2676	11.9	+0 54	9.0	+ .01	+ .01	+ .748	- .030	+1.27	- .18	.4	- .06	+ .21	.8
139	+2 2526	12.8	+2 8	8.0	+ .01	+ .01	+ .780	+ .133	- .56	- .01	.6	+ .52	+ .47	.8
140	+0 2920	13.5	-0 14	6.0	- .01	+ .01	+ .806	- .183	- .66	- .42	.6	+ .28	+ .04	1
141	-2 3497	14.1	-2 46	9.3			+ .825	- .523	- .53	- .54	.4	+ .29	- .28	.4
142	+7 2534	14.3	+7 33	8.5			+ .829	+ .861	- .83	-1.30	.4	- .06	-1.34	.4
143	-3 3267	15.1	-3 57	8.3			+ .860	- .683	- .85	+ .03	.4	+ .05	- .33	.4
144	+4 2609	17.2	+3 51	9.2	+ .04	- .01	+ .924	+ .365	- .85	- .75	.4	-1.03	- .46	.4
145	+6 2599	17.5	+5 52	7.0	+ .03	+ .03	+ .934	+ .635	- .41	-1.21	.4	- .85	- .04	.6
146	+1 2689	18.4	+1 16	9.1	- .03	+ .07	+ .968	+ .019	(+1.55	-2.81	.4)	-1.25	+ .04	.4
147	-5 3487	18.3	-5 34	7.7			+ .968	- .903	- .03	+ .76	.4	- .33	- .38	.4

TABLE 5

RESIDUALS WALLAI-TAHITI FOR STARS OF CHECK FIELD

No.			M15-M7		Wt.	M17-M9		Wt.	N18-N11		Wt.	N20-N9		Wt.	Mean		Wt.
	x	y	v_x	v_y		v_x	v_y		v_x	v_y		v_x	v_y				
1c	-.995	+ .324	(+2°.05	+°.69	.7)	-.30	+°.49	.7	-.57	+°.52	1	+°.17	+°.34	1	-.23	+°.19	2.7
2c	-.985	-.217	+ .71	- .36	1	-.37	+ .01	1	+ .21	+ .19	1	- .27	- .25	1	+ .07	- .10	4
3c	-.976	+ .919	+ .52	+ .43	.4	-.19	- .87	.7	-1.12	- .62	.4	+1.03	- .11	.4	+ .02	- .38	1.9
4c	-.924	+ .716	+ .05	- .36	.8	+ .43	+ .14	.9	- .14	+ .62	1	- .04	+ .02	1	+ .07	+ .13	3.7
5c	-.901	-.783	- .60	- .17	1	- .57	+ .31	.8	+ .43	- .55	.7	- .16	+ .55	.7	- .27	+ .02	3.2
6c	-.855	-.929	+ .07	- .14	1	(+ .76	+3.51	.9)	+ .02	- .64	.7	+ .41	- .25	.8	+ .16	- .32	2.5
7c	-.826	+ .213	.00	+ .19	1	- .14	+ .21	1	.00	- .26	1	- .20	+ .11	1	- .08	+ .06	4
8c	-.810	-.502	- .07	+ .60	1	+ .39	+ .19	1	+ .17	+ .26	1	- .26	- .28	1	+ .06	+ .19	4
9c	-.766	+ .008	- .05	+ .14	1	+ .18	- .40	1	+ .38	- .01	.7	+ .05	+ .58	1	+ .12	+ .08	3.7
10c	-.752	+ .641	- .12	+ .24	.8	- .23	- .52	.9	+ .02	+ .22	1	- .16	- .45	1	- .12	- .14	3.7
11c	-.706	- .375	- .31	- .14	1	.00	+ .08	1	- .02	+ .41	1	- .49	- .05	1	- .20	+ .08	4
12c	-.698	+ .375	- .12	- .26	1	+ .25	+ .62	1	+ .50	+ .10	1	- .08	+ .04	1	+ .14	+ .12	4
13c	-.582	- .299	- .05	+ .10	1	+ .47	+ .35	1	- .05	- .06	1	+ .07	+ .30	1	+ .11	+ .17	4
14c	-.565	-.958	(+ .52	-2.03	1)	(+ .99	+4.54	1)	- .31	+ .39	1	+ .36	+ .86	1	+ .02	+ .62	2
15c	-.533	+ .681	.00	+ .41	1	- .18	+ .52	1	+ .43	- .19	1	- .18	- .01	1	+ .02	+ .18	4
16c	-.507	+ .926	+ .19	- .67	.4	- .25	+1.03	1	+1.00	+ .36	.4	- .13	+ .45	.8	+ .05	+ .49	2.6
17c	-.494	+ .364	- .67	+ .24	1	+ .54	+ .61	1	+ .33	- .40	1	- .13	- .09	1	+ .02	+ .09	4
18c	-.457	+ .143	- .05	- .19	1	+ .37	+ .13	1	+ .14	- .06	1	+ .15	- .47	1	+ .15	- .15	4
19c	-.436	-.044	- .26	- .36	1	+ .04	- .09	1	- .02	+ .19	1	- .38	- .19	1	- .16	- .11	4
20c	-.424	-.679	- .14	- .05	1	+ .37	- .34	1	+ .14	+ .30	1	+ .05	+ .05	1	+ .10	- .01	4
21c	-.344	+ .137	- .50	- .53	1	- .66	- .59	1	+ .21	+ .50	1	+ .15	- .30	1	- .20	- .23	4
22c	-.300	+ .448	+ .17	+ .31	1	+ .06	+ .52	1	- .33	+ .17	1	- .06	- .22	1	- .04	+ .20	4
23c	-.301	-.627	.00	- .07	1	+ .48	- .37	1	+ .07	+ .18	1	- .14	- .18	1	+ .10	- .11	4
24c	-.285	-.478	+ .38	- .43	1	+ .53	+ .56	.9	- .07	- .45	1	+ .72	+ .15	1	+ .39	- .06	3.9
25c	-.263	- .121	- .12	- .26	1	+ .31	+ .01	1	- .40	- .25	1	+ .51	+ .95	1	+ .08	+ .11	4
26c	-.225	-.851	+ .07	+ .52	1	- .02	- .21	1	- .12	- .36	1	+ .14	- .31	1	+ .02	- .09	4
27c	-.222	-.313	+ .19	- .31	1	- .26	- .42	1	- .14	+ .29	1	- .13	- .20	1	- .08	- .16	4
28c	-.196	+ .200	- .12	+ .21	1	- .05	- .42	.8	- .24	- .28	.8	+ .45	+ .29	1	+ .03	- .02	3.6
29c	-.161	+ .029	- .17	- .24	1	- .51	.00	1	+ .05	+ .36	1	- .14	+ .30	1	- .19	+ .10	4
30c	-.130	-.139	+ .93	+ .55	1	- .74	- .11	1	+ .12	+ .12	.4	+ .49	- .29	1	+ .21	+ .06	3.4
31c	-.122	+ .486	+ .21	- .17	1	+ .96	-1.21	.8	+ .57	- .48	1	- .35	- .06	1	+ .32	- .44	3.8
32c	-.118	+ .128	- .40	+ .19	.7	+ .16	- .02	.7	+ .21	- .12	.6	- .05	+ .02	1	- .03	+ .02	3.0
33c	-.110	-.196	+ .17	+ .50	1	- .14	+ .45	1	- .31	- .45	1	+ .16	+ .01	1	- .03	+ .13	4
34c	-.105	+ .075	+ .05	- .10	1	- .26	- .23	1	- .19	+ .22	.6	- .07	+ .02	1	- .11	- .05	3.6

TABLE 4—(Continued)

OBSERVED DISPLACEMENTS IN x AND y FOR STARS OF ECLIPSE FIELD

D_x	M17-M9 D_y	Wt.	D_x	N18-N11 D_y	Wt.	D_x	N19-N13 D_y	Wt.	D_x	N20-N9 D_y	Wt.	Mean obs. displacement D_x	D_y	Wt.	No.
+1°.05	—°.09	1	+°.68	+°.32	1	+°.06	—°.04	1	+°.13	—°.07	1	+°.45	+°.07	5.2	127
—°.54	—1.07	.3	—°.76	+°.59	.4	+°.11	+°.08	1	—°.10	—°.33	.8	—°.14	+°.07	2.9	128
+°.03	+°.07	.8	—°.17	—°.04	1	.00	—°.24	1	+°.24	+°.38	1	+°.04	+°.02	5.5	129
—°.10	+°.52	.8	+°.80	—°.77	.9	—°.74	+°.78	1	+°.01	+°.28	1	—°.06	+°.07	5.2	130
+°.15	—°.49	1	—°.47	+°.01	.8	—°.13	—°.44	1	—°.39	—°.48	1	—°.14	—°.24	5.4	131
+°.18	+°.42	.7	—°.01	+°.38	1	+°.01	+°.20	1	+°.45	—°.29	.7	+°.09	+°.07	4.3	132
+°.45	—°.44	.4	+°.51	—°.38	.7	+°.20	—°.44	1	+°.35	—°.12	1	+°.34	—°.25	3.85	133
+°.12	—°.67	.4	—°.12	+°.52	.6	—°.06	+°.97	.7	+°.54	—°.18	.7	+°.11	+°.14	3.3	134
—°.16	+°.41	.4	—°.42	+°.70	.7	—°.30	—°.20	1	+°.13	—°.59	1	—°.23	—°.06	3.5	135
—°.93	—1.10	.4	+°.74	+°.29	.7	—°.12	+°.16	1	+°.28	+°.14	.7	+°.08	—°.05	3.7	136
—°.07	—1.14	.4	+1.02	+°.41	.6	—°.12	+°.49	1	—°.10	+°.09	.7	+°.35	+°.04	3.6	137
.00	+°.66	.7	—°.57	+°.94	.7	+°.14	—°.50	.9	+°.32	+°.13	1	+°.11	+°.20	4.5	138
+°.14	+°.02	.7	—°.39	+°.26	1	+°.14	—°.24	1	+°.01	—°.13	1	—°.01	+°.20	4.5	139
+°.42	+°.25	1	—°.22	.00	1	—°.04	+°.13	1	—°.13	+°.09	1	—°.02	+°.05	5.6	140
+°.44	—°.16	.4	+°.42	—°.77	.7	—°.10	—°.26	.9	—°.01	+°.17	1	+°.07	—°.26	3.8	141
—°.55	+°.96	.4	+°.69	+°.63	.4	—°.64	—°.19	1	—°.12	+°.26	.5	—°.32	—°.15	3.1	142
+°.36	—°.08	.6	+°.20	—°.55	.8	+°.18	+°.22	.8	—°.26	+°.13	1	—°.02	—°.08	4	143
—°.79	—°.15	.4	+°.31	—°.88	.9	+°.19	—°.42	1	—°.48	—°.07	1	—°.26	—°.44	4.1	144
—°.30	+°.42	.4	—°.15	—°.58	1	—°.30	—°.08	1	—°.55	—°.58	1	—°.41	—°.36	4.4	145
+°.79	—°.93	.4	+°.48	+°.50	.4	—°.03	+°.28	.8	+°.26	—°.34	.4	+°.04	—°.03	2.4	146
(—°.43	+1.94	.4)	—°.33	—°.27	1	+°.33	—1.01	.8	(—°.26	+1.59	.8)	—°.08	—°.36	2.6	147

TABLE 5—(Continued)

RESIDUALS WALLAI-TAHITI FOR STARS OF CHECK FIELD

No.	x	y	v_x	M15-M7 v_y	Wt.	v_x	M17-M9 v_y	Wt.	v_x	N18-N11 v_y	Wt.	v_x	N20-N9 v_y	Wt.	v_x	Mean v_y	Wt.
35c	—°.065	+°.758	—°.17	—°.45	1	+°.09	—°.39	1	+°.02	—°.41	1	—°.12	—°.16	1	—°.04	—°.35	4
36c	—°.028	—°.524	+°.10	—°.10	1	+°.29	—°.35	1	—°.05	+°.43	1	—°.16	—°.47	1	+°.04	—°.12	4
37c	—°.016	+°.130	+°.29	—°.33	1	+°.10	—°.55	1	—°.24	+°.30	1	—°.03	+°.19	1	+°.03	—°.10	4
38c	+°.004	—°.084				—1.01	—1.21	.5				—°.18	—°.24	.25	—°.73	—°.89	.75
39c	+°.032	—°.064				—1.20	—°.07	.25							—1.20	—°.07	.25
40c	+°.044	+°.305	+°.24	—°.12	1	+°.15	+°.38	1	—°.05	+°.05	1	—°.02	+°.13	1	+°.08	+°.11	4
41c	+°.061	—°.225	+°.33	+°.48	.6	—°.10	+°.29	1	—°.10	+°.34	.7	—°.10	+°.05	1	—°.02	+°.26	3.3
42c	+°.079	—°.890	+°.29	—°.62	1	—°.11	+°.97	1	+°.36	—°.12	1	—°.15	+°.03	1	+°.10	+°.06	4
43c	+°.084	+°.102	+°.14	—°.19	.4	—°.60	+°.61	.7	+°.38	+°.43	.25	+°.43	+°.97	.8	+°.03	+°.58	2.15
44c	+°.117	+°.638	—°.10	—°.31	1	—°.42	—°.07	1	—°.02	—°.26	1	+°.20	+°.30	1	—°.08	—°.08	4
45c	+°.121	—°.672	—°.29	+°.17	1	—°.29	—°.35	1	+°.14	—°.17	1	+°.06	+°.16	1	—°.10	—°.05	4
46c	+°.126	—°.366	+°.12	—°.31	1	+°.08	—°.02	1	—°.14	—°.05	1	—°.20	+°.13	1	—°.04	—°.06	4
47c	+°.171	—°.115	+°.02	—°.14	1	+°.11	+°.22	1	—°.52	—°.17	1	—°.01	—°.40	1	—°.10	—°.12	4
48c	+°.224	+°.178	+°.38	+°.14	1	+°.24	+°.27	1	—°.17	+°.02	1	+°.24	+°.06	1	+°.17	+°.12	4
49c	+°.244	+°.979	(—°.17	+1.62	.4)	+°.53	+°.08	.6	—°.33	—°.02	1	—°.24	—°.09	1	—°.10	—°.02	2.6
50c	+°.246	—°.539	—°.05	+°.07	1	—°.26	+°.25	1	—°.21	—°.24	1	—°.22	+°.07	1	—°.18	+°.04	4
51c	+°.256	—°.005	+°.24	+°.24	1	+°.14	—°.13	1	—°.52	+°.36	1	+°.19	—°.19	1	+°.01	+°.07	4
52c	+°.284	—°.410	—°.02	—°.12	1	—°.07	—°.05	1	+°.33	—°.59	1	+°.02	—°.14	1	+°.06	—°.22	4
53c	+°.308	—°.038	—°.52	+°.10	.7	—°.10	+°.12	1	—°.31	+°.06	.7	+°.61	—°.15	1	—°.02	+°.02	3.4
54c	+°.330	+°.831	—°.17	+°.29	1	—°.26	—°.26	1	—°.52	+°.12	1	—°.45	+°.07	1	—°.35	+°.06	4
55c	+°.384	+°.310	—°.02	+°.21	1	—°.04	+°.04	1	—°.12	+°.24	1	—°.46	+°.17	1	—°.16	+°.16	4
56c	+°.405	+°.586	+°.19	+°.19	1	—°.02	+°.18	1	+°.14	+°.15	1	—°.03	—°.13	1	+°.07	+°.10	4
57c	+°.431	—°.300	—°.36	+°.02	1	—°.27	+°.33	1	+°.19	—°.17	1	—°.14	+°.24	1	—°.14	+°.10	4
58c	+°.485	—°.620	—°.43	+°.02	1	+°.12	+°.45	1	+°.05	—°.48	1	—°.53	+°.03	.7	—°.17	.00	3.7
59c	+°.489	+°.060	—°.12	+°.14	1	—°.21	+°.07	1	+°.02	+°.22	1	—°.17	+°.09	1	—°.12	+°.13	4
60c	+°.546	+°.936	.00	+°.17	.7	+°.20	—°.13	.9	+°.29	+°.24	1	+°.02	+°.27	1	+°.14	+°.14	3.6
61c	+°.593	—°.113	—°.21	+°.26	1	—°.03	—°.25	1	+°.45	.00	1	—°.50	—°.20	1	—°.07	—°.05	4
62c	+°.608	+°.177	—°.22	—°.29	.7	+°.80	—°.42	.7	+°.33	—°.43	.25	—°.27	—°.24	1	+°.08	—°.32	2.65
63c	+°.617	+°.361	—°.26	+°.05	1	—°.23	+°.07	1	—°.29	+°.15	1	+°.12	—°.34	1	—°.16	—°.02	4
64c	+°.622	+°.688	+°.21	+°.10	1	+°.06	+°.33	1	+°.24	—°.05	1	—°.07	—°.45	1	+°.11	—°.02	4
65c	+°.681	—°.675	—°.26	—°.31	.5	—°.24	+°.21	.6	+°.05	+1.07	.7	+1.01	—°.05	1	+°.28	+°.24	2.8
66c	+°.691	—°.909	—°.83	+°.41	.6	(—2.55	+°.96	.9)	+°.31	+°.12	1	(—°.74	+1.65	1)	—°.12	+°.23	1.6
67c	+°.751	—°.275	—°.24	+°.10	.8	+°.76	+°.12	1	+°.29	—°.25	1	—°.18	—°.02	1	+°.18	—°.02	3.8
68c	+°.805	+°.969	+°.31	+°.26	.4	—°.13	+°.09	.5	+°.45	+°.45	1	+°.27	+°.02	1	+°.27	+°.21	2.9

TABLE 5—(Continued)

No.	x y		v_x v_y		Wt.	v_x v_y		Wt.	v_x v_y		Wt.	v_x v_y		Wt.	v_x v_y		Wt.	Mean v_x v_y		Wt.
69c	+.843	+.278	-.26	+.14	.6	+.57	-.33	.6	-.48	-.09	1	+.07	+.04	1	-.07	-.05	3.2			
70c	+.907	+.522	-.29	-.95	.5	-.46	+.02	.7	-.02	-.55	1	+.03	+.02	1	-.14	-.31	3.2			
71c	+.913	+.754	+.02	-.10	.6	-.19	+.91	.6	-.60	-.31	1	-.11	+.11	1	-.25	+.09	3.2			
72c	+.927	-.015	+.64	+.07	.5	+.40	-.63	.7	+.50	+.10	1	+.40	+.27	1	+.47	-.01	3.2			
73c	+.942	-.183	+.45	-.19	.7	-.45	-.68	.7	+.52	-.18	1	+.06	-.06	1	+.17	-.25	3.4			
74c	+.945	-.790	+.60	-.05	.4	-.15	+.57	.4	-.52	.00	1	+.02	+.27	1	-.11	+.17	2.8			
75c	+.968	-.573	+.55	+.19	.4	-.68	-.21	.6	-.79	-.45	1	-.21	-.44	1	-.40	-.31	3.0			

for the steel of the camera tubes and of 0.0000089 for the glass of the photographic plate it is found that 0".30 (0".040 per 1°) of our scale difference must be due to temperature expansion. We have then:

	5-foot cameras	15-foot cameras
Scale diff. between ecl. and check field.....	0".153 per 1°	0".415 per 1°
Light deflection of ref. stars (Einstein).....	-.009	-.064
Temperature expansion.....	-.040	-.040
Unaccounted for.....	0".104	0".311

The discrepancy is considerably reduced but not removed. However, our assumption that all parts of the telescope were of the same temperature as the air is rather improbable. Most likely the telescope did not follow the rapid cooling of the air (about 2°C) during the half hour preceding totality. Moreover it should be remembered that the observing house had to be opened about 15 minutes before totality so that the telescope was for some time exposed to the rays of the partly eclipsed Sun. This must have heated the telescope tube more than the photographic plate, and a difference of 2.5°C between the temperature of the tube and the plate is sufficient to explain the rest of our scale difference and does not seem at all unreasonable.

The plates of the 15-foot cameras exhibit the same effect even to a larger degree, as is shown in the comparison given above; it may be explained in a similar way, if the tube temperature is assumed to be 7°C higher than that of the plate. In any case there can be no question that the effect is instrumental. For, if the scale value of the check field (corrected for temperature expansion with the air temperature) were used for the reduction of the eclipse field, the two instruments would furnish quite incompatible results for the light deflections of the stars situated between 2° and 3° from the Sun's center (5-foot cameras :0".48, 15-foot cameras :0".94).

The terms of the second order of our reduction formulae are mainly due to differences in the inclination of the plates to the optical axis, for their coefficients (Table 3: "Check F. corr.", or "Eclipse F. corr.")

approximately fulfill the conditions: $d=e'$, $e=f'$, $f=0$, $d'=0$, which are characteristic of inclination corrections. On account of the imperfections of the plate-holders the inclinations vary somewhat for the different pairs of plates of the same camera. In the mean we have:

M-plates: $d=-3".60$ $e'=-3.09$	N-plates: $d=-6".20$ $e'=-5.93$
$e=+2.05$ $f'=+2.17$	$e=+3.12$ $f'=+3.02$
$f=+0.09$ $d'=+0.03$	$f=-0.16$ $d'=-0.24$

or the plate inclination at Wallal differs from that at Tahiti on the average by 3'.8 for objective M, by 6'.6 objective N.

About two-thirds of the coefficients of the third order terms amount to more than twice their probable errors and have probably some real significance, although there does not seem to be much agreement among the three plates of each objective. However, if they are due to optical distortions of the objectives, they depend on the focus setting and plate inclination and may be different for every pair of plates. Whatever their interpretation the use of third order terms in the reduction formulae is necessary to bring each pair of check field exposures to close agreement and thus eliminate any systematic errors. For the study of light deflections the third order terms are of great importance, especially the terms in x^3 and xy^2 for the Δx measures, and the terms in x^2y and y^3 for the Δy measures. On both sides of the plate they produce apparent displacements of the stars towards or away from the center and are thus acting in a somewhat similar sense as the light deflections; if they are not well eliminated, they may seriously affect the law according to which the observed light deflections depend on the star's angular distance from the Sun's center. With the reduction method employed and the great care taken in the check field exposures there can be no doubt that these terms were determined with sufficient accuracy; but without the check field exposures it would hardly have been possible to separate

the effects of these third order correction terms from the light deflections. By chance it happened that the means of the four important constants:

$$g = +".22, \quad k' = -".10; \quad i = -".47, \quad h' = +".06$$

are small, so that our final results were little influenced by them.

7. THE RESULTS OF THE 5-FOOT CAMERAS

Considering the unavoidable errors of observation with an instrument of only 5 feet focal length and the

arrows. In all of the four central areas the displacements are essentially pointed away from the Sun. A small systematic difference between the means of the central and the outer areas is, however, apparent; similar means formed for the check field do not reveal such a difference. To investigate whether this error might have been introduced by a magnitude equation (guiding error), the stars brighter than photographic magnitude 9.0 were separated from the fainter ones. The mean difference between the two groups is:

$$\text{bright-faint } \Delta x = -".08 \quad \Delta y = -".02$$

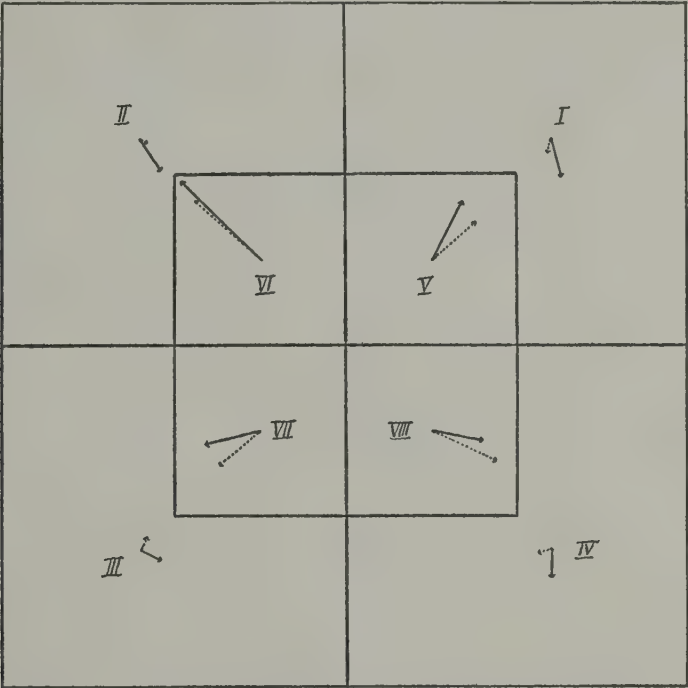


Fig. 1. Average star displacements (relative to reference stars) of 8 areas.
The full line arrows give the observed values.
The dotted arrows give the corrected values.

smallness of the light deflections for the majority of the stars observed, it is evident that group means of several stars are needed to bring out the typical features of the light deflections. The field of our plates was thus divided into 8 areas as outlined in Fig. 1, and the weighted means of the observed star displacements (last four cols. of Table 4) were computed for each area; they are represented in the figure by the full line

About half of this amount would have to be applied to the central areas which contain more faint stars, and this would indeed reduce the discrepancy a little. It is evident that a systematic error of this character would also be produced by errors in the coefficients d, f, d', f' of the second order terms of our reduction formulae which were taken over from the check field and have in the mean of the four check field plates a probable error of $\pm".05$. If these coefficients were corrected by:

Outer areas				Central areas			
Area	Stars	Average displ. D_x	D_y	Area	Stars	Average displ. D_x	D_y
I	16	−".03	−".11	V	18	−".09	+".17
II	15	−".06	−".08	VI	21	+".23	+".23
III	18	−".06	−".03	VII	24	+".16	−".04
IV	17	.00	−".08	VIII	16	−".15	−".03
Mean		−".04	−".08	Mean		+".04	+".08
Check field:		−".01	.00	Check field:		−".02	+".02

$$\Delta d = +".03 \quad \Delta d' = +".10$$

$$\Delta f = +".06 \quad \Delta f' = +".20$$

with a corresponding change in zero point of:

$$\Delta a = .00 \quad \Delta a' = -".07$$

and applying the above determined magnitude equation, the discrepancy would be eliminated. The resulting figures are:

Outer areas			Central areas		
	D_x	D_y		D_x	D_y
I	+".01	-.04	V	-.13	+.11
II	-.02	-.01	VI	+.19	+.17
III	-.02	+.04	VII	+.12	-.10
IV	+.04	-.01	VIII	-.19	-.09

They are marked in Fig. 1 by the dotted arrows which, for the four central areas, are almost exactly pointed away from the Sun's center.

It appears then that the systematic difference between the four central and the four outer areas can be mainly attributed to the accidental errors of the second order plate constants. With the symmetrical

distribution of our stars, however, these terms have practically no influence on the light deflections derived in the mean from all stars.

Our observations having confirmed the assumption that the light deflections operate radially away from the Sun's center, the radial components D_r of the observed star displacements D_x and D_y were computed according to the formula:

$$D_r = D_x \frac{x}{r} + D_y \frac{y}{r} \quad (r = \sqrt{x^2 + y^2})$$

They are given for the eclipse field in Table 6 in which the stars are arranged according to their angular distances from the Sun's center.

When group means are formed, as in Table 7, the phenomenon of star displacement in the neighborhood of the Sun stands out plainly for every single plate.

TABLE 6

RADIAL COMPONENTS OF OBSERVED DISPLACEMENTS FOR STARS OF ECLIPSE FIELD

No.	Distance from Sun's center	M15-M7 D_r Wt.	M16-M11 D_r Wt.	M17-M9 D_r Wt.	N18-N11 D_r Wt.	N19-N13 D_r Wt.	N20-N9 D_r Wt.	Mean D_r Wt.
74	0.55	+ .70 1	+ .82 .7	-.11 .4	+ .27 .7	+ .17 .4	+ .82 .9	+ .54 4.1
81	0.62		-.24 .25	+ .79 .25				+ .28 .5
62	0.66			+ .58 .3			+ .38 .25	+ .49 .55
73	0.75		+ .04 .25	-1.74 .25				-.48 .8
65	0.92	+1.05 .25	-.61 .25	+1.27 .25	+1.13 .25	+1.31 .25		+ .83 1.25
83	0.96	+ .12 1	+ .11 1	+ .35 1	+ .01 .8	+ .19 1	+1.10 .7	+ .28 5.5
56	1.11			+1.05 .25		+1.61 .3		+1.35 .55
85	1.13	+ .84 1	+ .96 1	+ .43 1	-.20 1	+ .31 1	+ .35 1	+ .45 6
71	1.15	+ .09 .8	+ .50 1	+1.14 .7	-.09 .7	+ .89 1	-.03 .8	+ .44 5
52	1.27	+ .40 .3	-.55 .4	+ .67 .4	+1.63 .25	-.85 .4	-.83 .5	-.08 2.25
88	1.32	+ .90 .4	-.11 .4	+ .74 .3	+ .35 .3	+ .39 .5	+ .14 .5	+ .38 2.4
61	1.47		+1.09 .4	+ .28 .3		+2.07 .25	+ .85 .3	+1.03 1.25
92	1.49	+1.52 .7	+ .37 .7	+ .08 .5	+1.13 .25	+1.87 .9	-.67 .8	+ .72 3.85
58	1.54	+ .37 1	+ .10 1	-.11 1	+ .59 1	+ .28 1	+ .21 1	+ .24 6
68	1.63	-.22 .8	+ .13 1	+1.05 .4	+ .27 .7	-.22 1	+ .04 1	+ .08 4.9
50	1.66	+ .36 1	+ .03 1	-.06 1	+ .71 1	+ .45 1	+ .44 1	+ .32 6
89	1.73	+ .70 1	+ .63 1	-.10 1	+ .92 1	+1.18 .4	+ .12 1	+ .51 5.4
75	1.73	-.20 .5	+1.46 .9	-.88 .5		+ .74 .8		+ .50 2.7
99	1.80	+1.01 .5	+ .15 .6	+1.33 .3	+2.04 .25	-.00 1	+ .28 .7	+ .51 3.35
64	1.90		+ .80 .5		-.49 .4	+ .37 .5		+ .28 1.4
100	1.91	+ .29 1	-.30 1	+ .40 1	-.10 1	-.08 1	+ .25 1	+ .08 6
101	1.91	+1.83 .25	+ .95 .6	+ .05 .4	+ .40 .25	-.72 .4	+ .37 .8	+ .43 2.7
80	1.93	-.13 .5	+1.13 .7	+1.58 .9	+1.44 .4	+1.40 .6	-.35 .7	+ .87 3.8
93	1.95	+1.47 .8	+ .03 .8	+ .24 .5	+ .27 .3	+1.01 1	-.04 .7	+ .58 4.1
102	1.95	+1.60 .8	+1.25 1	+ .14 .5	+ .20 .4	+ .46 .9	+ .78 .7	+ .85 4.3
48	1.97	+1.13 .5	-.24 .8	+ .77 .5	+1.12 .4	+ .17 .8	+1.23 .25	+ .51 3.25
96	1.97	+ .69 .8	-.06 .8	+ .15 .6	-.28 .7	+ .60 1	-1.31 .7	+ .02 4.6
97	1.98	+ .22 1	-.40 .9	-.06 1	-.18 .7	+ .61 1	-.39 .8	-.01 5.4
69	2.03	+ .17 1	+ .22 1	-.03 1	+ .09 1	+ .14 1	-.43 1	+ .03 6
53	2.04	+ .27 1	+ .17 1	-.04 1	+ .26 1	+ .22 1	-.12 1	+ .13 6
94	2.06	+1.44 .5	-.24 .9	+ .74 .4	+ .73 .4	+ .47 1	+ .34 .25	+ .48 3.45
76	2.06	+ .12 .8	-.19 1	-.44 .9	+ .16 1	+ .38 1	+ .10 1	+ .03 5.7
103	2.06	-.17 .8	-.26 1	+ .66 1	+ .71 .7	+ .14 1	+ .57 1	+ .27 5.5
47	2.08	-1.13 .25	-1.12 .3	-.33 .3	+ .65 .4	+ .69 .5	+1.61 .25	+ .14 2
55	2.12	-.74 .5	+ .10 .8	-.08 .4	+1.26 .4	-.26 .9	-.54 .4	-.08 3.4
54	2.13		-.02 .6	+ .12 .4	+ .01 .4	+ .54 .7	-.05 .25	+ .17 2.35
42	2.20	+1.03 .5	+ .91 .9	+ .96 .8	-.20 .4	+ .19 1	-.20 1	+ .44 4.6
49	2.28	+ .18 1	+ .48 .8	-.18 1	+ .60 .7	+ .37 1	-.33 1	+ .15 5.5
41	2.29	+ .08 .6	+ .41 .9	-.81 .4	+1.35 .25	+ .14 .6	+ .04 .5	+ .17 3.25
36	2.39	+ .17 .7	+ .63 1	-.12 .4	+ .91 .4	+ .33 .9	+ .95 .8	+ .51 4.2

TABLE 6—(Continued)

RADIAL COMPONENTS OF OBSERVED DISPLACEMENTS FOR STARS OF ECLIPSE FIELD

No.	Distance from Sun's center	M15-M7		M18-M11		M17-M9		N18-N11		N19-N13		N20-N9		Mean	
		D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.
98	2.48	+ .75	.7	+ .45	1	+ .07	.8	+ .89	.7	- .23	1	+ .12	1	+ .30	5.2
111	2.53	+ .20	.5	+ .59	.8	- .65	.5	+ .14	.4	+1.42	.7	+ .64	.7	+ .49	3.6
63	2.58	- .06	1	- .18	1	+ .69	.9	+ .57	1	+ .12	1	- .09	1	+ .17	5.9
110	2.58	+ .25	1	+ .34	1	+ .08	1	+ .04	1	+ .12	1	+ .02	1	+ .14	6
35	2.63	- .27	1	- .02	1	+ .01	1	+ .70	1	.00	1	+ .34	1	+ .13	6
72	2.63	+ .63	1	- .01	1	+ .28	1	+ .09	1	+ .17	1	+ .07	1	+ .20	6
91	2.80	+ .04	1	- .58	1	- .09	1	- .34	1	+ .67	1	- .04	1	- .06	6
29	2.84	- .08	1	+ .27	1	- .09	1	- .04	1	+ .74	1	+ .14	1	+ .16	6
95	2.86	+ .29	1	+ .17	1	- .15	1	- .10	1	+ .44	1	- .14	1	+ .08	6
105	2.89	- .16	.6	- .60	.7	- .17	.7	+ .04	.4	+ .95	.7	+ .44	.7	+ .09	3.8
46	2.92	+ .69	1	+ .38	1	- .03	.8	+ .39	.9	- .67	1	+ .25	1	+ .17	5.7
116	2.93	+ .47	.7	+ .30	.8	+ .90	.7	+ .40	.7	- .63	.4	+ .74	.8	+ .44	4.1
70	2.96	- .04	1	+ .01	1	+ .40	.4	+ .20	.7	+ .87	1	- .66	1	+ .10	5.1
84	3.05	+ .08	1	+ .26	1	+ .38	1	+ .40	1	+ .42	1	+ .59	1	+ .36	6
37	3.07	- .27	1	- .38	1	- .42	1	+ .91	.9	+ .04	1	+ .52	.7	+ .03	5.6
113	3.10	+ .52	1	- .17	1	- .02	1	+ .47	1	+ .01	1	- .60	1	+ .04	6
114	3.10	+ .05	1	- .01	1	- .12	1	+ .32	1	+ .40	1	+ .02	1	+ .11	6
106	3.10	+ .35	.7	- .24	.7	+ .36	.7	+1.64	.7	+ .64	1	- .68	1	+ .30	4.8
39	3.14	+1.39	1	+ .62	1	- .01	1	+ .35	1	+ .44	1	- .14	1	+ .44	6
31	3.18	+ .05	.6	- .03	1	- .21	.6	+ .79	.4	+ .13	1	- .76	.4	.00	4
57	3.18	- .15	1	+ .18	1	- .16	1	+ .19	1	+ .05	1	+ .41	1	+ .09	6
119	3.37	- .07	1	+ .08	1	+ .05	1	- .04	1	- .24	1	+ .19	1	.00	6
118	3.39	+ .45	1	- .20	1	+ .13	1	+ .33	1	+ .35	1	+ .23	1	+ .22	6
115	3.41	- .36	1	- .39	1	- .61	.8	+ .32	.9	- .27	1	+ .79	1	- .08	5.7
34	3.45	+ .24	1	+ .01	1	+ .84	1	- .61	1	+ .02	1	+ .49	1	+ .17	6
28	3.52	+ .30	1	+ .23	1	- .03	1	+ .18	1	- .01	1	+ .02	1	+ .12	6
77	3.52	+ .79	1	- .41	1	+ .18	1	- .31	.7	+ .62	1	- .02	1	+ .17	5.7
25	3.55	+ .26	1	+ .22	1	- .02	1	+ .19	1	+ .45	1	+ .07	1	+ .19	6
33	3.57	- .21	1	- .57	1	- .40	.7	+ .95	.4	- .60	1	-1.39	.4	- .41	4.5
27	3.64	+ .23	1	+ .62	1	+ .50	1	+ .49	1	+ .02	1	- .04	1	+ .30	6
107	3.77	- .11	1	+ .10	1	- .08	1	- .23	1	+ .36	1	- .40	1	- .06	6
112	3.92	- .34	1	- .02	1	+ .07	1	+ .58	1	+ .53	1	- .25	1	+ .10	6
123	4.01	+ .04	1	+ .26	1	- .14	1	- .30	1	+ .01	1	+ .24	1	+ .02	6
87	4.08	+ .06	1	+ .25	1	- .66	.7	+ .21	1	- .06	1	+ .01	.8	.00	5.5
43	4.13	- .57	1	+ .27	1	- .06	1	+ .35	.9	+ .35	1	+ .32	1	+ .11	5.9
104	4.13	+ .05	.4	- .37	.9	+ .68	1	+ .84	1	+ .36	1	- .34	1	+ .23	5.3
38	4.23	+ .03	.6	+ .57	1	- .29	.8	+ .49	.4	- .12	.8	+ .59	.7	+ .20	4.3
127	4.24	+ .45	.7	+ .17	.5	+1.06	1	+ .65	1	+ .06	1	+ .14	1	+ .44	5.2
122	4.35	- .67	.4	- .28	1	+1.29	.7	- .51	.4	+ .30	1	+ .63	1	+ .24	4.5
121	4.35	- .27	.7	- .22	.9	- .13	.9	+ .40	1	- .11	1	+ .05	1	- .03	5.5
67	4.36			+ .06	1	- .15	1	- .13	1	- .55	1	- .22	1	- .20	5
22	4.38	+ .01	1	+ .24	1	+ .11	1	- .53	.9	+ .90	1	- .15	1	+ .11	5.9
18	4.48	- .11	1	+ .14	1	- .30	1	- .52	.9	+ .55	1	- .18	1	- .06	5.9
26	4.66	+ .93	.7	+ .36	1	- .31	.9	- .13	.7	+ .24	1	- .53	.8	+ .09	5.1
20	4.69	- .11	1	- .12	1	+ .25	1	+ .06	.8	+ .32	1	+ .36	1	+ .13	5.8
117	4.69	+ .06	1	- .16	1	- .73	1	+ .22	1	- .83	1	- .52	1	- .33	6
45	4.75	- .09	1	- .17	1	- .09	1	+ .30	1	- .02	1	.00	1	- .01	6
51	4.80	- .01	1	+ .25	1	- .70	1	- .37	1	+ .04	1	- .21	1	- .17	6
79	4.80	+ .90	1	+ .07	1	- .70	.7	- .18	1	+ .10	1	- .13	1	+ .05	5.7
17	5.04	+1.92	.25	+ .85	.6	+1.29	.6			- .68	.8	+1.05	.4	+ .62	2.65
131	5.10	- .13	.7	+ .05	.9	+ .31	1	- .44	.8	+ .04	1	- .19	1	- .04	5.4
78	5.22	+ .51	.8	+ .04	1	+ .69	1	+ .78	.4	- .93	1	+ .45	.8	+ .17	5
130	5.47	- .21	.7	+ .22	.8	- .37	.8	+1.09	.9	-1.05	1	- .15	1	- .09	5.2
109	5.48	- .18	.7	+ .44	1	- .43	1	- .24	1	+ .06	1	- .11	1	- .07	5.7
129	5.52	- .56	.7	+ .48	1	+ .07	.8	- .16	1	- .15	1	+ .43	1	+ .05	5.5
13	5.57	- .08	1	+ .17	1	+ .32	1	- .46	1	+1.03	1	- .06	1	+ .15	6
138	5.61	+1.28	.4	- .07	.8	- .03	.7	- .61	.7	+ .16	.9	+ .31	1	+ .10	4.5
12	5.65	+ .66	1	+ .91	1	+ .28	1	- .75	1	- .32	1	+ .42	1	+ .20	6
32	5.66	+ .49	1	- .05	1	- .09	1	+ .26	1	+ .30	1	+ .16	1	+ .18	6
24	5.76	+ .42	.7	+ .06	1	- .45	1	- .16	1	+ .44	1	+ .21	1	+ .07	5.7
16	5.82	+ .60	1	- .51	1	- .24	1	+ .41	.8	+ .30	1	+ .13	1	+ .11	5.8
59	5.83	+1.13	.5	- .18	1	- .20	.7	- .03	.9	- .32	1	- .80	1	- .18	5.1

TABLE 6—(Continued)

RADIAL COMPONENTS OF OBSERVED DISPLACEMENTS FOR STARS OF ECLIPSE FIELD

No.	Distance from Sun's center	M15-M7		M16-M11		M17-M9		N18-N11		N19-N13		N20-N9		Mean	
		D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.	D_r	Wt.
139	5.93	-.755	.6	+.759	.8	+.714	.7	-.734	1	+.710	1	-.701	1	.00	5.1
120	5.96	-.04	.8	-.01	1	+.14	1	+.45	1	-1.10	1	-.15	.8	-.712	5.6
60	5.98	-.21	1	+.27	1	+.14	1	+.30	1	+.19	1	-.60	1	+.02	6
132	6.16	-.46	.5	-.02	.4	+.36	.7	+.17	1	+.11	1	+.25	.7	+.11	4.3
140	6.19	-.55	.6	+.26	1	+.36	1	-.21	1	-.07	1	-.15	1	-.03	5.6
44	6.28	+.34	1	+.19	1	+.37	1	+.06	1	+.23	1	+.21	1	+.23	6
10	6.30	+.42	1	+.10	1	+.15	1	-.50	1	+.24	1	+.34	1	+.12	6
82	6.44	-.21	.4	-.68	1	+.84	.5	-.03	1	-.58	1	-.40	.8	-.27	4.7
6	6.46	+.45	1	+.20	.9	+.83	1	-.56	.8	+.27	1	-.08	1	+.21	5.7
137	6.46	+1.34	.4	+.64	.5	+.53	.4	+.66	.6	-.35	1	-.14	.7	+.28	3.6
108	6.47	+.21	1	-.06	1	-.20	1	+.06	1	-1.13	1	-.36	1	-.25	6
125	6.51	+.06	.7	-.05	1	-.52	1	+.10	1	-.11	1	-.07	1	-.11	5.7
23	6.69	+.70	.7	+.63	1	+.46	.7	+1.12	.4	+.29	.7	+.25	.8	+.53	4.3
21	6.71	+.22	.4	+.09	.8	-.60	.9	-.44	1	+1.10	1	-.29	1	.00	5.1
5	6.79	+.56	1	-.15	1	+.31	1	-.39	.9	-.24	1	-.38	1	-.04	5.9
40	6.92	+.29	.5	-.10	1	-.67	1	+.05	.9	-.20	1	-.19	.8	-.18	5.2
124	6.95	+.16	.7	-.10	.9	-.51	.8	-.13	1	-1.05	1	-.18	1	-.32	5.4
4	7.00	+.27	.8	+.14	1	+.27	1	-.98	1	+.13	1	+.17	1	-.01	5.8
9	7.05	-.09	1	-.50	1	-.51	1	-.70	1	-.62	1	+.17	1	-.37	6
19	7.13	+.43	.5	+.04	1	-1.15	1	-.27	.8	+.23	1	+.23	1	-.20	4.3
146	7.24			-1.25	.4	+.77	.4	+.49	.4	-.02	.8	+.25	.4	+.04	2.4
141	7.31	-.16	.4	+.39	.4	+.45	.4	+.76	.7	+.06	.9	-.10	1	+.20	3.8
86	7.33	-.76	1	-1.25	1			+.29	.9	+1.02	1			-.19	3.9
134	7.41	-.27	.4	+.44	.5	+.55	.4	-.44	.6	-.70	.7	+.51	.7	-.02	3.3
15	7.41	+.75	1	+.39	1	+.07	1	+.15	.9	-.14	1	+.19	1	+.24	5.9
144	7.43	-1.06	.4	-1.13	.4	-.79	.4	-.03	.9	+.03	1	-.48	1	-.41	4.1
30	7.72	+.23	1	-.98	1			-.33	1	-.74	1			-.45	4
133	7.82	-.48	.25	+.62	.5	-.01	.4	+.08	.7	-.18	1	+.15	1	+.05	3.85
3	7.88	-.64	1	-.04	1	-.60	1	-.19	1	+.53	1	+.48	1	-.08	6
128	8.00			+1.07	.4	-1.19	.3	+.10	.4	+.13	1	-.37	.8	-.02	2.9
7	8.09	+.54	1	+.30	1	+.42	1	+.14	1	+.02	1	+.27	1	+.28	6
143	8.20	-.69	.4	+.24	.4	+.33	.6	+.50	.8	.00	.8	-.28	1	+.04	4
126	8.25			-.52	.4			-.01	.7	+2.20	.8			+.81	1.9
136	8.44	+.05	.4	+.40	.5	+.24	.4	+.26	.7	-.20	1	+.07	.7	+.09	3.7
145	8.44	-1.02	.4	-.72	.6	-.01	.4	-.45	1	-.29	1	-.78	1	-.54	4.4
135	8.69			-.58	.4	+.22	.4	+.29	.7	-.35	1	-.38	1	-.19	3.5
14	8.84	+.88	.8	+.43	1			+1.11	.7			-.09	.8	+.56	3.3
142	8.91	-1.51	.4	-1.01	.4	+.31	.4	+.93	.4	-.58	1	+.11	.5	-.33	3.1
1	8.95	+.37	.4	-1.57	.5	+.93	.6	+.91	.4	+.60	.8	+.98	.7	+.43	3.4
11	9.03	+.50	1	+.44	1			+.38	1	+1.24	1	+.21	1	+.55	5
8	9.35	-.26	.4	-.18	.9	-.72	.7	-.62	.7	-.89	1	-.12	1	-.47	4.7
147	9.85	-.54	.4	+.02	.4			-.06	1	+.93	.8			+.18	2.6
2	10.33	-2.17	.4	-.60	.5	-.59	.4	+.40	.4	-1.62	.8	+.53	.5	-.76	3.0

TABLE 7

GROUP MEANS OF RADIAL STAR DISPLACEMENTS

M15-M7			M16-M11			M17-M9			Mean for object. M		
Dist. f. center	Rad. displ.	Weight	Dist. f. center	Rad. displ.	Weight	Dist. f. center	Rad. displ.	Weight	Dist. f. center	Rad. displ.	Weight
0.62	+.77	1.25	0.65	+.26	1.45	0.68	+.15	1.45	0.65	+.37	4.15
1.12	+.43	3.5	1.12	+.35	3.8	1.12	+.64	3.65	1.12	+.47	10.95
1.65	+.49	5.5	1.66	+.48	7.1	1.64	+.05	5.0	1.65	+.36	17.6
2.17	+.36	17.5	2.17	+.20	21.6	2.17	+.20	17.6	2.17	+.25	56.7
3.07	+.21	18.6	3.08	-.01	19.2	3.07	+.05	17.7	3.07	+.08	55.5
3.98	+.06	15.5	4.03	+.09	18.3	4.04	+.08	17.7	4.02	+.07	51.5
5.31	+.24	15.05	5.33	+.13	17.9	5.32	-.05	17.0	5.32	+.10	49.95
6.54	+.20	14.0	6.57	.00	17.9	6.54	.00	17.4	6.55	+.06	49.3
8.25	-.15	11.45	8.30	-.16	14.2	8.26	-.01	8.8	8.28	-.12	34.45

TABLE 7—(Continued)

N18-N11			N19-N13			N20-N9			Mean for object, N		
Dist. f. center	Rad. displ.	Weight	Dist. f. center	Rad. displ.	Weight	Dist. f. center	Rad. displ.	Weight	Dist. f. center	Rad. displ.	Weight
0°64	+°50	.95	0°71	+°46	.95	0°57	+°73	1.15	0°64	+°57	3.05
1.11	+°09	3.05	1.13	+°41	4.2	1.14	+°22	3.5	1.13	+°26	10.75
1.65	+°65	4.6	1.66	+°58	6.85	1.63	+°13	5.8	1.65	+°45	17.25
2.18	+°39	15.3	2.17	+°32	22.0	2.16	+°07	18.8	2.17	+°26	56.1
3.07	+°25	17.6	3.07	+°24	19.1	3.07	+°10	18.6	3.07	+°20	55.3
4.03	+°14	16.3	4.05	+°18	18.8	4.05	+°04	17.7	4.04	+°09	52.8
5.36	+°06	16.3	5.34	+°07	18.7	5.34	+°03	18.2	5.35	+°03	53.2
6.56	+°10	17.8	6.53	+°16	18.5	6.57	+°07	18.0	6.55	+°11	54.3
8.28	+°13	16.6	8.31	+°04	19.6	8.29	+°01	15.7	8.29	+°06	51.9

MEAN OF ALL PLATES

Dist. f. center	Radial displ.	P. e.	Weight	Stars	Einstein -°009r	Observ. -Theory
0°65	+°46	±°12	7.2	5	+°71	-°25
1.13	+°36	±°07	21.7	6	+°40	-°04
1.65	+°40	±°06	34.85	9	+°27	+°13
2.18	+°25	±°03	112.8	25	+°20	+°05
3.08	+°14	±°03	110.8	20	+°12	+°02
4.03	+°08	±°03	104.3	19	+°08	.00
5.33	+°03	±°03	103.15	19	+°04	-°01
6.56	+°03	±°03	103.6	20	+°01	-°04
8.29	+°01	±°04	86.35	22	+°02	+°01

(All radial displacements are relative to the reference stars.)

If the residuals of the check field measures are treated in a similar way, we obtain the data of Table 8 in which no evidence of systematic displacements is found. The group means of the radial components of the check-field residuals amount only to a few hundredths of a second and have all the characteristics of accidental errors of observation: their distribution is irregular and four of the seven values are smaller, three larger than the probable error.

TABLE 8

GROUP MEANS OF RADIAL COMPONENTS OF CHECK FIELD
RESIDUALS
(Mean of four plates)

Distance from center	Wallal- Tahiti	Weight
1°35	+°01	36.45
2.71	+°05	39.0
3.99	+°06	39.7
5.26	+°07	42.05
6.30	+°03	39.0
7.33	+°03	33.6
8.73	.00	33.0

(Probable error of each group mean : ±°04)

The mean results of all plates of the five-foot cameras are illustrated graphically in Fig. 2. While the stars of both the eclipse and the check field were photographed on the same plate, measured quite intermingled, and reduced with the same reduction formula (except for linear terms), only those of the eclipse field exhibit radial deflections of considerable size. The latter can, therefore, not be due to any error introduced

by the instrument, the photographic process, or the measurement and reduction of the photographs, and we must conclude that at the eclipse the stars in the immediate neighborhood of the Sun actually appear displaced from their normal positions.

A comparison of the observed radial star displacements of the eclipse field with the prediction of Einstein's Generalized Theory of Relativity is given in Table 7 for the mean of all plates. It was for this purpose necessary to convert the absolute light deflections of the theory into relative ones by a scale correction of $-°009r$, where r is the star's angular distance from the Sun's center in degrees. The agreement between observation and theory is quite as good as can be expected from the accidental measuring errors. The only larger differences are found among the first three groups, which contain fewer stars and have less weight. More than half of the residuals: "Obs.-Theory" are smaller than the probable error of the corresponding group mean.

Assuming with Einstein that the light deflections are inversely proportional to the stars' angular distances from the Sun's center, we can determine for every plate the light deflection at the Sun's limb (β) and the additional scale correction (a) to be applied to our measures. The two unknowns a and β were obtained from the data of Table 6 by least squares solutions with the following equations of condition:

$$-a + \beta \frac{R}{r} = D_r; R = \text{radius of Sun's disk} = 0°2661.$$

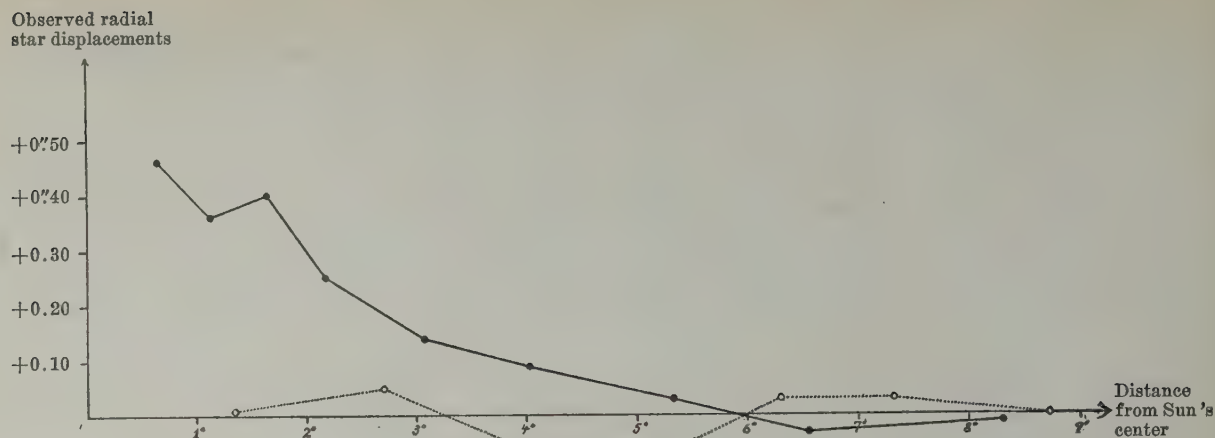


Fig. 2. Radial components of observed star displacements plotted as functions of the star's angular distance from the Sun's center.

Full circles joined by full line: Group means of eclipse field stars.
Open circles joined by dotted line: Group means of check field stars.

TABLE 9
LIGHT DEFLECTION AT SUN'S LIMB

Plates	Defl. at limb β	Weight	Scale corr. α
M15-M7.....	+2".38	0.85	+".003
M16-M11.....	+1.66	0.96	+".012
M17-M9.....	+1.36	0.83	+".011
Mean of obj. M.....	+1".80		
N18-N11.....	+2.00	0.73	+".005
N19-N13.....	+2.54	0.89	+".017
N20-N9.....	+1.01	0.88	+".008
Mean of obj. N.....	+1".85		
Mean of all plates.....	+1".82$\pm$".15		+".010

	Defl. at Sun's limb	
15-foot cameras.....	1".72 $\pm$ ".11	(weight 2)
5-foot cameras.....	1.82 $\pm$.15	(weight 1)
Mean.....	1".75 $\pm$.09	(weight 3)
Einstein's Theory.....	1.75	

The mean taken with weights adopted according to the probable errors coincides exactly with the prediction of Einstein's Generalized Theory of Relativity, and the probable error of the final result of our observations is only 5 per cent.

It should not be overlooked that the two series of observations which thus lead to quite concordant results are not repetitions of the same method, but differ in a great many respects, such as:

1. The use of entirely different types of objectives.
2. Different focal length, scale of photographs, size of field.
3. Seed 23 plates used for 5-foot cameras have considerably blackened sky background; Seed 30 plates used with 15-foot cameras are much less fogged.
4. Different methods were employed in measuring the plates.
5. The method of reduction differs by including third order terms in the reduction formula of the 5-foot camera plates which were neglected in case of the larger instrument.

6. The results of the 5-foot cameras are mainly based on numerous small displacements of stars between 1° and 3° from the Sun's center compared with reference stars between 5° and 10° distant, those of the 15-foot cameras on a few large displacements of stars between 0.5 and 1.5 compared with reference stars between 2° and 3.5.

The constant β is extrapolated a long way from relatively small observed star displacements and has merely the significance of the numerical coefficient in an interpolation formula. For this reason the individual values of β have small weight and show a considerable range. If their differences from the mean, 1".82, are used for computing the probable error of unit weight we obtain $\pm$ ".37, which is very little in excess of the value $\pm$ ".33 derived on page 138 from the mean displacements of each star. This shows that the range of the individual plate results for β is not much larger than would be expected from the unavoidable errors of observation.

The mean result 1".82 agrees quite closely with the value 1".75 predicted by Einstein's Theory.

8. COMBINED RESULTS OF THE TWO INSTRUMENTS

The results of the 5-foot cameras confirm in every way those of the 15-foot cameras and this justifies combining the data of the two instruments. For the light deflection at the Sun's limb we have:

The most important contribution of the observations with the 5-foot cameras is that of testing the law according to which the light deflections depend on the star's angular distance from the Sun's center; but it is for this purpose advantageous to combine them with those of the 15-foot cameras which are more accurate for stars near the Sun. For the differential measure (radial component) of a pair of good star images the two instruments furnished the following probable errors:

15-foot cameras $\pm .20$ ($\pm .0044$ mm)

5-foot cameras $\pm .34$ ($\pm .0025$ mm)

which show that the accuracy of the measures is not proportional to the focal length of the telescope; the images of the shorter cameras being more sharply defined were measured with higher linear accuracy. According to these figures we have to divide the weights of the 5-foot camera measures by 3 to make them comparable with the weight system of the 15-foot cameras.

The star displacements observed with the 15-foot cameras are relative to a set of reference stars situated between 2° and $3^\circ.5$ from the Sun's center. Most of these (26) were also observed with the 5-foot cameras which give for them an average radial displacement of $+0''.15$ while their average distance from the Sun's center is $2^\circ.52$. A scale correction of $+''060$ per 1° is, therefore, needed to reduce the results of the 15-foot cameras to the system of the 5-foot cameras, i. e., to express them as relative displacements referred to the reference stars of the latter instrument situated between 5° and 10° from the Sun's center.

The directly observed values⁷ D_1 were adopted for the radial displacements of the 15-foot cameras which had always been considered by the observers as the final results of this instrument. The values D_2 corrected for the residuals of the check field stars were merely given for an estimate of the possible influence of certain systematic errors. The conception that the check-field residuals of the 15-foot cameras represent systematic errors which in a similar way affected the measures of the eclipse field seemed, *a priori*, open to doubt, but the observations of the second instrument now available speak decidedly against it. If the systematic arrangement of the check-field residuals of the 15-foot cameras (see *l. c.* fig. 3) were in any way related to the blackening of the plate by the corona, as had been suspected, we should find a similar effect in the check-field observations of the 5-foot cameras. This is, however, not the case; between $0^\circ.5$ and 2° from the Sun's center where the larger instrument had furnished negative residuals, we now find a small positive one. It should further be noted that the directly observed

displacements D_1 of the 15-foot cameras are in better agreement with the results of the other instrument than the corrected ones D_2 .

TABLE 10
RADIAL COMPONENTS OF OBSERVED STAR DISPLACEMENTS
(Mean results of the two instruments)

Number 15-foot camera	Number 5-foot camera	Distance from Sun's center	Observed radial displ.	Weight	Einstein Theory -''009 ^r
63	74	0.55	+0.70	5.27	.84
48		0.55	+ .50	.22	.84
73	81	0.62	- .05	.86	.74
55		0.65	+ .76	.44	.71
41	62	0.66	+ .74	1.49	.70
66		0.71	+ .59	.25	.65
60	73	0.75	+ .52	2.33	.61
76		0.84	+1.63	.22	.55
47	65	0.92	+ .65	1.62	.50
56		0.93	+ .74	.55	.49
78	83	0.96	+ .46	5.58	.48
57		0.99	- .47	.66	.46
79		1.03	+ .55	1.39	.44
77		1.08	+ .28	.44	.42
35	56	1.11	+ .61	1.49	.41
52		1.11	+ .44	1.2	.41
81	85	1.13	+ .41	5.9	.40
59	71	1.15	+ .43	5.42	.40
29		1.21	+ .06	1.27	.37
27	52	1.27	+ .21	3.36	.36
88		1.30	+ .23	1.71	.35
84	88	1.32	+ .45	3.68	.34
90		1.37	+ .32	.73	.33
85		1.40	+ .33	1.2	.32
70		1.43	+ .48	1.94	.31
37	61	1.47	+ .52	3.6	.30
91	92	1.49	+ .55	3.56	.30
82		1.51	+ .14	1.86	.29
36	58	1.54	+ .33	4.73	.29
30		1.55	+ .52	1.76	.29
51	68	1.63	+ .23	5.23	.27
25	50	1.66	+ .25	5.9	.27
26		1.67	+ .28	.72	.26
65		1.67	+ .61	2.13	.26
93		1.67	+ .10	1.68	.26
86	89	1.73	+ .17	5.7	.25
64	75	1.73	+ .31	4.65	.25
101	99	1.80	+ .34	3.21	.24
19		1.88	+ .34	1.06	.23
46	64	1.90	+ .16	3.35	.23
102	100	1.91	+ .24	5.75	.23
103	101	1.91	+ .40	3.78	.23
72	80	1.93	+ .24	3.58	.22
80		1.93	+ .42	1.86	.22
22		1.94	+ .55	1.39	.22
92	93	1.95	+ .43	4.25	.22
104	102	1.95	+ .31	4.46	.22
18	48	1.97	+ .18	3.47	.22
98	96	1.97	+ .38	4.53	.22
99	97	1.98	+ .20	5.13	.22

⁷ *L.O.Bull.*, 11, 49; Table II.

TABLE 10—(Continued)

RADIAL COMPONENTS OF OBSERVED STAR DISPLACEMENTS

Number 15-foot camera	Number 5-foot camera	Distance from Sun's center	Observed radial displ.	Weight	Einstein Theory —".009r
14		2.03	+0.38	1.79	.21
58	69	2.03	+ .21	5.75	.21
28	53	2.04	+ .18	5.9	.21
95	94	2.06	— .06	3.96	.21
68	76	2.06	+ .21	5.38	.21
105	103	2.06	+ .26	4.86	.21
17	47	2.08	+ .38	2.61	.21
33	55	2.12	+ .02	3.59	.20
31	54	2.13	+ .13	2.34	.20
12	42	2.20	+ .53	4.41	.19
23		2.21	+ .12	1.06	.19
106		2.25	— .10	.47	.19
20	49	2.28	+ .31	4.56	.18
11	41	2.29	+ .06	2.69	.18
94		2.38	+ .52	.77	.17
8	36	2.39	+ .21	3.86	.17
32		2.44	+ .37	.44	.17
100	98	2.48	+ .28	4.28	.17
112		2.48	— .61	.76	.17
117	111	2.53	+ .41	1.53	.16
118		2.54	— .94	.25	.16
75		2.57	+ .29	1.05	.16
43	63	2.58	+ .22	4.55	.16
110	110	2.58	— .28	5.75	.16
7		2.58	+ .44	1.27	.16
87		2.61	+ .44	.87	.15
5	35	2.63	+ .16	5.9	.15
	72	2.63	+ .20	2	.15
109		2.68	+ .35	1.16	.15
97		2.72	— .04	1.35	.15
6		2.78	+ .22	.87	.14
89	91	2.80	— .03	5.18	.14
	29	2.84	+ .16	2	.14
113		2.85	— .34	.66	.14
96	95	2.86	+ .15	5.9	.14
107	105	2.89	+ .14	2.57	.14
16	46	2.92	+ .24	4.21	.13
21		2.93	— .05	1.12	.13
	116	2.93	+ .44	1.37	.13
	70	2.96	+ .10	1.7	.13

TABLE 10—(Continued)

RADIAL COMPONENTS OF OBSERVED STAR DISPLACEMENTS

Number 15-foot camera	Number 5-foot camera	Distance from Sun's center	Observed radial displ.	Weight	Einstein Theory —".009r
4		2.98	0.00	.66	.13
	84	3.05	+ .36	2	.13
9	37	3.07	+ .12	3.37	.12
	113	3.10	+ .04	2	.12
	114	3.10	+ .11	2	.12
108	106	3.10	+ .33	2.47	.12
10	39	3.14	+ .37	4.46	.12
2	31	3.18	— .02	2.46	.12
	57	3.18	+ .09	2	.12
	119	3.37	.00	2	.11
	118	3.39	+ .22	2	.11
	115	3.41	— .08	1.9	.11
3	34	3.45	+ .19	3.5	.10

(The radial displacements are relative to the reference stars of the 5-foot camera plates.)

The combined results of the two instruments are given in Table 10 for all stars situated within 3.5 of the Sun's center. The first column contains the number assigned to each star in Table II of the former publication, the second column that of our present Table 4. The angular distance of the star from the center of the eclipsed Sun in the third column is expressed in degrees. The observed radial star displacements of the fourth columns are the weighted means of the results of the two instruments those of the 15'-cameras having received a scale correction of +".060r. The weights of the fifth column are based on the weight system of the 15-foot cameras in which unit weight corresponds to a probable error of $\pm .20$. The continuation of Table 10 for greater distances from the Sun, where only observations with the 5-foot cameras are available, is identical with the later parts of Table 6 (last two columns), except that the weights there given have to be divided by 3. The group means of Table 11 are derived from the data of Tables 10 and 6.

TABLE 11

GROUP MEANS OF OBSERVED RADIAL STAR DISPLACEMENTS

(Mean of the two instruments)

Distance from Sun's center	Obs. radial star displacements	P. e.	Weight	Stars	Einstein Theory —".009r	Obs. —Th.	Courvoisier Yearly R. —".0735r	Obs. —Y. R.
0.675	+".63	$\pm .07$	8.92	10	+".68	—".05	+".46	+".17
1.22	+ .41	.03	43.1	17	+ .37	+ .04	+ .40	+ .01
1.71	+ .27	.03	47.7	14	+ .26	+ .01	+ .36	— .09
1.97	+ .29	.03	45.9	12	+ .22	+ .07	+ .33	— .04
2.17	+ .22	.03	41.0	14	+ .20	+ .02	+ .31	— .09
2.67	+ .10	.03	45.3	18	+ .15	— .05	+ .27	— .17
3.14	+ .17	.03	41.8	18	+ .12	+ .05	+ .23	— .06
4.03	+ .08	.03	34.8	19	+ .08	.00	+ .15	— .07
5.33	+ .03	.03	34.4	19	+ .04	— .01	+ .04	— .01
6.56	— .03	.03	34.5	20	+ .01	— .04	— .07	+ .04
8.29	— .01	.04	28.8	22	— .02	+ .01	— .21	+ .20

9. COMPARISON WITH EINSTEIN'S THEORY

The results of our observations given in Tables 10 and 11 are, like those of Tables 6 and 7, relative star displacements referred to the reference stars of the 5-foot camera plates between 5° and 10° from the Sun, i. e., the unknown scale value was so chosen that the observed radial displacements are zero for the reference stars on the average. This presentation of the results has the advantage of stating the quantities actually observed free from any assumptions. If a comparison is desired between the observations and any theory, it is necessary first to determine a scale correction by a

represents the plotted group means as well as the unavoidable errors of our observations permit. The residuals between observation and theory are in distribution and size (6 being smaller, 5 larger than the probable errors) such as would be expected from the theory of errors. The four successive residuals between 1° and 2.5° can hardly claim any real significance, as even their mean amounts only to $+0.035$, and any arbitrary smooth interpolation curve drawn close to the plotted group means will not differ from the theoretical curve by more than $0.03-0.04$.

The results of the individual stars differ widely in accuracy, as is shown by the weights of Table 10. The best observed stars, those which had well exposed images and were measured on all of the 10 plates, seem

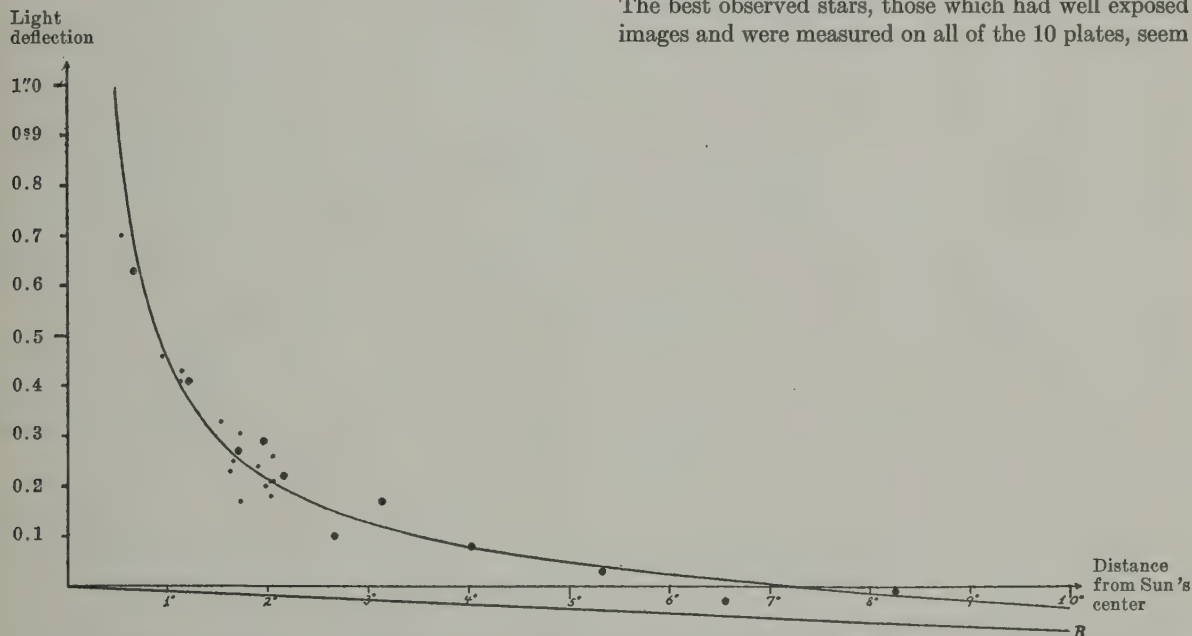


Fig. 3. Radial components of observed light deflections (mean of the two instruments) compared with Einstein's theory.

Large dots: Observed group means.

Small dots: Individual results of 15 best measured stars.

The light deflections, plotted as functions of the star's angular distance from the Sun's center, are relative to the reference stars of the 5-foot camera plates (between 5° and 10° from the Sun's center). The curve gives the light deflections predicted by Einstein's theory. The straight line B indicates the scale correction deduced from the theoretical values; if the ordinates are measured from B as base line, they represent absolute light deflections.

least squares solution which will bring the observed values to best agreement with those of the theory.

For Einstein's Generalized Theory of Relativity the scale correction was found to be $+0.009r$ and this was deducted from the data of Einstein's formula:

$$1.75 \frac{R}{r} \quad R = \text{radius of Sun's disk} = 0.2661 \\ r = \text{star's angular distance from Sun's center}$$

to obtain the relative displacements of the theory in the 6th column of Table 10 and the 6th column of Table 11. The comparison between our observations and Einstein's theory is found in Table 11 and is illustrated graphically in Fig. 3. Einstein's curve

to be best suited to illustrate how closely the observed star displacements agree in amount and direction with the prediction of Einstein's theory. All stars of weight 4.6 or more within 2.5° from the Sun's center are listed in Table 12. For these 15 stars which include all those brighter than photographic magnitude 9.0 as well as two fainter ones the mean observed displacements in both coordinates (cols. 3 and 4) were computed, combining the results of the two instruments in the same manner as was described for the radial components of Table 10. The data of Table 12 are, therefore, relative star displacements as compared with the reference stars of the 5-foot camera plates (5° to 10° from the Sun's center), and a scale correction of $-0.009r$

TABLE 12
OBSERVED DISPLACEMENTS OF THE 15 BEST MEASURED STARS
(Mean of the two instruments)

Number 15-foot cameras	Number 5-foot cameras	Observed displacement		Einstein Theory —scale corr.		Observation —Theory	
		D_x	D_y				
63	74	+".25	+".65	+".30	+".78	—".05	—".13
78	83	+".43	+".16	+".45	+".14	—".02	+".02
81	85	+".41	—".01	+".40	—".02	+".01	+".01
59	71	+".18	—".41	+".03	—".39	+".15	—".02
36	58	—".14	+".32	—".18	+".22	+".04	+".10
51	68	—".03	—".23	—".02	—".27	—".01	+".04
25	50	—".25	—".06	—".22	—".14	—".03	+".08
86	89	+".19	+".06	+".18	+".18	+".01	—".12
64	75	+".12	+".30	+".03	+".25	+".09	+".05
102	100	+".25	—".01	+".22	—".06	+".03	+".05
99	97	+".23	.00	+".19	—".11	+".04	+".11
58	69	+".22	—".20	+".01	—".21	+".21	+".01
28	53	—".15	+".11	—".12	+".17	—".03	—".06
68	76	+".11	—".19	+".04	—".20	+".07	+".01
105	103	+".25	—".06	+".20	—".05	+".05	—".01

degree was deducted from the absolute deflections of Einstein's theory to convert them into relative ones (cols. 5 and 6). The agreement between observation and theory is surprisingly close.

In Fig. 4 the observed relative star displacements are plotted as arrows. The larger displacements fall almost exactly in the direction of the radius vector. The radial components of the deflections for these fifteen stars are entered in Fig. 3 as small dots.

The combined observations of the two instruments thus confirm that the light deflections are directed radially away from the Sun's center, while their amount is inversely proportional to the angular distance from the Sun's center, and they give for the numerical coefficient (deflection at the Sun's limb) the exact value 1".75 predicted by Einstein's Theory.

In one point perhaps the results of the 1922 eclipse need to be supplemented by future observations. It is undoubtedly desirable to establish by observation the light deflections of stars between 0°.3 and 0°.6 from the Sun's center and to increase the accuracy of the observed values between 0°.6 and 1°. This can only be done at a few exceptionally favorable eclipses, when stars brighter than the sixth magnitude are situated in close proximity to the Sun's edge.

10. COMPARISON WITH COURVOISIER'S YEARLY REFRACTION

After the publication of the results of the 15-foot cameras the question was brought up⁸ and discussed whether the observed light deflections might not be identified with Courvoisier's "yearly refraction."⁹ This effect which was essentially derived from daylight observations with the meridian circle also consists in

⁸ J. Hopmann, *Phys. Zeitschr.* **24**, 476, 1924; A. Kopff, *Scientia*, June 1924.

⁹ *Beob. Ergeb. d. Kgl. Sternw. zu Berlin*, No. 15, 1913.

an apparent displacement of the stars away from the Sun; but the amount of the displacement decreases very gradually from about ½" near the Sun to 0".09 at 90° from the Sun, and Courvoisier interpolated his observations by the formula¹⁰

$$0".55 \left(1 - \sqrt{\sin \frac{r}{2}} \right)$$

The Courvoisier effect so far has no theoretical explanation, and Kienle¹¹ in a critical discussion showed that the observational data on which it rests are not very convincing.

Since the Einstein-effect and the Courvoisier-effect differ widely in the law according to which the star displacements diminish with increasing angular distance from the Sun, it is only necessary to extend the Sun in order to decide between the two interpretations. It was mainly for this purpose that the 5-foot cameras were included in the equipment of the 1922 eclipse expedition. A comparison of the combined results of the two instruments with the Courvoisier formula is given in Table 11 (last two columns) and in Fig 5. A scale correction of ".073 per 1°, determined by a least squares solution to reduce the group residuals to a minimum, was deducted from the values of the Courvoisier formula. The representation of our observations by the Courvoisier formula is quite unsatisfactory. Two of the group means differ from the curve by inadmissibly large amounts (5–6 times the probable error), and all seven group means between 1°.5 and 6° have large negative residuals. When Figs. 3 and 5 are compared, there can be no question that our observations confirm Einstein's prediction and contradict Courvoisier's "yearly refraction," and the conclusion imposes itself that the latter cannot have, in the imme-

¹⁰ *A.N.*, **209**, 356, 1919.

¹¹ *Phys. Zeitschr.*, **25**, 1, 1924.

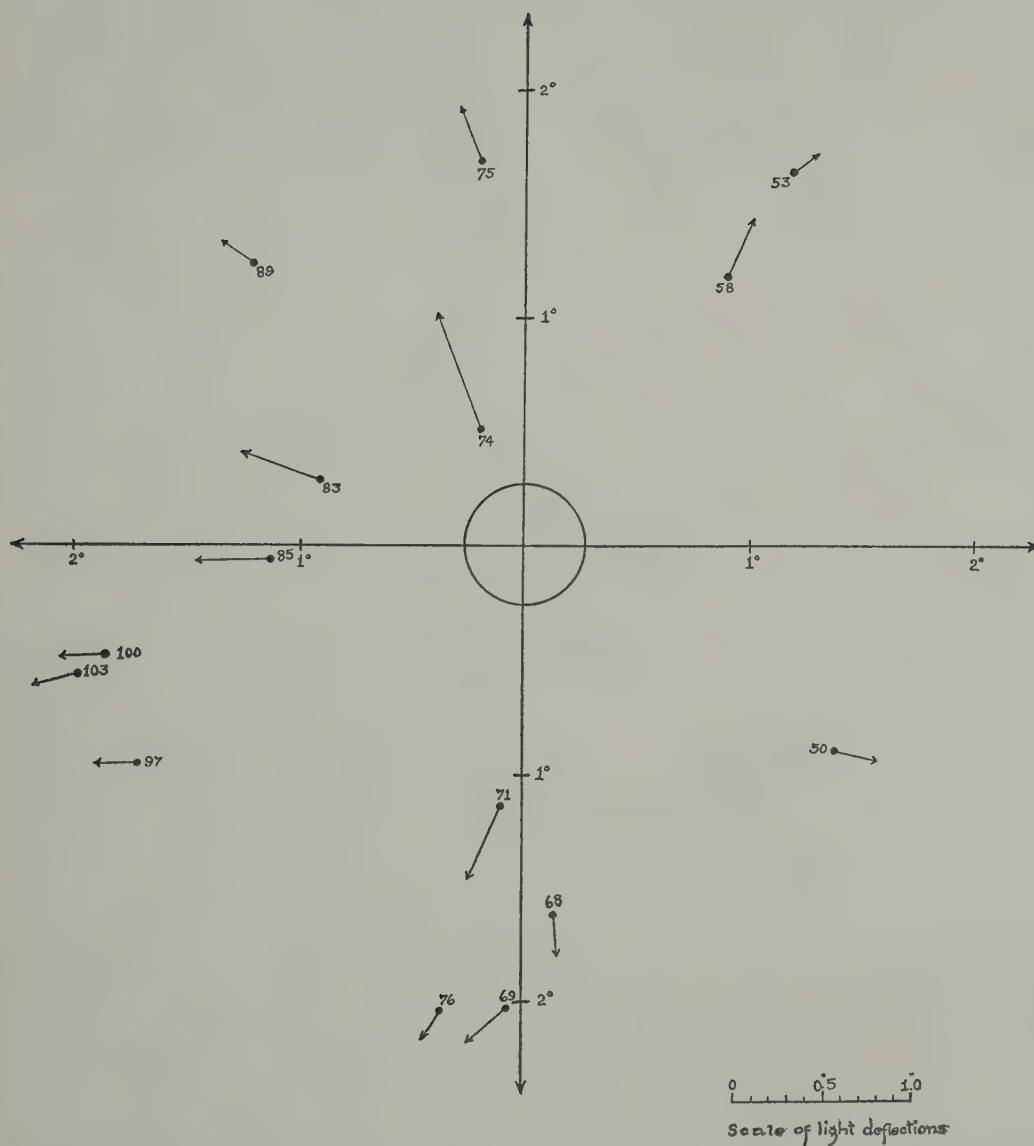


Fig. 4. Observed light deflections (mean of the two instruments) of the 15 best measured stars within $2^{\circ}5'$ from the Sun's center.

The arrows represent in size and direction the observed light deflections relative to the reference stars (5° to 10° from the Sun's center). The numbers of the stars are those of Table 4.

diate surroundings of the Sun, the amounts stipulated by Courvoisier's formula. The observations of *Venus* near upper conjunction, on which this part of Courvoisier's curve is based, must be affected by systematic errors peculiar to meridian circle observations. If, however, the existence of the Courvoisier effect is rejected in the neighborhood of the Sun, it loses its character as a "cosmical refraction," and his results derived from star observations at greater distances from the Sun are then more probably related to the refraction anomalies investigated by Varnum¹².

Furthermore exposures during the first part of totality should give different results from those taken at the middle or toward the end of totality, and finally the light deflections would not be the same at different eclipses when the Sun is at different zenith-distances. The eclipse observations, on the contrary indicate none of these variations, except such small differences as might be expected from the errors of observation, and speak against interpretation as refraction effects. The fact that the two first plates of the 5-foot cameras give a large, the two last ones a small value for the light

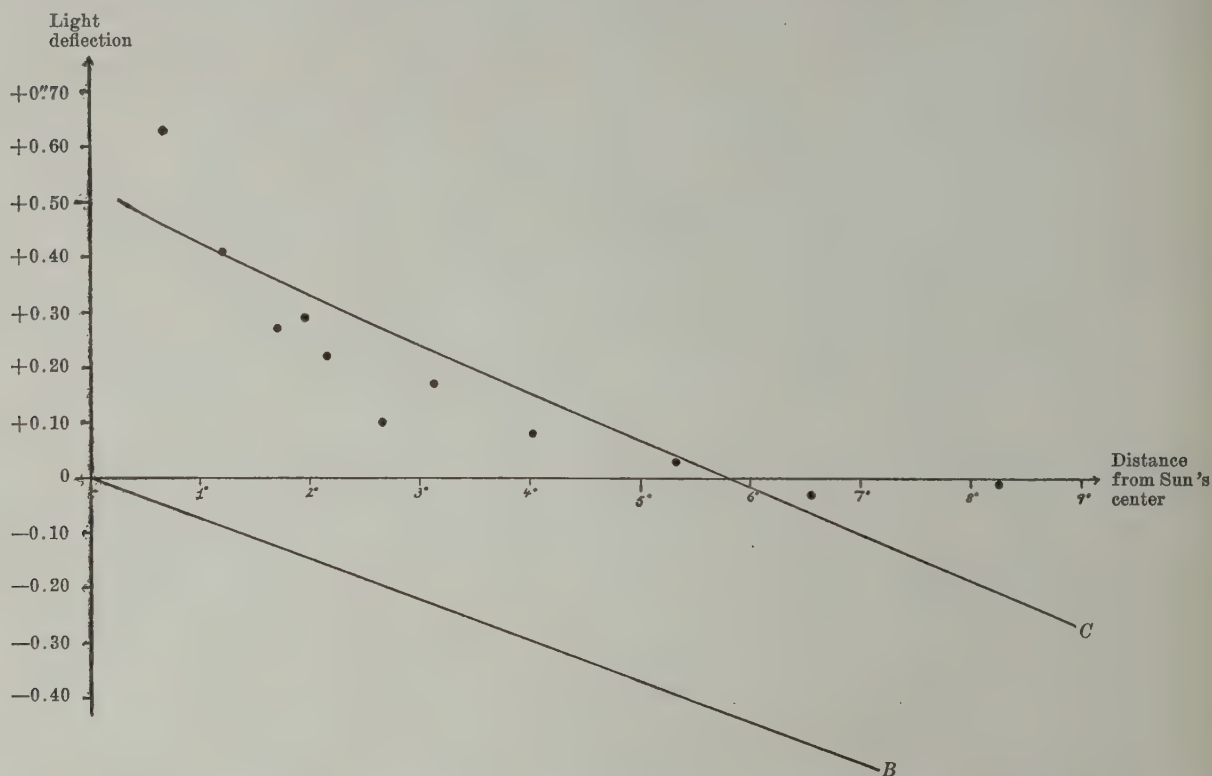


Fig. 5. Radial components of observed light deflections (mean of the two instruments) compared with Courvoisier's yearly refraction.

The full circles are the observed relative light deflections plotted as functions of the star's angular distance from the Sun's center. The curve *C* gives the star displacements of the Courvoisier formula after the scale correction indicated by the straight line *B* had been deducted from them. If the ordinates are measured from *B* as base line, they represent absolute light deflections.

11. INFLUENCE OF TERRESTRIAL REFRACTION

The suggestion has been repeatedly made that the light deflections observed at total solar eclipses might be due to anomalous refraction in the Earth atmosphere. If this were the case, we should *a priori* expect them to depend on the relative position of the line of sight to the zenith and to the axis of the Moon's shadow. Accordingly they should vary with the position angle of the star referred to the Sun's center, being different in the direction of the zenith than perpendicular to it.

¹² *A.J.*, 34, 61, 1922.

deflection at the Sun's limb must be accidental, since the more accurate results of the 15-foot cameras show close agreement between the first and second exposure.

Refraction anomalies produced by the cooling effect of the Moon's shadow are accessible to theoretical investigation and were discussed in 1919-1920 by British astronomers and physicists¹³ (A. Anderson, Eddington, Crommelin, Cave, Dines, Richardson, Schuster) with the conclusion that they could not have an appreciable influence at the 1919 eclipse. Nevertheless it seemed desirable to make a careful calculation

¹³ *Nature*, 104, 354, 372, 393, 413, 436, 468, 563.

of the refraction in the shadow cone for the peculiar conditions of the 1922 eclipse at Wallal.

According to the records¹⁴ the temperature minimum took place about five minutes later than the end of totality and the total temperature drop amounted to 4°4C, with a subsequent rise of 4°0 up to the fourth contact; the temperature gradient was fairly uniform, about -0°075C and +0°069C per minute immediately before and after totality; the barometer showed no variation. The thermometer readings in the observing house indicate a drop of 1°5F (0°83C) for 7 minutes including the period of totality, the corresponding gradient of 0°12C per minute, however, has an uncertainty of about 30%. If we neglect the time lag in the temperature minimum, and make the latter coincide

760 mm of mercury) with altitude (col. 2) is taken from the *Standard Atmosphere Chart* of the Bureau of Standards. The temperature drop with altitude (col. 3) was supposed to have been 10°C per km (adiabatic) up to 3 km and 8° above this level before the eclipse; by the cooling effect of the shadow it was reduced to 8° for the lower levels at totality. The air density δ (col. 4) and the refracting index μ (col. 5) were calculated by the formulae:

$$\delta = p \frac{273}{T}, \quad (T = \text{absolute temperature})$$

$$\mu = 1 + .000292 \delta$$

The horizontal temperature gradient per 50 km was assumed to diminish proportionally to the altitude from

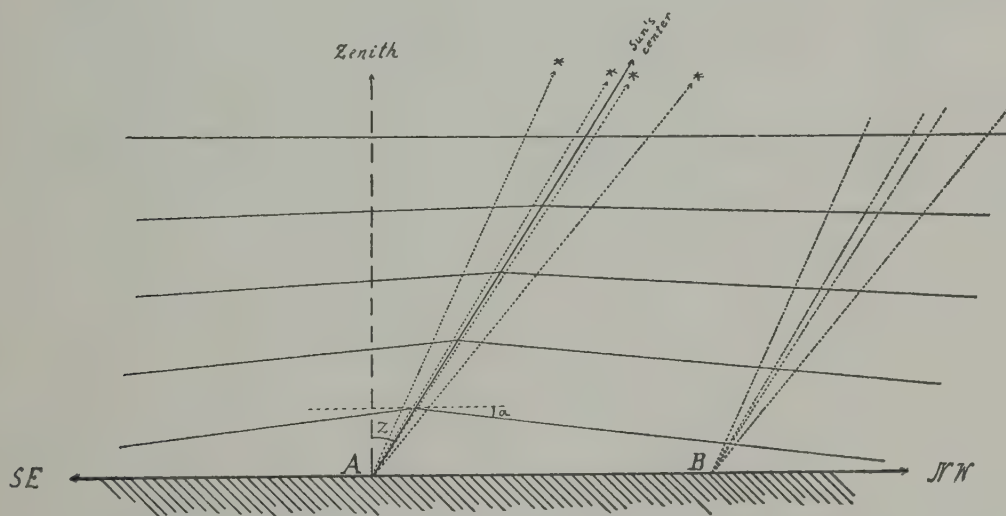


Fig. 6. Conical surfaces of equal air density in the Moon's shadow cut by a plane passing through the zenith and the Sun.

The inclination of the layers is much exaggerated. In the scale of the figure the shadow of totality would extend about 4 feet to the right and the left of A.

with the center of the shadow, we may assume that in the region of the Earth's surface covered at a given moment by the Moon's shadow the temperature rose from +25°9C at the center evenly in all directions at the rate of about 0°1C per 50 km (the shadow moved at a speed of 47 km per minute). Such a temperature distribution must have caused the layers of equal air density and refracting index to be tilted, forming conical surfaces with the apices in the axis of the shadow. Fig. 6 shows a cut through these layers made by a plane passing through the zenith and the Sun. The assumptions about the temperature and pressure distribution in the atmosphere above the Earth's surface necessary to calculate the inclination of the layers are given in Table 13. The decrease in air pressure (unit

0°1C at the surface (as observed) so as to vanish at 5 km. If we keep in mind that even the temperature variation between day and night decreases in the free air rapidly with the altitude and is small at 1 km, there can be no question that the small and short variation of the eclipse does not reach the higher atmospheric levels, probably not above 1 km. We have, however, made all assumptions so that within the possible limits they produce the largest refraction effect.

If we designate by α the varying inclination of the atmospheric layers, the refraction integral becomes, since α is a small angle:

$$R = \int \tan(i \pm \alpha) \frac{d\mu}{\mu} = \int \tan i \frac{d\mu}{\mu} \pm \int \alpha \sec^2 i \frac{d\mu}{\mu}$$

where i is the angle between the light ray and the

¹⁴ A. D. Ross, *Report of the Australasian Assoc. Adv. of Science*, 16, 196, 1923.

TABLE 13

Altitude in km.	Air pressure	Temp. C	Air dens. μ	Refr. ind. μ	Horiz. temp. grad.	Horiz. change of μ	Incl. of layers
0.0	1.011	+25.9	.924	1.000271	-°10C	-.090×10 ⁻⁸	12".4
0.5	.952	+21.9	.881	257	.09	-.078	12.4
1.0	.897	+17.9	.841	245	.08	-.068	12.2
1.5	.843	+13.4	.803	234	.07	-.058	11.3
2.0	.793	+ 9.0	.767	224	.06	-.048	9.9
3.0	.700	0.0	.700	204	.04	-.030	6.4
4.0	.615	- 8.0	.634	185	.02	-.014	3.1
5.0	.539	-16.0	.572	167	.00	.000	0.0

vertical. The first integral represents the normal refraction already applied to the observations; the second gives the abnormal refraction effect produced by the Moon's shadow. On account of the small factor a we may set $\mu=1$ and $i=z$ (the zenith distance of the star), and have

$$dR = \pm \sec^2 z f a d\mu$$

the integral to be extended from the Earth's surface to an altitude of 5 km. The + sign applies to stars of smaller, the - sign to stars of larger zenith distance than the Sun. With this formula and the data of Table 13 the following abnormal refraction effects dR were computed for two reference stars at 7°5 and two stars at 1° from the Sun's center, all situated at the same azimuth as the Sun.

Zenith distance	Distance from Sun	dR
24°5	7°5	+".001
31.0	1.0	+.001
33.0	1.0	-.001
39.5	7.5	-.001

An apparent displacement of the stars away from the Sun takes place, but of such small amount that it does not affect our results, and no reasonable assumption about the pressure and temperature distribution of the atmosphere within the Moon's shadow will increase this amount materially.

Even this negligible disturbance is not produced unless the apices of the conical surfaces are exactly in line between the observer and the Sun as for point A in Fig. 6. If this ever happens, it can only be for a brief moment; at any other time the conditions are similar to those for an observer at point B, when all stars of a photograph are affected alike, so that their relative positions are not changed.

A study of the systematic errors of meridian circle observations, on the other hand, led W. B. Varnum¹⁵ to the conclusion that considerable refraction anomalies (shifts in the refraction zenith), must be caused by the Earth's atmosphere and that part of these have a diurnal variation, depending on the hourly change of air density. He suggests then¹⁶ that during a total

solar eclipse similar refraction anomalies occur as during the change of day and night. With numerical coefficients derived from observations at Albany and the meteorological data of the eclipse, he figures out that the abnormal refraction of a star near the eclipsed Sun may have amounted to 1".82 and should have been directed radially away from the Sun. We find it impossible to agree with Mr. Varnum in this calculation (he seems to use too large values for ρ and F''). Let us take up for example the case of a star immediately above or below the eclipsed Sun. Only the shift in zenith distance then enters into consideration, for which Varnum gives the formula:

$$dR = f' \sec^2 z \cdot \mu \cdot \rho \quad \begin{array}{l} \mu = \text{air density} \\ \frac{1}{100} \rho = \text{hourly change of } \mu \end{array}$$

For the eclipse we have:

$$\begin{aligned} z &= 32^\circ 0 \\ \mu &= 0.965 \\ \rho &= 1.4 \end{aligned}$$

ρ was calculated with the actual temperature change during the half hours preceding and following totality. Adopting for the empirical coefficient f' the average, regardless of sign, of the 22 stretches of Albany observations:

$$f' = 0".117,$$

we obtain:

$$dR = 0".22$$

and this is the "prismatic refraction" which according to Varnum's suggestion would apparently displace the stars immediately above or below the eclipsed Sun. It is important to note that this effect would not be inversely proportional to the angular distance from the Sun's center and would apply to our reference stars in nearly the same amount as to the stars close to the Sun; it could thus in no way explain the star displacements observed at the eclipse, although it would affect the numerical data to some extent.

Mr. Varnum's assumptions are, however, open to serious objections: In the first place, the origin of these refraction anomalies is not understood sufficiently to justify the suggestion that at an eclipse similar effects take place as are occasioned by the diurnal meteorolog-

¹⁵ *A. J.*, 34, 61, 1922.

¹⁶ *Publ. Amer. Astr. Soc.*, 5, 96, 1923; *Pop. Astr.*, 32, 35.

ical changes. While there may be some evidence that the discrepancies of meridian circle observations are caused by the atmosphere, they can hardly be attributed to a general inclination of the layers of equal density (prismatic effect), as such inclinations would have to be much larger than is compatible with the actually observed temperature and pressure gradients. Still less does it seem justified to assume that, at an eclipse, refraction anomalies of this kind would operate in all directions away from the Sun, with a sharp discontinuity at the Sun, so that they would even affect differential measures of stars separated by less than 2° . It must be remembered that at a height of one mile above the ground the light rays which reached the telescope from two stars situated on opposite sides of the Sun, one degree from its center, were only 160 feet apart as compared with a diameter of 150 miles for the shadow. It seems out of question that in this small interval there could be such a sharp discontinuity in atmospheric conditions as to deflect the two light bundles in opposite ways. In order to influence all our observations alike, this discontinuity would have to be accurately centered on the Sun and remain so during totality, following the apparent daily motion of the Sun ($1\frac{1}{4}^\circ$) and taking no part in the passing of the shadow. This sounds quite absurd. If there was any circular prismatic effect, it was most likely centered around the temperature minimum which followed the shadow center by at least 200 miles. The light rays of all stars on our photographs would then have passed on the same side of the center and suffered approximately the same abnormal refraction, so that their relative positions would not have been changed.

Our investigation shows that the probable arrangement of the atmospheric layers during the total solar eclipse of Sept. 21, 1922, as based on the meteorological records, has no appreciable influence on our results. Even the insufficiently founded hypothesis that, at an eclipse, refraction effects take place similar to the diurnal refraction anomalies affecting the absolute star positions measured with the meridian circle, could in no way explain the observed relative star displacements, and would probably not have influenced them at all.

12. SUMMARY

1. Six photographs of the star field surrounding the eclipsed Sun were obtained with the five-foot cameras, and four of these contain also an exposure of a check star field.

2. The eclipse plates were compared by differential measurement with photographs of the same stars taken at Tahiti four months before the eclipse.

3. On every plate the stars near the eclipsed Sun appear displaced away from the Sun's center as com-

pared with the reference stars situated in the outer portions of the plate, and the observations of this instrument lead to a value of $1''.82 \pm .15$ for the light deflection at the Sun's limb. The check field stars show no such displacements.

4. Although the two series of observations made at the 1922 eclipse differ in the type of instrument used as well as in the methods of measuring and reduction, their results are in the best agreement.

5. Combining the measures of the two instruments, the relative displacements of stars between $0^\circ.5$ and 10° from the Sun's center are established with a high degree of accuracy (Table 11) independently of any theoretical assumptions.

6. Within the limits set by the unavoidable errors of observation the star displacements are directed radially away from the Sun's center.

7. Einstein's law that the light deflections are inversely proportional to the angular distance from the Sun's center represents the observations best. With this law the combined observations of the two instruments furnish a value of $1''.75 \pm .09$ for the deflection at the Sun's limb which agrees exactly with Einstein's prediction.

8. The eclipse observations contradict the existence of Courvoisier's "yearly refraction" at least in the immediate neighborhood of the Sun.

9. Abnormal refraction in the Earth's atmosphere caused by the cooling effects of the Moon's shadow could not have affected these measures appreciably.

Having guarded our results, by means of the check-field observations, against any systematic errors of instrumental character, or errors due to the photographic process, or to the method of measurement, and having found that abnormal refraction in the Earth's atmosphere must be rejected as a possible explanation, the conclusion seems inevitable, that the observed star displacements are due to a bending of the light rays in the space immediately surrounding the Sun. As to the amount of the light deflections and the law according to which these diminish with increasing angular distance from the Sun's center, the observations agree within the limits of accidental observing errors with the prediction of Einstein's Generalized Theory of Relativity, and the latter seems at present to furnish the only satisfactory theoretical basis for our results.

Mr. E. A. Kholodowsky, Mrs. D. Havens Chappell, and Miss P. Hayford assisted in recording the measures and in making some of the computations.

LICK OBSERVATORY,

February, 1928.

Issued March 27, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 398

MOTION OF MATTER IN THE TAIL OF COMET 1908 III (MOREHOUSE)

BY

N. T. BOBROVNIKOFF

Comet Morehouse showed an unusual activity in the formation of the tail. Many violent outbursts with complete changes in the shape of the tail were photographed at different observatories. They are commonly described as explosions, although in some cases a sudden stoppage of emission would explain the fact as well.

One of the most striking explosions occurred on October 14, 1908, with the consequent motion of cloud formations in the tail. It has been the subject of several investigations, but the problem of the motion of these cloud-like masses cannot be regarded as solved. Indeed, K. Pokrowski¹ and A. J. Orlov² who computed orbits for these masses treated them as a single point, whereas it is apparent that the condensations were moving along the tail with very unequal speeds and showed many peculiarities worthy of notice.

The idea of a more thorough investigation of the motion of the tail formations suggested itself to the present writer during his stay at the Yerkes Observatory in 1926. The late Professor E. E. Barnard collected there a remarkable series of the photographs of this comet of which only few have been discussed and published. The explosion of October 14 which forms the subject of this paper was described by Barnard in the *Astrophysical Journal* (28, 384, 1908) and three of the best plates were reproduced there.

DATA

The condensations out of which five were selected for detailed study are first noticeable on the plates of October 14. Barnard in his paper does not mention

them on this date, although they are clearly visible on the negatives in spite of overexposure and all peculiar features of their appearance on October 15 can be identified on the plates of October 14. They were followed on October 16 and 17 and they are still visible on the Lick plates of October 18, but identification is impossible. The notation is as shown in Fig. 1.

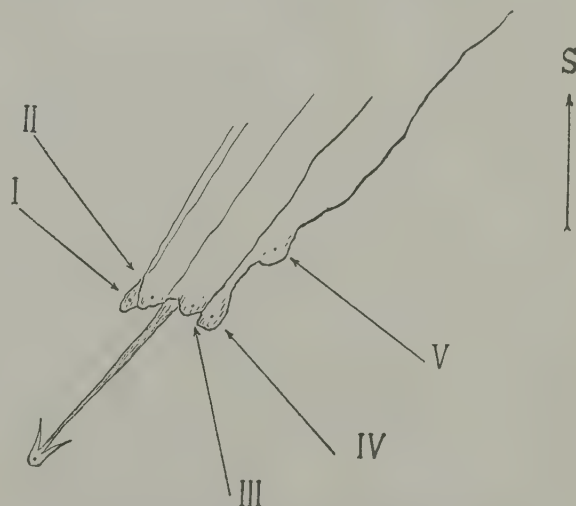


Fig. 1. Morehouse's Comet on October 15, 1908.
Astrophysical Journal, 28, plate 32, 1908.

It is at once apparent from the comparison of photographs that condensations I and II were moving much faster than III and IV. On October 14, I and II were between the nucleus and condensations III and IV. On October 15, all condensations were almost on a straight line perpendicular to the axis of the tail and on October 16, I and II were farther from the nucleus

¹ *A.N.*, 184, 3, 1910.

² *Publ. der K. U.-Sternwarte zu Jurjew*, 21, Heft 3, 1910.

than III and IV. A. Kopff³ has found different velocities for different condensations measured on these dates. In his notation condensations *d* and *e* apparently correspond to I and II of the present study. M. Wolf⁴ has also derived similar results.

There were altogether twenty-five measureable plates of this series obtained by Barnard on four nights. Some of them were taken simultaneously or almost simultaneously and, accordingly, the mean of the results of individual plates was taken. In addition to this four excellent reproductions of negatives obtained by F. Quénisset⁵ at Juvisy were found among Barnard's papers and kindly placed at my disposal by Miss Mary R. Calvert, Professor Barnard's niece. They furnished intermediate points between the Yerkes plates and therefore were used, at some sacrifice of the homogeneity of the material. Thus nine points for the orbit of condensation I, sixteen points for II, III and IV, and five points for V have been obtained. The plates were denoted as follows: *a*—Bruce 10-inch telescope, *b*—Bruce 6-inch, *c*—Bruce 3.5-inch, *Q*—Quénisset Voigtländer photographs. The instruments have been described in the original articles for which references are given.

The measurement presented many difficulties. The exposure was sometimes as long as two hours during which time the condensations were moving. This introduced fuzziness at the edges of the condensations. The stars on the plates were long, wavy lines and a precise bisection of them was impossible. In addition to this the condensations showed unquestionable rapid changes in shape and brightness and also frequent deviations from the smooth line of motion which were several times larger than the uncertainty of measurement. These deviations could be ascertained sometimes on two or even three plates taken simultaneously, which all gave the same result. The oscillatory motion of the condensations showed all the characteristics of the undulatory cometary tail, that is, increasing amplitude and period with the recession from the nucleus.

Finally, the following method was adopted. First, condensations were located among the stars on the B. D. charts and their positions to within one minute of arc were determined. Then for greater precision the position of the densest part of every condensation was measured in the measuring machine differentially with respect to the nucleus and thus $\Delta\alpha$ and $\Delta\delta$ were found. One-tenth of a minute of arc was the utmost precision of measurement to be hoped for, especially on later dates. The position of the nucleus was therefore computed from the elements by Kobold⁶ which gave more precise results than could possibly be obtained under

the conditions of measurement. This also gave at once the geocentric position for 1908.0 and thus spared considerable time and labor. The final results of the measurement are given in Table 1, the times being corrected for aberration.

The usual assumption has been made, namely, that the condensations were moving in the plane of the orbit of the nucleus. For the reduction to the plane of the orbit a method developed by A. J. Orlow⁷ has been employed. The necessary equatorial elements of the orbit of the nucleus were obtained from Kobold's elements as follows:

$$\begin{aligned} T &= \text{Dec. } 26.7858 \text{ G.M.T.} \\ \omega' &= 202^\circ \quad 9' \quad 37'' \\ \Omega' &= 125^\circ \quad 10' \quad 32'' \\ i' &= 130^\circ \quad 17' \quad 7'' \\ \log q &= 1.97532 \end{aligned}$$

The results of this reduction are presented in Table 2. *R* is the heliocentric distance of the condensation and *w* is the true anomaly of the condensation in the orbit of the nucleus. *r* and *v* are the corresponding quantities for the nucleus.

DETERMINATION OF THE REPULSIVE FORCE OF THE SUN AND OF THE ORBITS OF CONDENSATIONS

Several methods for the computation of the repulsive force of the Sun acting on the particles in the comet's tail are in existence. They are either extremely laborious (Socolow's method) or are not reliable in the present case, as *w* varied but little (Jaegermann's method). A. J. Orlow⁸ has devised a short and efficient method which was used in the present paper. Since this method has been published only in Russian, in a hardly accessible publication, it seems desirable to reproduce it here in its main features.

In the following the quantities pertaining to the orbit of the nucleus will be denoted as usual by small letters, those pertaining to the orbit of a particle or a condensation in the tail by the same capital letters. Thus we shall have two sets of quantities for the orbit of a particle: one referring to the orbit of the nucleus, the other referring to the particle's own orbit.

Let *E* be the excentricity of the condensation's orbit, *P* the parameter, *M*_π the time of perihelion passage, *w*_π the true anomaly of the axis of the particle's orbit. Under assumptions that

- (1) the condensation is moving in the plane of the orbit of the nucleus;
- (2) it is acted upon by a central repulsive force *f* originating in the Sun and varying inversely as the

³ *A.N.*, 180, 122, 1909.

⁴ *A.N.*, 180, 1, 1909.

⁵ *Bull. Soc. Astron. de France*, 23, 32, 1909.

⁶ *A.N.*, 179, 273, 1908.

⁷ *Bull. de l'Acad. Imp. des Sci. de St. Pétersbourg*, No. 4, p. 299, 1909.

⁸ *Publ. der K. U.-Sternwarte zu Jurjew*, 21, Heft 3, 1910.

TABLE 1

1908.0

 $\alpha = 19^h +$

Point	Plates	Time Oct. G.M.T.	Nucleus		Condensation I		Condensation II		Condensation III		Condensation IV		Condensation V	
			δ	α	δ	α	δ	α	δ	α	δ	α	δ	α
1	161a	14.535	$+51^\circ 37' 3$	$30^m 7^s 0$	$+51^\circ 37' 0$	$30^m 11^s 7$	$+51^\circ 37' 0$	$30^m 13^s 7$	$+51^\circ 37' 3$	$30^m 16^s 4$	$+51^\circ 38' 6$	$30^m 21^s 1$		
2	162b	.583	33.4	29 57.0	33.2	8.1	32.2	10.1	33.9	13.0	34.7	18.9		
	163a													
	164c													
3	Q ₁	15.359	50 10.7	27 12.1	50 10.7	29 21.5	50 12.2	29 41.3	50 15.3	0.7	50 21.4	30.1		
4	Q ₂	.430	4.7	26 57.2	7.3	40.5	9.4	30 9.2	12.4	18.7	18.5	47.6		
5	165b	.433	49 51.8	35.0	49 48.7	30 13.8	49 50.9	32.2	49 57.1	59.8	1.4	31 5.8	$+50^\circ 3' 1$	$32^m 43^s 0$
	166a	.560	48.8	31.3	45.4	13.3	47.9	34.5	54.6	31 10.1	49 59.0	12.1	1.1	49.3
6	167c													
7	168a													
	169b	.598	45.2	23.3	41.7	5.3	43.7	47.7	50.5	7.7	56.2	9.7	49 57.8	55.3
8	170c	.629	42.0	15.9			37.0	51.5	43.2	11.9	49.7	11.1		
9	171b	.700	37.0	3.4	31.6	41.4	32.8	31 4.6	42.8	26.2	48.6	25.4	50.2	33 30.2
	172a	.723	31.5	25 57.9			27.5	4.3	36.0	30.3	42.5	26.7	46.3	45.9
10	173c													
11	174a													
	174b	.745	23.9	52.8			24.6	7.2	34.1	43.2	41.4	32.8		
12	174 $\frac{1}{2}$ a	.793	24.3	45.5			19.9	20.5	28.0	51.1	35.6	43.5		
	174 $\frac{1}{2}$ b													
13	Q ₃													
	175a	16.311	29.2	24 5.3	48 25.9	33 34.1	48 33.8	34 10.7	48 44.5	34 10.7	48 50.8	33 34.1		
14	175b	.532	48 6.5	23 25.6			6.0	45.6	15.0	35 14.8	26.1	34 54.0		
	175c													
	176a													
15	176b	.641	47 52.2	2.3			47 53.8	57.9	1.5	14.3	17.3	31 1.9		
	176c	17.448	46 27.9	20 41.7	46 34.3	40 31.8					47 3.3	37 26.5		
16	Q ₄													
	Q ₄													
17	179a	.568	16.3	20.4			46 35.2	42 21.2	46 45.7	39 22.8				
	179b													
	180c													

TABLE 2

Point	Mid-Exp. G.M.T. October	Nucleus		I		II		III		IV		V	
		r	v	R	w	R	w	R	w	R	w	R	w
1	14.535	1.5368	$-76^\circ 45' 0$	1.5372	$-76^\circ 43' 7$	1.5374	$-76^\circ 43' 7$	1.5372	$-76^\circ 45' 0$	1.5377	$-76^\circ 44' 8$		
2	.583	.5364	42.5			.5372	42.8	.5373	43.3	.5377	43.8		
3	15.359	.5268	15.4	.5367	14.3	.5382	14.8	.5393	17.3	.5409	21.4		
4	.430	.5259	12.9	.5380	14.4	.5400	15.7	.5404	17.8	.5421	21.9		
5	.433	.5246	9.3	.5417	5.0	.5420	12.9	.5445	10.7	.5446	13.8	1.5520	$-76^\circ 14' 2$
6	.560	.5243	8.3	.5418	3.8	.5432	5.4	.5453	10.0	.5450	13.1	.5525	13.9
7	.598	.5239	7.0	.5414	2.4	.5445	3.5	.5454	8.2	.5450	12.3	.5531	12.7
8	.629	.5234	5.9			.5453	75 59.8	.5463	4.0	.5455	8.8	.5565	8.0
9	.700	.5227	3.5	.5446	75 58.6	.5468	57.0	.5474	4.7	.5463	10.6	.5580	8.9
10	.723	.5223	2.5			.5466	56.2	.5478	2.6	.5469	7.3		
11	.745	.5220	1.8			.5470	55.1	.5489	2.0	.5473	7.2		
12	.793	.5215	0.2			.5483	53.7	.5498	75 59.3	.5484	4.8		
13	16.311	.5152	75 41.9	.5615	33.0	.5636	38.4	.5607	51.3	.5549	2.6		
14	.532	.5126	34.1			.5697	25.8	.5697	32.0	.5666	75 40.7		
15	.641	.5112	30.2			.5698	21.2	.5704	26.7	.5674	38.5		
16	17.448	.5014	1.3	.6043	49.2					.5835	13.0		
17	.568	.5000	74 56.9			.6138	74 52.2	.5956	2.0				

square of the distance we have for the motion of the condensation the following equations:

$$(1) \quad \begin{cases} R^2 \frac{dw}{dt} = C \\ \frac{d^2 R}{dt^2} = \frac{f}{R^2} + \frac{C^2}{R^3} \end{cases}$$

where C is the constant of areas, equal to $\sqrt{fP}$ and R and w the quantities used in Table 2. From equation 1 we easily derive:

$$(2) \quad f = R^2 \frac{d^2 R}{dt^2} - \frac{C^2}{R}$$

The idea of the method is to determine for the mean moment $t=t_0$ of all observations the corresponding radius vector R_0 , its first and second derivatives, and the constant of areas from the whole series of observations. Since C is usually very small we can neglect it altogether in the first approximation and put

$$(3) \quad f = R_0^2 \frac{d^2 R_0}{dt^2}$$

For the determination of the derivatives of R at $t=t_0$ we have Taylor's series (4):

$$(4) \quad \begin{cases} R = R_0 + \frac{dR_0}{dt} t + \frac{1}{2} \frac{d^2 R_0}{dt^2} t^2 + \frac{1}{6} \frac{d^3 R_0}{dt^3} + \dots \\ w = w_0 + \frac{dw_0}{dt} t + \frac{1}{2} \frac{d^2 w_0}{dt^2} t^2 + \frac{1}{6} \frac{d^3 w_0}{dt^3} + \dots \end{cases}$$

Taking in the series for R only three first terms we can easily find approximate values of R_0 and its derivatives from three or more observations well distributed through the whole period of observation. The approximate value of f can be found then from (3).

Differentiating (1) in succession and putting the obtained values of the derivatives in (4) we have, neglecting the terms with the derivatives of higher order than three,

$$(5) \quad \begin{cases} R = R_0 + \left(t - \frac{ft^3}{3R_0^2} \right) \frac{dR_0}{dt} + \frac{1}{2} \frac{d^2 R_0}{dt^2} t^2 \\ w = w_0 + \frac{Ct}{R_0^2} - \frac{Ct^2}{R_0^3} \frac{dR_0}{dt} - \frac{Ct^3}{3R_0^3} \frac{d^2 R_0}{dt^2} \end{cases}$$

In the series for R the coefficient $t - \frac{ft^3}{3R_0^2} = t'$ differs only slightly from t , if the whole period of observation covers only a few days, since f is usually a small number. Its approximate value and R_0 can be taken from (3).

Now we have from the measurements n values of R and w . Subtracting from each equation,

$R = R_0 + \frac{dR_0}{dt} t' + \frac{1}{2} \frac{d^2 R_0}{dt^2} t'^2$, the mean of all equations and denoting the residuals with Δ , we have:

$$(6) \quad \begin{cases} \Delta R_1 = \frac{dR_0}{dt} \Delta t'_1 + \frac{1}{2} \frac{d^2 R_0}{dt^2} \Delta t_1^2 \\ \Delta R_2 = \frac{dR_0}{dt} \Delta t'_2 + \frac{1}{2} \frac{d^2 R_0}{dt^2} \Delta t_2^2 \\ \dots \dots \dots \Delta R_n = \frac{dR_0}{dt} \Delta t'_n + \frac{1}{2} \frac{d^2 R_0}{dt^2} \Delta t_n^2 \end{cases}$$

From these equations $\frac{dR_0}{dt}$ and $\frac{d^2 R_0}{dt^2}$ are obtained by the method of least squares. R_0 is then easily found from the mean of all equations. Using in the same way the second equation of (5), $w = w_0 + \frac{Ct''}{R_0^2}$, where $t'' = t' - \frac{t^2}{R_0} \frac{dR_0}{dt}$, we find C and w_0 . For the final check on the convergence of series (4), we have, neglecting only terms with derivatives of higher order than six,

$$(7) \quad R = R_\pi + \frac{1}{2} \frac{d^2 R_\pi}{dt^2} t_\pi^2 + \frac{1}{24} \frac{d^4 R_\pi}{dt^4} t_\pi^4$$

For $t = -t_\pi$, $R = R_0$ in equation (7); substituting $t = -t_\pi$ in the expression of R and its derivatives, we therefore obtain:

$$(8) \quad \begin{cases} R_0 = R_\pi + \frac{1}{2} \frac{d^2 R_\pi}{dt^2} t_\pi^2 + \frac{1}{24} \frac{d^4 R_\pi}{dt^4} t_\pi^4 \\ \frac{dR_0}{dt} = -\frac{d^2 R_\pi}{dt^2} t_\pi - \frac{1}{6} \frac{d^4 R_\pi}{dt^4} t_\pi^3 \\ \frac{d^2 R_0}{dt^2} = +\frac{d^2 R_\pi}{dt^2} + \frac{1}{2} \frac{d^4 R_\pi}{dt^4} t_\pi^2 \end{cases}$$

From (1) we have by successive differentiations at $R = R_\pi$

$$(9) \quad \begin{cases} \frac{d^2 R_\pi}{dt^2} = \frac{f}{R_\pi^2} + \frac{C^2}{R_\pi^3} \\ \frac{d^4 R_\pi}{dt^4} = -\frac{2f}{R_\pi^3} \frac{d^2 R_\pi}{dt^2} - \frac{3C^2}{R_\pi^4} \frac{d^2 R_\pi}{dt^2} \end{cases}$$

Heretofore we have not made any hypothesis concerning the kind of the conic section in which the particle or the condensation moves. Bredichin⁹ has shown that the kind of the orbit depends entirely upon the value

⁹ R. Jaegermann, *Bredichin's Mechanische Untersuchungen über Cometenformen*. St. Petersburg, 1903, pp. 283-291.

of the repulsive force $(1-\mu)$ of the Sun acting on the particle. For $(1-\mu) > 1$ it will be a hyperbola convex to the Sun. As a fact of observation we can assume that in the tails of the comets of Bredichin's first type, to which the tail of Comet Morehouse evidently belongs, $(1-\mu)$ is much larger than unity. Therefore the resulting orbit of the condensation should be the branch of a hyperbola convex to the Sun. Considering the equation of such hyperbola

$$(10) \quad R = \frac{P}{E \cos V - 1}$$

we derive for R_π

$$(11) \quad R_\pi = \frac{P}{E-1} \text{ or } E = 1 + \frac{P}{R_\pi}$$

Remembering that $C = \sqrt{fP}$ we obtain from (9) and (11):

$$(12) \quad \begin{cases} \frac{d^2 R_\pi}{dt^2} = \frac{fE}{R_\pi^2} \\ \frac{d^4 R_\pi}{dt^4} = -\frac{f^2 E(3E-1)}{R_\pi^3} \end{cases}$$

In the last equation of (12) f is a small quantity and its approximate value from (6) will serve. P and therefore E are known from the relation $P = \frac{C^2}{f}$ where C is already known. The approximate value of R_π is also known. Indeed, we have from (5):

$$(13) \quad \frac{dR}{dt} = \frac{dR_o}{dt} \left(1 - \frac{f}{R_o^3} t^2\right) + \frac{d^2 R_o}{dt^2} t$$

For $t = t_\pi$, $\frac{dR}{dt} = 0$, and we obtain:

$$(14) \quad t_\pi = -\frac{\frac{dR_o}{dt}}{\frac{d^2 R_o}{dt^2}} \left(1 - \frac{f}{R_o^3} t_\pi^2\right)$$

Here $\frac{f}{R_o^3} t_\pi^2$ is a small factor and it is easy to find t_π by successive approximations or by solving the quadratic equation (14) directly for t_π . The choice between the roots is easy as they differ very much. Knowing t_π we find R_π from the mean of equations (5). Therefore $\frac{d^4 R_\pi}{dt^4}$ is known with sufficient accuracy from (12) and we have to solve (8) only for R_π , $\frac{d^2 R_\pi}{dt^2}$ and t_π . The result is:

$$(15) \quad \begin{cases} \frac{d^2 R_\pi}{dt^2} = \frac{d^2 R_o}{dt^2} - \frac{1}{2} \frac{d^4 R_\pi}{dt^4} t_\pi^2 \\ t_\pi = -\frac{\frac{dR_o}{dt} + \frac{1}{2} \frac{d^4 R_\pi}{dt^4} t_\pi^3}{\frac{d^2 R_\pi}{dt^2}} \\ R_\pi = R_o - \frac{1}{2} \left(\frac{d^2 R_\pi}{dt^2} + \frac{1}{12} \frac{d^4 R_\pi}{dt^4} t_\pi^2 \right) t_\pi^2 \end{cases}$$

In the right side of equation (15) the approximate value of t_π should be taken from (14). Finally, from (12) we have

$$(16) \quad f = \frac{R_\pi^2}{E} \frac{d^2 R_\pi}{dt^2}$$

This value of f should coincide with that from equations (6) and (2) if the convergence of the series employed is rapid enough.

We have found already t_π and E . The remaining elements can be obtained by simple considerations in the following way:

$$(17) \quad \begin{cases} M_\pi = t_o + t_\pi \\ A = \frac{P}{E^2 - 1} \\ w_\pi = w_o - V_o \end{cases}$$

V_o , the true anomaly of the condensation in its own orbit, is computed for the moment t_o together with the ephemeris for all points used in the calculation. It is easy to compute the ephemeris using tables for hyperbolic motion (given also by A. J. Orlow). We have the following equations for the computation of the ephemeris:

$$(18) \quad \begin{cases} N = \frac{\sqrt{f}}{A^{\frac{3}{2}}} t & x = E - 1 \\ \tan F - \log \tan \left(45^\circ + \frac{F}{2}\right) = N - x \tan F \\ \tan \frac{V}{2} = \sqrt{\frac{E-1}{E+1}} \tan \frac{F}{2} \\ \frac{1}{R} = \frac{1}{R_\pi} - \frac{2E}{P} \sin^2 \frac{V}{2} \end{cases}$$

Since, however, the intervals of time in the case of motion of the condensations are usually small, a simplified formula for V may be used. Expanding w as a function of w_π and noting that $w - w_\pi = V$ we come by simple considerations to the formulae

$$(19) \quad \begin{cases} n = N \sqrt{\frac{E-1}{(E+1)^3}} \\ V = n - \frac{n^3 E}{3x} \end{cases}$$

where t is counted from the perihelion of the condensation.

Suppose now that the gaseous mass was ejected from the nucleus with an initial velocity g and under an angle G with the radius vector; counting G positive in the direction opposite to the motion of the nucleus. The vector of the initial impulse is the geometric sum of two vectors: one, of the velocity of ejection and the other, of the orbital velocity of the nucleus at that point. Considering the projection of these vectors on the prolongation of the radius vector and the perpendicular to it at the nucleus, we obtain:

$$(20) \quad \begin{cases} g \cos G = \frac{dr}{dt} - \frac{dR}{dt} \\ g \sin G = \frac{c - C}{R} \end{cases}$$

where $\frac{dr}{dt} = \frac{k \sin v}{\sqrt{2q}}$, $\frac{dR}{dt} = \frac{E\sqrt{f} \sin V}{\sqrt{P}}$, $c = k\sqrt{2q}$, $C = \sqrt{fP}$ at the moment of ejection. From equations (20) g and G may be found. The moment of ejection is determined graphically or by comparing the ephemeris of the nucleus and the condensation at their nearest approach.

This method is short and efficient in practice. Once the normal equations for (6) are formed, the equations for the following approximations are obtained in a very short time. If several condensations are observed at the same time the coefficients of the unknowns in the least squares solutions remain almost the same, which simplifies the work. The chief drawback of the method is a rather lengthy reduction to the plane of the orbit of the nucleus and the derivation of the heliocentric polar coordinates.

CONDENSATION I

In the first approximation, four equations, $R = R_o + \frac{dR_o}{dt}t + \frac{1}{2} \frac{d^2R_o}{dt^2}t^2$, were formed making use of point 1; mean from 5, 6, 7; point 13 and point 16. They gave:

$$\begin{aligned} R_o &= 1.5510 \\ \frac{dR_o}{dt} &= 0.0221 \\ \frac{d^2R_o}{dt^2} &= 0.0180 \\ \log f &= \bar{2}.63190 \end{aligned}$$

Comparing f with the Gaussian constant k , we find the effective repulsive force of the Sun $\mu = -145$ and its absolute value $(1-\mu) = 146$ in the units of k^2 , that is 146 times the force of gravitation acting on the same condensation. For the second approximation all nine equations, $R = R_o + \frac{dR_o}{dt}t + \frac{1}{2} \frac{d^2R_o}{dt^2}t^2$, were formed. They gave the following normal equations¹⁰ in the least squares solution:

$$\left. \begin{aligned} +99324 \frac{dR_o}{dt} + 37252 \frac{d^2R_o}{dt^2} &= 2546 \\ +41788 \frac{d^2R_o}{dt^2} &= 1490 \end{aligned} \right\}$$

$$50459 \frac{C}{R_o} = 585$$

which yielded:

$$\begin{aligned} R_o &= 1.5443 \\ \frac{dR_o}{dt} &= 0.0184 \pm 0.0006 \\ \frac{d^2R_o}{dt^2} &= 0.0192 \pm 0.0010 \text{ (probable error)} \\ w_o &= -75^\circ 58'.1 \\ \log C &= \bar{2}.44119 \\ (1-\mu) &= 154 \pm 8.1 \end{aligned}$$

The next approximation (16) gave the same value of $(1-\mu)$, which is also true for the rest of the condensations.

This results in the elements:

$$\begin{aligned} A &= 0.7632 \\ E &= 1.0110 \\ R_\pi &= 1.5357 \\ w_\pi &= -76^\circ 36'.1 \\ M_\pi &= \text{October 14.764, 1908, G. M. T.} \end{aligned}$$

The orbit represents the observations as shown in Table 3. The subscripts mean: c , the value computed and o , the value observed. It was not considered necessary to compute α and δ for the condensations as the heliocentric coordinates give a good idea of the degree of correspondence. $1'$ in w corresponds approximately to $2'.5$ and 0.0001 in R would give about $0'.5$ on the original plates. It is seen from this that the orbit satisfies the observations fairly well, taking into account the difficulty of measurement.

¹⁰ In this and later normal equations a factor of 10000 has been introduced as a matter of convenience.

TABLE 3

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5362	1.5372	+0.0010	-76°45'2	-76°43.7	+1.5
2	.5392	.5667	- .0025	12.2	14.3	-2.1
3	.5400	.5380	- .0020	9.3	14.4	-5.1
5	.5401	.5417	+ .0016	9.2	5.0	+4.2
6	.5418	.5418	.0000	4.1	3.8	+0.3
7	.5425	.5414	- .0011	2.5	2.4	+0.1
9	.5443	.5446	+ .0003	75 58.4	75 58.6	-0.2
13	.5593	.5615	+ .0022	33.8	33.0	+0.8
16	.6076	.6043	- .0033	74 48.8	74 49.2	-0.4

CONDENSATION II

The first approximation gave $(1-\mu)=143$. The second approximation, including all sixteen equations, resulted in the following normal equations:

$$\left. \begin{aligned} +163510 \frac{dR_o}{dt} + 35781 \frac{d^2 R_o}{dt^2} &= +3970 \\ +58337 \frac{d^2 R_o}{dt^2} &= +1795 \\ +78067 \frac{C}{R_o^2} &= +823 \end{aligned} \right\}$$

which are satisfied by

$$\begin{aligned} R_o &= 1.5475 \\ \frac{dR_o}{dt} &= 0.0203 \pm 0.0003 \\ \frac{d^2 R_o}{dt^2} &= 0.0183 \pm 0.0006 \\ w_o &= -75^\circ 57'7 \\ \log C &= 2.40223 \\ (1-\mu) &= 148 \pm 4.7 \end{aligned}$$

The representation of observations is given in Table 4.

TABLE 4

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5367	1.5374	+0.0007	-76°42'0	-76°43'7	-1'7
2	.5365	.5372	+ .0007	40.2	42.8	-2.6
3	.5408	.5382	+ .0026	11.8	14.8	-3.0
4	.5416	.5400	- .0016	9.2	15.7	-6.5
5	.5417	.5420	+ .0003	9.1	12.9	-3.8
6	.5437	.5432	- .0005	4.4	5.4	-1.0
7	.5442	.5445	+ .0003	3.0	3.5	-0.5
8	.5445	.5453	+ .0008	1.9	75 59.8	+2.1
9	.5464	.5468	+ .0004	75 59.2	57.0	+2.2
10	.5467	.5466	- .0001	58.4	56.2	+2.2
11	.5471	.5470	- .0001	57.6	55.1	+2.5
12	.5481	.5483	+ .0002	55.9	53.7	+2.2
13	.5616	.5636	+ .0020	36.9	38.4	-1.5
14	.5690	.5697	+ .0007	28.9	25.8	+3.1
15	.5730	.5698	- .0032	24.9	21.2	+3.7
17	.6166	.6138	- .0028	74 51.7	74 51.2	-0.5

The elements were derived as follows:

$$\begin{aligned} A &= 0.7643 \\ E &= 1.0096 \\ R_\pi &= 1.5365 \\ w_\pi &= -76^\circ 37'6 \\ M_\pi &= \text{October 14.669} \end{aligned}$$

CONDENSATION III

The first approximation gave $(1-\mu)=77$. From the second approximation, including all sixteen equations, there resulted the normal equations:

$$\left. \begin{aligned} +164516 \frac{dR_o}{dt} + 35994 \frac{d^2 R_o}{dt^2} &= +3258 \\ +58337 \frac{d^2 R_o}{dt^2} &= +1263 \\ +81402 \frac{C}{R_o^2} &= +812 \end{aligned} \right\}$$

which gave:

$$\begin{aligned} R_o &= 1.5488 \\ \frac{dR_o}{dt} &= 0.0174 \pm 0.0004 \\ \frac{d^2 R_o}{dt^2} &= 0.0109 \pm 0.0010 \\ w_o &= -76^\circ 2'1 \\ \log C &= 2.38830 \\ (1-\mu) &= 88 \pm 8.0 \end{aligned}$$

The following elements were obtained:

$$\begin{aligned} A &= 0.7612 \\ E &= 1.0151 \\ R_\pi &= 1.5353 \\ w_\pi &= -76^\circ 58'2 \\ M_\pi &= \text{October 14.188} \end{aligned}$$

This orbit satisfies the observations in the following way:

TABLE 5

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5360	1.5372	+0.0012	-76°45'7	-76°45'0	+0'7
2	.5362	.5373	+ .0011	44.0	43.0	+1.0
3	.5430	.5393	- .0037	16.3	17.3	-1.0
4	.5436	.5404	- .0032	13.9	17.8	-3.9
5	.5437	.5447	+ .0010	13.8	10.7	+3.1
6	.5450	.5443	- .0007	9.3	10.0	-0.7
7	.5465	.5454	- .0011	7.8	8.2	-0.4
8	.5469	.5463	- .0006	6.7	4.0	+2.7
9	.5479	.5474	- .0005	4.3	4.7	-0.4
10	.5485	.5478	- .0007	3.4	2.6	+0.8
11	.5488	.5489	+ .0001	2.7	2.0	+0.7
12	.5492	.5498	+ .0006	0.9	75 59.3	+1.6
13	.5607	.5607	.0000	75 42.4	51.3	-8.9
14	.5665	.5697	+ .0032	34.5	32.0	+2.5
15	.5693	.5704	+ .0011	30.7	26.7	+4.0
17	.6009	.5956	- .0053	74 57.9	2.0	-4.1

CONDENSATION IV

The first approximation gave $(1-\mu)=79$. The normal equations of the second approximation are as follows:

$$\left. \begin{aligned} +156506 \frac{dR_o}{dt} + 26078 \frac{d^2R_o}{dt^2} &= +2542 \\ +46850 \frac{d^2R_o}{dt^2} &= +882 \\ +78330 \frac{C}{R_o^2} &= +707 \end{aligned} \right\}$$

which yielded

$$\begin{aligned} R_o &= 1.5473 \\ \frac{dR_o}{dt} &= 0.0145 \pm 0.0005 \\ \frac{d^2R_o}{dt^2} &= 0.0108 \pm 0.0010 \\ w_o &= -76^\circ 7'.7 \\ \log C &= \bar{2}.33294 \\ (1-\mu) &= 88 \pm 8.1 \end{aligned}$$

from which the following elements have been derived:

$$\begin{aligned} A &= 0.7812 \\ E &= 1.0118 \\ R_x &= 1.5377 \\ w_x &= -76^\circ 47'.7 \\ M_x &= \text{October } 14.424 \end{aligned}$$

This orbit represents the observations as shown in Table 6.

TABLE 6

Point	R_c	R_o	$R_o - R_c$	w_c	w_o	$w_o - w_c$
1	1.5376	1.5377	+0.0001	-76°44'.4	-76°44'.8	-0'.4
2	.5376	.5377	+ .0001	43.0	43.8	-0.8
3	.5420	.5409	- .0011	19.6	21.4	-1.8
4	.5427	.5421	- .0006	17.4	21.9	-4.5
5	.5428	.5446	+ .0018	17.3	13.8	+3.5
6	.5443	.5450	+ .0007	13.5	13.1	+0.4
7	.5449	.5450	+ .0001	12.3	12.3	0.0
8	.5452	.5455	+ .0003	11.4	8.8	+2.6
9	.5460	.5463	+ .0003	9.2	10.6	-1.4
10	.5462	.5469	+ .0007	8.6	7.3	+1.3
11	.5466	.5473	+ .0007	7.9	7.2	+0.7
12	.5473	.5484	+ .0011	6.4	4.8	+1.6
13	.5559	.5549	- .0010	75 50.8	2.6	-11.8
14	.5607	.5666	+ .0059	44.0	75 40.7	+3.3
15	.5630	.5674	+ .0044	40.8	38.5	+2.3
16	.5853	.5835	- .0018	16.6	13.0	+3.6

CONDENSATION V

Only five positions of this condensation could be measured with certainty on the plates of October 15. The orbit based on such a short arc is indeterminate. But since all four orbits computed above intersect the

orbit of the nucleus approximately in the same place, $R=1.5365$ and $w=-76^\circ 43'.0$, we may reasonably assume that orbit V also passed through this point. This gives us an additional point corresponding to $t=\text{October } 14.535$.

The resulting normal equations are as follows:

$$\left. \begin{aligned} +191220 \frac{dR_o}{dt} - 62200 \frac{d^2R_o}{dt^2} &= 3310 \\ +22420 \frac{d^2R_o}{dt^2} &= 1057 \end{aligned} \right\}$$

which give the approximate value $(1-\mu)=90$. This value, however, is rather uncertain owing to the small difference between two quantities in the numerator of the expression for $\frac{d^2R_o}{dt^2}$. This may influence $(1-\mu)$ for about ten units either way. Owing to this uncertainty and to the assumption mentioned above the orbit has not been computed.

GENERAL REMARKS ON THE ORBITS

The computed orbits agree with the observations satisfactorily. Indeed, taking into account the conditions of measurement, no better correspondence could be expected. Any method of variation of the elements might possibly improve them, but such a procedure is clearly not justified by the nature of the problem as all measurements were necessarily approximate. Moreover, this would not change the value of $(1-\mu)$ which depends only on the quantities derived from the least squares solution, principally upon R_o and its second derivative. The time of perihelion passage depends almost entirely on the ratio of the first and second derivatives of R at $t=t_o$, and therefore is also obtained from the least squares solution.

Figure 2 represents the orbits of condensations II and III and of the nucleus. The deviations from the smooth line are far too large to be explained by the errors of measurement as has been mentioned before. The path of the condensations resembles a cork-screw, which is typical for the structure of the whole tail of this comet.

If we consider the differences $R_o - R_c$ we see that at the beginning of the motion the values are positive which may be accounted for by the repulsive action of the nucleus on the condensations. It is difficult to ascertain the existence of such action between the condensations themselves, owing to the undulatory motion shared by all condensations. The fact that they drew apart from each other with recession from the nucleus

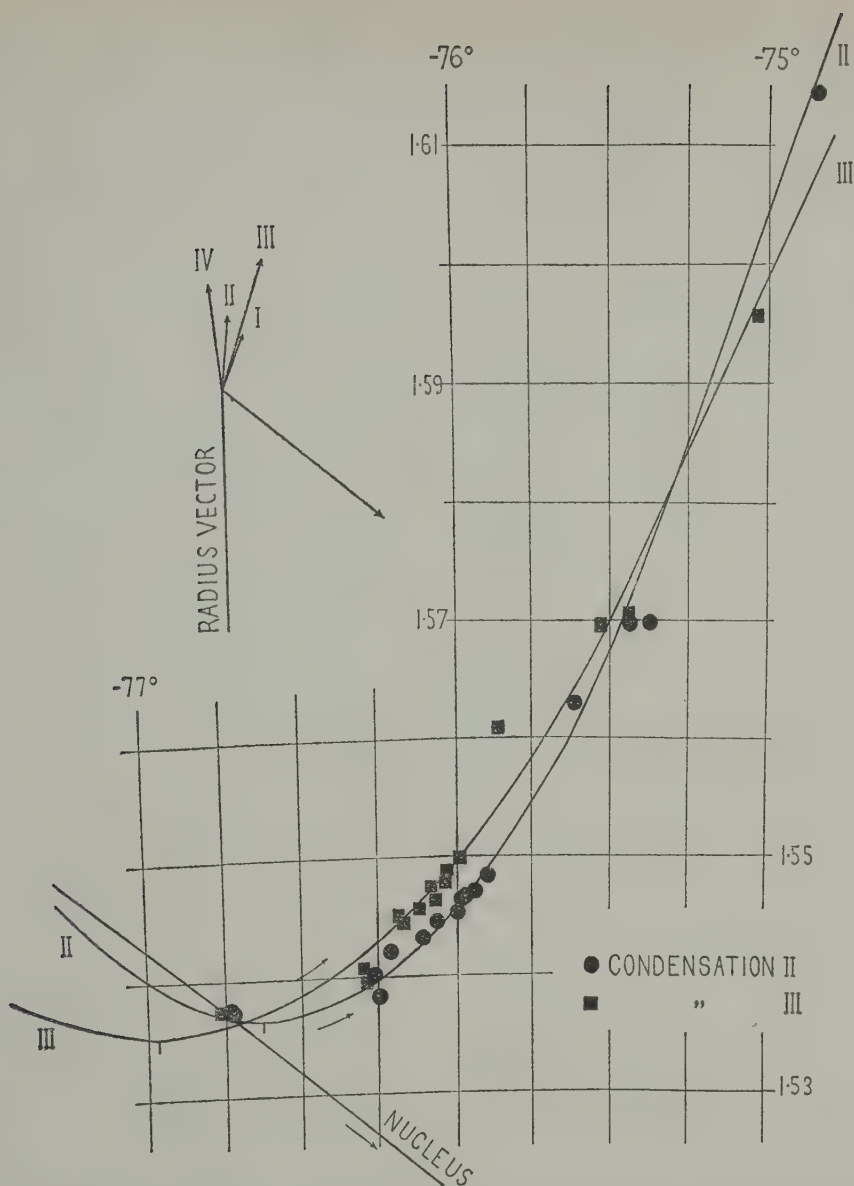


Fig. 2. Orbits of condensations II and III and of the nucleus. Heliocentric coordinates R and w . The insert shows the vectors of the velocity of ejection of condensations with the velocity of the nucleus. Perihelion passage is marked by short vertical lines.

cannot be taken as an evidence for mutual repulsion. It is of course a consequence of the different values of $(1-\mu)$ and the different initial conditions.

It is clear furthermore that on the last date, October 17, all condensations lag considerably behind the computed points although they do not deviate much from the orbit. This may be interpreted as a diminution of acceleration due to the dissipation of these cloud-like formations and consequent increase of resist-

ance on the part of matter filling the tail and moving with smaller velocities. Such a decrease of acceleration was actually found by M. Wolf¹¹ from his measurements of this comet. Analogous phenomena in the case of comet 1882 II in the motion of the so-called Schmidt's Clouds (*Die Schmidtschen Wolken*) has been studied in detail by Bredichin and others.¹²

¹¹ loc. cit.

¹² Jaegermann, *Bredichin's Mechanische Untersuchungen über Cometenformen*, St. Petersburg, 1903, p. 408.

INITIAL CONDITIONS

The moment of explosion cannot be determined with great accuracy on account of the perturbative action of the nucleus on the condensations immediately after the moment of explosion. This changes the moment of the closest approach of the condensations to the nucleus which is computed backwards from the ephemeris.

We cannot assume the moment of emission earlier than the first observation, that is October 14.535, although some condensations were later than this at the nearest distance from the nucleus. Apparently the first photograph was obtained almost at the moment of ejection. Comparison of different orbits fixes the moment of ejection at October 14.500, which cannot be far from the truth.

We can compute, however, the velocity of ejection g and the angle of projection G with considerable accuracy. Indeed, in (20), $\frac{dR}{dt}$ in our case is a small quantity

in comparison with $\frac{dr}{dt}$, as the explosion naturally occurred near the perihelion of the condensations. R also does not change appreciably in the limits of uncertainty as to the time of explosion. C does not depend immediately on the time of ejection but is determined from the whole orbit. Therefore even a large variation in the time of the explosion will not change the results materially. It is also clear from (20) that G can be determined with greater accuracy than g . Indeed, the change of the moment of explosion in Case II by as much as 0.340, which greatly exceeds the limits of uncertainty, gave $G=184^\circ$ instead of 186° previously obtained and $g=21$ km/sec instead of 12 km/sec.

Assuming then October 14.5 as the moment when the gases in the nucleus were suddenly released we have for g and G the values given in Table 7. The insert in Figure 2 shows the vectors of the initial velocities of the condensations and of the velocity of the nucleus on an arbitrary scale. It will be seen that the cometary gases were thrown from the nucleus in a very narrow fan (about 27°) in a direction almost opposite to the direction of the radius vector. The initial velocities fall into two groups: I and II with the mean 11.2 km/sec and III and IV with the mean 20.3 km/sec.

DISCUSSION OF RESULTS

There is little doubt that condensations I and II formed one group with the same physical properties, while condensations III, IV and V formed another. Table 7 gives a summary of the results obtained. Quantities Ψ are the asymptote angles of the hyperbolas.

TABLE 7

Condensation	I	II	III	IV	V
$(1-\mu)$	154	148	88	88	90
g in km/sec.....	10.0	12.4	22.8	17.8	
G	$200^\circ 48'$	$186^\circ 33'$	$198^\circ 21'$	$174^\circ 2'$	
A	0.7632	0.7643	0.7612	0.7812	
E	1.0110	1.0095	1.0151	1.0118	
ψ	$8^\circ 27'$	$7^\circ 53'$	$9^\circ 54'$	$8^\circ 48'$	
R_π	1.5357	1.5365	1.5353	1.5377	
w_π	$-76^\circ 36' 1$	$-76^\circ 37' 6$	$-76^\circ 58' 3$	$-76^\circ 47' 7$	
M_π 1908, Oct. 14.764 G.M.T.	14.669	14.118	14.424		

An examination of this table reveals several interesting facts. The mean of $(1-\mu)$ for condensations I and II is 151, exactly the value of the repulsive force of the Sun calculated recently for the CO^+ molecule by Baade and Pauli.¹³ The spectrum of the tail of this comet has been studied extensively by many observers and identified with the so-called Third Negative Carbon Group belonging to the CO^+ molecule. Indeed, the intensity of these radiations in the tail of Morehouse's comet constituted its peculiar feature. The spectrum of condensations such as have been discussed here was found to be identical with the spectrum of the tail,¹⁴ that is the condensations consisted almost exclusively of CO^+ , which is in perfect accordance with our results.

The good agreement of the theoretical value of $(1-\mu)$ with its dynamical value for condensations I and II shows that these condensations, at least, were moving in the plane of the orbit of the nucleus, that is, the assumption made at the beginning of this investigation is justified. On the other hand, it means also that the Comet Tail bands are a resonance radiation, since Baade and Pauli treated the CO^+ molecule as a resonator. This conclusion can be arrived at by a quite different way of reasoning.¹⁵ It may be of interest to note that the value $(1-\mu)=156$ has been obtained by S. V. Orlov¹⁶ for this comet in the case of motion of the condensations on October 1, 1908.

The question arises as to the significance of the other value of the repulsive force, namely $(1-\mu)=88$, obtained for condensations III, IV and V. We know that the only other constituent of the tail was the N_2^+ molecule which gave a band at $\lambda 3914$. However, this radiation was insignificant in comparison with the CO^+ group, whereas in the tail condensations III, IV and V form the bulk.

The possibility of the inclination of the plane of the orbits III and IV to that of I and II has been considered. By tipping the plane of the orbit III through rather large angles (about 40°) to the plane of the orbit

¹³ *Naturwissenschaften*, 15, 49, 1927.

¹⁴ F. Baldet, *Ann. de l'Observ. d'Astron. Phys. de Paris*, 7, 28, 1926.

¹⁵ Bobrovnikoff, *Astroph. Jour.*, 66, 439, 1927.

¹⁶ *Russian Astron. Jour.*, I, 1, 71, 1924.

II a fair degree of correspondence of the points of III with those of II may be reached. But such an inclination is hardly possible. Again, although orbit II is quite different in shape from orbit I, both give the same $(1-\mu)$. This is also true for the orbits of the second group as compared with each other. Condensations I and II on one side and III and IV on the other were symmetrically situated with respect to the axis of the tail, and it is difficult to see why I and II should move exactly in the plane of the orbit of the nucleus, while III and IV move far away from it.

As is well known, the stereoscope in such cases gives striking results. Some observers reported that the south part of the condensations (that is I and II) was nearer to us than the northern part (III and IV), the inference being that they did not lie in the same plane. This is, however, the natural result of the inclination of the orbit of the nucleus to the ecliptic and the situation of the comet in its orbit with reference to the Earth. On the other hand, the stereoscope, in the case of moving and rapidly changing condensations, may give quite erroneous information as has been pointed out by Barnard.¹⁷

There is another line of evidence which favors the view that all condensations were moving in the same plane. The number 88 is not due to chance. The numbers $(1-\mu)=151$ and $(1-\mu)=88$ are approximately simple multiples of 22, their ratio being 7:4. S. V. Orlov¹⁸ has determined by an approximate method the values of $(1-\mu)$ from the motion of receding masses in this comet as measured by M. Wolf.¹⁹ The frequency curves of $(1-\mu)$ show sharp maxima for the multiples of 22 or 23, among them being $(1-\mu)=89$. The value $(1-\mu)=89$ was also obtained by R. Jaegermann²⁰ for the motion of condensations in the tail of comet 1903 IV (Borrelly). This can hardly be a coincidence. It may depend on the peculiarities of the structure of the CO^+ molecule in cometary tails.

The gaseous masses were ejected in the direction almost opposite to the Sun. The larger initial velocity of condensations III and IV and the smaller value of the repulsive force as compared with condensations I and II explains the deformation and twists in the tail in perfect agreement with observations. Indeed, since G is nearly 180° we can apply to the motion of condensations in reference to the nucleus the simple formula of uniformly accelerated motion with the initial velocity g . We have then for velocity v

$$(21) \quad v = g + ht,$$

where t is the time from the moment of ejection and h is the acceleration due to the repulsive force $(1-\mu)$. For condensations II and III we have the following values of v :

TABLE 8

Time	II	III
October 14.50	12	23 km/sec
14.75	26	32
15.00	41	41
15.25	72	58

We see that at the beginning III was moving faster and accordingly covered a greater distance than II. Condensation II soon overtook III owing to its larger $(1-\mu)$. The resulting figure was investigated in fuller detail by S. V. Orlov²¹ based on Kopff's measurements.²² The agreement between the computed and observed values of v is quite satisfactory. The mechanical theory of cometary forms can well explain the resulting shape of the tail without resorting to the hypothesis of a resisting medium in space or currents in the ether.

The results obtained here are hardly in favor of the Bessel-Bredichin theory of cometary forms. The values of $(1-\mu)$ and of g are not only much larger than ever admitted by Bredichin, but also there is no correlation between their values as is required by Bredichin's theory. According to Bredichin, the formation of the tail is intimately connected with the existence of envelopes. The envelopes are formed by continuous emission of matter from the nucleus on the sunward side. Different matter is ejected with different g 's forming successive envelopes in such a way that to each value of $(1-\mu)$ corresponds its own value of g . $(1-\mu)$ is approximately proportional to g , or conversely. Jaegermann²³ gives the results of Bredichin's investigations of more than fifty comets, which were divided by Bredichin into three types (Table 9):

TABLE 9

	First type	Second type	Third type
$1-\mu$	18	from 2.2 to 0.5	from 0.3 to 0
g (mean).....	6.5 km/sec	1.5	0.4

It is seen from Table 7 that in our case we have the converse relation, namely, the smaller values of g correspond to the larger values of $(1-\mu)$. We might derive a larger value of g for condensations I and II than that for condensations III and IV assuming only that I and II were ejected considerably later than III and IV, by at least 0^d5 . This would be in flat contra-

¹⁷ *Mon. Not., R.A.S.*, **69**, 624, 1909.

¹⁸ *Publ. Russ. Astroph. Obs.*, **1**, 231, 1922.

¹⁹ *A.N.*, **180**, 1, 1909.

²⁰ *Mem. de l'Acad. des Sci. de St. Pétersbourg*, **16**, No. 12.

²¹ *Bull. de l'Acad. Imp. des Sci. de St. Pétersbourg*, No. 18, p. 1800, 1916.

²² *Loc. cit.*

²³ Bredichin's *Mechanische Untersuchungen über Cometenformen*, p. 392.

diction with the computed orbits for the condensations as well as with the direct observation on October 14. There seems to be no escape from the conclusion that these condensations belong to the same isochrone, according to Bredichin's terminology (*Isochronengebilde*). Being ejected almost directly away from the Sun they never formed any envelopes around the nucleus. It may be added here that in this comet, as well as in other comets, notably Halley's, studied by the present writer, the CO^+ spectral images do not show the envelopes in contrast with the radiations of cyanogen and Swan.

The straight stem or streamer connecting the condensations with the nucleus was found to be exactly on the prolongation of the radius vector on all three days of observation. Measurements by Miss E. Glancy²⁴ show that even at a considerable distance from the nucleus (on the average 9°) the tail hardly deviated from the radius vector. This is what we might expect from the direction of the initial impulse and large $(1-\mu)$ as derived in this investigation for the condensations. This would show that the condensations did not differ essentially in their behavior from the rest of the matter in the tail, which was confirmed by the spectral analysis of the tail. Apparently the whole tail was formed in the same manner, that is by ejection of matter almost in the direction of the prolonged radius vector.

In such a case the very idea of explosion is not applicable. There was apparently a continuous emission of matter into the tail and the condensations were only more than usually copious emission. After this abundant emission the matter in the nucleus was spent to a considerable degree which was evidenced by a faint and slender tail until a new supply was formed.

The case under investigation gives us an idea of the magnitude of the internal forces in the comets. If the matter ejected on October 14 did not consist of molecules of CO^+ but of solid particles subject to Newtonian gravitation, the resulting orbit would have been a hyperbola concave to the Sun and a new comet would have been formed. Bredichin²⁵ pointed this out as a possible cause of the disruption of comets, which in many instances cannot be explained by the action of the planets. In the case of Comet 1899 I (Swift) he found²⁶ that the secondary nucleus, which afterward formed an independent comet, was expelled from the

main body with $g=12.5$ km/sec and $G=180^\circ$, that is under almost exactly the same conditions as condensations I and II in the present study.

We derive from this investigation one more surprising fact. Matter, in this case CO^+ , may show the same spectrum but possess quite different physical properties and experience different repulsive action of the Sun, which would mean some sort of isotope effect. This is in direct connection with recent observations²⁷ on comets having tails of different Bredichin types but showing the same spectrum for both tails.

SUMMARY

1. The motion of five condensations in the tail of Morehouse's comet on October 14, 15, 16 and 17 has been investigated. The resulting orbits are hyperbolas convex to the Sun. The values of the repulsive force of the Sun fall into two groups:

$$(1-\mu)=151 \text{ and } (1-\mu)=88.$$

2. The first value is exactly what has been found by Baade and Pauli for the CO^+ molecule under the radiation pressure of the Sun.

3. The second value cannot possibly be explained by the inclination of the orbits of the second group to that of the first group.

4. There exists a simple relation between these values: $7:4$, as both numbers are approximate multiples of 22. This is in agreement with the results of S. V. Orlov for this comet in general.

5. The computed orbits and the velocities derived therefrom well explain the observed motion of the condensations and the deformation of the tail without the aid of additional hypotheses, such as, for instance, a resisting medium in space.

6. The moment of ejection of the matter was approximately October 14.500, with the direction of impulse almost directly away from the Sun and initial velocities which fall into two groups: $g=11$ km/sec, and $g=21$ km/sec, corresponding to $(1-\mu)=151$ and $(1-\mu)=88$.

7. The results obtained here are not in agreement with the accepted ideas of the formation of cometary tails.

²⁷ Baldet, *loc. cit.*; Bobrovnikoff, *loc. cit.*

²⁴ *Lick Obs. Bull.*, No. 161, 1909.

²⁵ *Études sur l'Origine des Météores Cosmiques, etc.*, St. Petersburg, 1903, p. 222.

²⁶ *Bull. de l'Acad. Imp. des Sci. de St. Pétersbourg*, 1901, p. 483.

LICK OBSERVATORY, February 14, 1928.

Issued March 28, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 399

ELEMENTS AND EPHEMERIS OF COMET *a* 1928 (REINMUTH)

BY

L. BERMAN and F. L. WHIPPLE

Comet *a* 1928 was detected on a Heidelberg plate of February 22 by Dr. K. Reinmuth. It has been described by some observers as having a round coma with a "starlike condensation." The estimated magni-

tude at discovery was 12.5. The comet has since become fainter, later estimates giving the magnitude as 13-14.

The following positions have been received:

1928 U. T. †	α (1928.0)	δ (1928.0)	Observer	Authority
(1) Jan. 29.10757†	9 ^h 23 ^m 35. ^s 60	+17°41'19".1	Reinmuth, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(2) Feb. 22.96160	9 15 07.45	+21 44 54.7	Reinmuth, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(3) Feb. 23.96368	9 14 54.93	+21 52 39	Mündler, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(4) Feb. 24.82611	9 14 45.61	+21 59 02.3	Chofardet, Besancon	Circ. B. C. A. No. 191, 1928
(5) Feb. 24.95063	9 14 43.60	+22 00 00.1	Mündler, Heidelberg—phot.	Beob. Zirk. No. 9, 1928
(6) Feb. 24.95736	9 14 42.2	+22 00 06	G. Struve, Berlin—Babelsberg	Beob. Zirk. No. 9, 1928
(7) Feb. 25.07288	9 14 42.97	+22 00 46.4	Van Biesbroeck, Yerkes—40", visual	Letter
(8) Feb. 25.93181	9 14 33.93	+22 06 57	G. Struve, Berlin—Babelsberg	Beob. Zirk. No. 9, 1928
(9) Feb. 25.95344	9 14 33.79	+22 07 06.1	Mündler, Heidelberg—phot.	Beob. Zirk. No. 10, 1928
(10) Feb. 26.17611*	9 14 31.2	+22 08 37	Crawford, Lick—12", visual	Telephone
(11) Feb. 26.18323	9 14 31.94	+22 08 44.2	Van Biesbroeck, Yerkes—40", visual	Letter
(12) Feb. 26.89764	9 14 26.0	+22 13 28	Volta, Turin	Circ. B. C. A. No. 191, 1928
(13) Feb. 26.99978	9 14 24.82	+22 14 19.5	Mündler, Heidelberg—phot.	Beob. Zirk. No. 10, 1928
(14) Feb. 27.01899	9 14 24.60	+22 14 26.7	G. Struve, Berlin—Babelsberg	Circ. B. C. A. No. 193, 1928
(15) Feb. 27.02935	9 14 24.87	+22 14 27.3	Chofardet, Besancon	Circ. B. C. A. No. 193, 1928
(16) Feb. 27.17776	9 14 23.75	+22 15 31.1	Van Biesbroeck, Yerkes—40", visual	Letter
(17) Feb. 28.00729**	9 14 18.6	+22 21 03	Bianchi, Milan	Circ. B. C. A. No. 192, 1928
(18) Feb. 28.08804	9 14 17.09	+22 21 24.5	Mündler, Heidelberg—phot.	Beob. Zirk. No. 10, 1928
(19) Feb. 28.19653*	9 14 18.96	+22 22 19.5	Crawford, Lick—12", visual	Telephone
(20) Feb. 28.92540	9 14 12.81	+22 26 40.7	G. Struve, Berlin—Babelsberg	Circ. B. C. A. No. 193, 1928
(21) Mar. 1.32824	9 14 07.40	+22 35 18.2	Van Biesbroeck, Yerkes—40", visual	Letter
(22) Mar. 2.19064	9 14 06.65	+22 40 11.2	Van Biesbroeck, Yerkes—40", visual	Letter
(23) Mar. 15.19074	9 16 22.41	+23 24 04.4	Van Biesbroeck, Yerkes—40", visual	Letter
(24) Mar. 15.27569	9 16 24.16	+23 29 13.5	Crawford, Lick—Crossley, phot.	Letter
(25) Mar. 17.35833	9 17 14.68	+23 32 35.4	Crawford, Lick—Crossley, phot.	Letter

* Position reported as unsatisfactory on account of unfavorable conditions.

† Found later on a plate taken before discovery.

** Apparent place.

A preliminary elliptic orbit was computed from the first three complete observations received, viz., (3) of February 23.96, (7) of February 25.07, and (10) of February 26.18. The last was reported as doubtful.

ELEMENTS, 1928.0

$T=1928 \text{ Feb. } 27.24 \text{ U. T.}$

$\omega = 18^\circ 43'7''$

$\Omega = 125 \ 44.8$

$i = 10 \ 55.9$

$e = 0.74$

$a = 8.6403$

$q = 2.269$

$P = 25.4 \text{ years}$

O-C	(3)	(7)	(10)
$\Delta\alpha$	+5"	0"	0"
$\Delta\delta$	0	0	-1

The elements and a short ephemeris were distributed on *Harvard College Observatory Card No. 57*.

An inspection of the critical determinant

$$N = (a')^3 \tan \delta - a'' (\tan \delta)' + a' (\tan \delta)''$$

revealed that the dominating term, $a''(\tan \delta)'$, had only the first figure fairly well determined and that the resulting geocentric distance might be expected to be uncertain by 50%.

On receipt of the complete Heidelberg positions, (1) of January 29.11 and (2) of February 22.96 by telegram, and of (19) of February 28.20 by telephone, a shift to (2) as new middle place was made preparatory to a differential correction. The residuals to be removed were the following:

O-C	$\Delta\alpha$	$\Delta\delta$
(1) Jan. 29	+14' 3".8	-39' 40".9
(19) Feb. 28	+68.9	+13.0

The practically complete removal of the above residuals was effected in one differential correction by the use of Leuschner's approximate formulae¹.

ELEMENTS, 1928.0

$T=1928 \text{ Feb. } 13.78 \text{ U. T.}$

$\omega = 16^\circ 00' 01''$

$\Omega = 124 \ 15 \ 39$

$i = 7 \ 17 \ 39$

$e = 0.4742$

$a = 3.4084$

$q = 1.7920$

$P = 6.29$

O-C	(1)	(19)
$\Delta\alpha$	+2".1	-0".6
$\Delta\delta$	-5.9	+0.8

On account of the doubtful character of the third place no further reduction of the residuals was attempted.

¹ *Publ. Lick Obs.* 7, 266, 1913.

This orbit and an ephemeris were distributed on *Harvard College Observatory Announcement Cards Nos. 59 and 60*. While the elements so obtained on the 30 day arc suggested identity with Taylor's comet, 1916 I, the application of Tisserand's criterion now renders the identity doubtful.

On receipt of (24) of March 15 by Crawford, this observation was substituted in place of the approximate position (19) for a further differential correction of the orbit. The residuals to be removed were as follows:

O-C	$\Delta\alpha$	$\Delta\delta$
(1) Jan. 29	+ 2".1	- 5".9
(24) Mar. 15	-389.5	+90.6

ELEMENTS, 1928.0

$T=1928 \text{ Jan. } 31.0727 \text{ U. T.}$

$\omega = 8^\circ 39' 05".3$

$\Omega = 124 \ 56 \ 05.8$

$i = 8 \ 03 \ 45.1$

$a = 3.74157$

$e = 0.5028$

$q = 1.86036$

$P = 7.237$

CONSTANTS FOR THE EQUATOR, 1928.0

$x = r[9.997110] \sin (223^\circ 51' 15".1 + v)$

$y = r[9.976493] \sin (136 \ 05 \ 49.7 + v)$

$z = r[9.531892] \sin (115 \ 12 \ 09.9 + v)$

EPHEMERIS 1928.0

1928 U. T.	α	δ	$\log \rho$
Mar. 19.0	9 ^h 18 ^m 01. ^s 63	+23°34'24".8	0.0264
Mar. 27.0	9 22 52.71	+23 33 11.1	0.0541
April 4.0	9 29 31.00	+23 16 21.2	0.0830
April 12.0	9 37 40.91	+22 45 46.4	0.1124
April 20.0	9 47 07.32	+22 03 08.2	0.1418
April 28.0	9 57 36.36	+21 09 58.9	0.1710
May 6.0	10 08 53.86	+20 07 51.5	0.1995
May 14.0	10 20 47.36	+20 58 10.7	0.2273
May 22.0	10 33 07.93	+17 42 06.3	0.2543
May 30.0	10 45 48.42	+16 20 44.5	0.2803

These elements are in substantial agreement with those obtained by Crommelin² and by Ebell³.

The complete representation, with the omission of the approximate positions, is as follows:

1928 U. T.	$\Delta\alpha$	$\Delta\delta$
(1) Jan. 29.11*	0".0	0".0
(2) Feb. 22.96*	0.0	0.0
(3) Feb. 23.96	+1	+3
(4) Feb. 24.83	+3.2	+2.4
(5) Feb. 24.95	-1.6	+5.4
(7) Feb. 25.07	+2.2	-1.9
(8) Feb. 25.93	-1	+1
(9) Feb. 25.95	+1.0	+0.8
(11) Feb. 26.18	+4.2	+1.4
(12) Feb. 26.90	+2	-9

² *B.A.A. Circular No. 62*, 1928.

³ *Beob. Zirk. No. 10*, 1928.

1928 U. T.	$\Delta\alpha$	$\Delta\delta$
(13) Feb. 27.00	+1".0	+2".0
(14) Feb. 27.02	+0.8	+2.1
(15) Feb. 27.03	+6.2	-2.3
(16) Feb. 27.18	+3.1	+0.6
(17) Feb. 28.01	0	+6
(18) Feb. 28.09	-0.4	-3.2
(20) Feb. 28.93	-2.5	-6.2
(21) Mar. 1.33	-3.5	+2.8
(22) Mar. 2.19	+1.5	-1.1
(23) Mar. 15.19	+3.0	+1.3
(24) Mar. 15.28*	+0.3	+0.1
(25) Mar. 17.36	+2.5	+0.2

The starred observations are those on which the solution has been based.

Two subsequent observations received by letter from Van Biesbroeck as the bulletin was going to press are represented as follows:

1928 U. T.	$\alpha(1928.0)$	$\delta(1928.0)$	$\Delta\alpha$	$\Delta\delta$
(26) Mar. 21.17665	9 ^h 19 ^m 08. ^s 68	+23°35'44".8	+6".7	+1".9
(27) Mar. 24.18553	9 20 56.01	+23 35 28.3	-1.9	-0.5

BERKELEY ASTRONOMICAL DEPARTMENT, March 30, 1928.

Issued April 12, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 400

THE RADIAL VELOCITY ORBIT OF α DRACONIS

BY

R. T. CRAWFORD

 α Draconis, Type F8

$$\alpha(1900) = 18^h 22^m 9^s : \delta(1900) = +72^\circ 41'$$

The calendar date of the first observation is 1898
July 25, and of the last observation 1899 Oct. 16.

The investigation of the radial velocity orbit of this
star is based upon the following observations:

SERIES I (1898-1899)

Obs. No.	Date J.D. 2414	Wt.	Vel. (observed)	Vel. ¹ (computed) _i	O-C Δ Vel.=v
1	496.79	1	+45.3	+44.3	+1.0
2	538.68	1	+45.7	+46.0	-0.3
3	552.68	$\frac{2}{3}$	+42.4	+43.5	-1.1
4	589.89	1	+14.2	+14.4	-0.2
5	595.65	$\frac{2}{3}$	+11.6	+11.6	0.0
6	599.60	$\frac{2}{3}$	+11.0	+10.7	+0.3
7	606.59	$\frac{2}{3}$	+10.4	+10.7	-0.3
8	612.59	$\frac{2}{3}$	+10.4	+11.7	-1.3
9	631.59	$\frac{2}{3}$	+18.1	+17.1	+1.0
10	641.10	$\frac{2}{3}$	+20.8	+19.9	+0.9
11	641.59	$\frac{2}{3}$	+19.8	+20.0	-0.2
12	693.09	1	+32.1	+32.0	+0.1
13	708.95	1	+34.0	+34.8	-0.8
14	721.01	1	+36.5	+36.8	-0.3
15	749.03	1	+41.0	+40.9	+0.1
16	754.03	1	+40.5	+41.5	-1.0
17	776.90	1	+44.6	+44.3	+0.3
18	804.89	$\frac{2}{3}$	+47.0	+46.3	+0.7
19	813.98	1	+46.2	+46.3	-0.1
20	820.95	1	+46.2	+45.9	+0.3
21	833.92	$\frac{2}{3}$	+42.4	+43.3	-0.9
22	841.74	1	+41.3	+39.9	+1.4
23	847.87	1	+35.6	+35.8	-0.2
24	852.88	1	+31.0	+31.4	-0.4
25	867.82	1	+17.1	+16.5	+0.6
26	890.75	1	+10.6	+11.2	-0.6
27	909.72	$\frac{2}{3}$	+17.2	+16.3	+0.9
28	944.71	$\frac{2}{3}$	+26.2	+25.9	+0.3

$$\Sigma pvv = 10.8$$

¹ The source of the computed velocities is explained later
in the text.

SERIES II (1922-1923)

Obs. No.	Date J.D. 2423	Vel. (observed)	Vel. ¹ (computed) _{iv}	(O-C) Δ Vel.=v
29	188.00	+43.3	+44.6	-1.3
30	245.78	+43.8	+44.6	-0.8
31	250.71	+42.1	+43.4	-1.3
32	257.72	+39.2	+40.8	-1.6
33	278.74	+25.5	+24.9	+0.6
34	295.76	+12.8	+13.1	-0.3
35	305.68	+11.4	+12.5	-1.1
36	306.67	+12.2	+12.6	-0.4
37	312.72	+14.2	+13.8	+0.4
38	320.73	+15.7	+16.2	-0.5
39	348.64	+24.5	+24.6	-0.1
40	388.59	+35.0	+33.6	+1.4
41	428.70	+39.5	+40.0	-0.5
42	472.05	+46.2	+44.9	+1.3
43	484.04	+44.0	+45.9	-1.9
44	496.03	+46.1	+46.5	-0.4
45	508.00	+48.3	+46.5	+1.8
46	522.00	+47.9	+45.4	+2.5
47	530.00	+44.6	+43.8	+0.8
48	532.89	+42.9	+42.9	0.0
49	550.91	+32.8	+32.7	+0.1
50	577.92	+12.8	+12.7	+0.1
51	595.81	+16.8	+14.5	+2.3
52	604.94	+16.6	+17.3	-0.7
53	628.74	+23.8	+24.4	-0.6

$$\Sigma vv = 32.8$$

The calendar date of the first observation of this
series is 1922 May 12, and of the last observation
1923 July 27.

SERIES III (1928)

Obs. No.	Date J.D. 2425	Vel. (observed)	Vel. ¹ (computed) _{iv}	(O-C) Δ Vel.=v
54	287.05	+16.6	+16.6	0.0
55	325.05	+27.7	+27.6	+0.1
56	345.94	+31.9	+32.2	-0.3

The calendar date of observation No. 54 is 1928
Feb. 10, and of No. 56 April 9.

The velocities are expressed in kilometers per second as usual. Hereafter the "per second" is omitted in writing the velocities.

The observed velocities tabulated above include all of the known systematic corrections² to render the observations homogeneous. It will be noted that all of the observed velocities in Series I are systematically smaller, by 0.25 km., than the values given by Wright³ in his investigation of this orbit. Corrections for flexure (amounting to 0.2 km. in the maximum) are available for the observations of Series II and III. These have not been applied, however, as the flexure corrections for the observations of Series I are unknown, and it is desired here to have all of the observations as nearly homogeneous as possible. This flexure correction is +0.2 km. for the observations numbered 29, 33, 34, 35, 36, 37, 38, 39, 40, 50, and 52. It is -0.2 km. for the other observations of Series II. It is -0.15 km. for the observations of Series III.

The weights assigned in Series I are those given by Wright³ for reasons set forth in his determination of the orbit based upon this series of observations. All of the observations of the other two series are considered to be of weight unity.

In the interval between, and including, the first two series of observations (1898-1923) the binary system completed thirty-two revolutions, so that material is now available for a more accurate determination of the period. The steps taken in this investigation are:

1.—A differential correction of Wright's orbit to represent the observations of Series II. This gives a first approximation to the periastron passage time for the year 1923, which is hereafter designated T_{II} .

2.—Evaluation of a second approximation to the period, P , (Wright's $P=281^d.8$ being the first approximation) by $P = \frac{T_{II} - T_I}{32}$ where T_{II} is that of Step 1 and

and T_I is the periastron passage time of 1898 (the first approximation for which is that from Wright's elements).

The interval between periastron passage times is used for the determination of P since T is quite accurately determined in an orbit of such eccentricity as this (here $e > 0.4$). P thus determined is, of course, the anomalistic period.

3.—With the P of Step 2 constant, a differential correction of Wright's other elements to represent the observations of Series I for the purpose of obtaining an improved T_I on the basis of this new P .

4.—With the P of Step 2 constant, a differential correction of the other elements resulting from Step 1 to represent the observations of Series II for the purpose of obtaining an improved T_{II} on the basis of this P .

² *Publ. Lick Obs.*, 16, xxi, 1928.

³ *Ap.J.*, 11, 131, 1900.

5.—The final value of P obtained as in Step 2, using now the values of T_I and T_{II} resulting from Steps 3 and 4 respectively.

6.—Representation of the observations of Series III.

The differential correction of Step 1 resulted in $T_{II} = \text{J. D. } 2423565.8$. From Wright's orbit $T_I = \text{J. D. } 2414582.5$. Hence (Step 2)

$$P = \frac{8983^d.3}{32} = 280^d.728$$

Step 3 resulted in the following set of elements:

ELEMENTS I

$$\begin{aligned} T &= \text{J.D. } 2414583.682 \pm 0^d.665 \\ K &= 17.97 \text{ km.} \pm 0.13 \text{ km.} \\ e &= 0.428 \pm 0.006 \\ \omega &= 119^\circ.41 \pm 1^\circ.23 \\ V &= +32.21 \text{ km.} \pm 0.22 \text{ km.} \\ P &= 280^d.728 \end{aligned}$$

$r_o = \pm 0.46$ km. (probable error of a single observation of weight unity)

$$\Sigma pvv = 10.8$$

It is from these elements that the "computed velocities" given in the table of observations of Series I have been derived.

(It will be noted that this Σpvv is larger than Wright's by an amount greater than would be expected as the elements are made to differ only because of the imposition of $P = 280^d.728$. An error was found in his residual for observation No. 22. Instead of +0.5, as published, it should be +1.7. Making this correction his Σpvv becomes 9.7 instead of 7.1.)

Step 4 resulted in the following set of elements:

ELEMENTS II

$$\begin{aligned} T &= \text{J.D. } 2423565.521 \pm 1^d.288 \\ K &= 17.17 \text{ km.} \pm 0.23 \text{ km.} \\ e &= 0.419 \pm 0.012 \\ \omega &= 126^\circ.03 \pm 2^\circ.38 \\ V &= +33.64 \text{ km.} \pm 0.38 \text{ km.} \\ P &= 280^d.728 \\ r_o &= \pm 0.85 \text{ km.} : \Sigma vv = 32.2 \end{aligned}$$

The final value of the anomalistic period resulting from this investigation is therefore (Step 5)

$$P = \frac{8981^d.839}{32} = 280^d.683 \pm 0^d.045$$

The differences between Elements I which represent the 1898-99 observations and Elements II which represent the 1922-23 observations seem to be too large to be accidental in a case such as this, where the probable error of a single observation of weight unity

is less than one kilometer. To test this a third set of elements was obtained by taking the weighted mean of Elements I and II with the period $P=280^d683$. This gives

ELEMENTS III

$$\begin{aligned} K &= 17.79 \text{ km.} \\ e &= 0.426 \\ \omega &= 120^{\circ}80 \\ V &= +32.57 \text{ km.} \\ P &= 280^d683 \end{aligned}$$

The periastron passage times are those of Elements I and Elements II. The observations of the two series are not at all satisfactorily represented by this set of elements. It was then considered that a satisfactory representation might be obtained by using these elements except V , and deriving a separate V for each series to render Σpv or Σv for each series equal zero. This resulted in $V = +32.31$ km. for the 1898-99 series, and $V = +33.27$ km. for the 1922-23 series. This procedure gave a fair representation of the first series of observations (Σpvv being 13.7 an increase of 2.9 over the $\Sigma pvv = 10.8$ for Elements I). But the observations of Series II were very poorly represented (Σvv being 66.4, more than twice that for the representation by Elements II, viz. $\Sigma vv = 32.8$). A least square solution to represent all of the observations undoubtedly would yield elements but little different from Elements III.

The evidence to date shows that, *unless there is some unknown systematic difference between the observations of*

Series I and those of Series II, all of the observations cannot be represented satisfactorily by one set of elements. This means that *the system of X Draconis consists of more than two bodies.*

The differences between Elements I and Elements II are not necessarily due entirely to perturbations by a third body, but are probably due, in large part, to changing values of $\sin i$ arising from the changing inclination as the binary system swings about the center of gravity of the three (or more) body system. After another interval of twenty to twenty-five years the question of the possible multiple character of the system can be settled definitely.

At this epoch (1928) the best set of elements is that of Elements II with $P = 280^d683$ instead of $P = 280^d728$, i. e.

ELEMENTS IV

$$\begin{aligned} T &= \text{J.D. } 2423565.521 \\ K &= 17.17 \text{ km.} \\ e &= 0.419 \\ \omega &= 126^{\circ}03 \\ V &= +33.64 \text{ km.} \\ P &= 280^d683 \\ a \sin i &= 60,170,000 \text{ km.} \end{aligned}$$

It is from these Elements IV that the "computed velocities" tabulated with the observations of Series II and Series III were obtained. It will be noted that the Series III (1928) observations are most satisfactorily represented (Step 6).

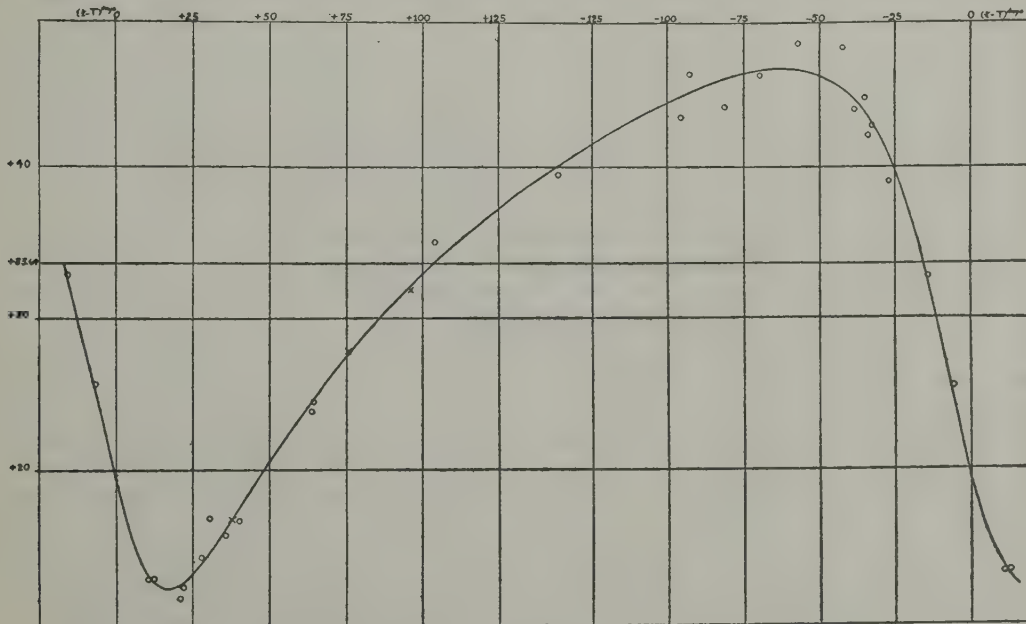


Figure 1. Graph of Elements IV, X Draconis.

The accompanying figure is the graph representing Elements IV with the observed velocities of Series II and Series III represented by the symbols $\circ$ and $\times$ respectively.

It will be instructive to see how observations made from now on are represented by these elements. For this purpose the following ephemeris has been com-

puted. This gives $\frac{dz}{dt}$, the velocity referred to the center of mass of the binary system, as a function of $(t-T)$, the interval of time from periastron passage.

Then $\text{Vel.} = \frac{dz}{dt} + V.$

EPHEMERIS FOR VELOCITIES REFERRED TO THE CENTER OF GRAVITY OF THE BINARY SYSTEM

(Computed from Elements IV)

$(t-T)$ days	$\frac{dz}{dt}$ km	$(t-T)$ days	$\frac{dz}{dt}$ km	$(t-T)$ days	$\frac{dz}{dt}$ km	$(t-T)$ days	$\frac{dz}{dt}$ km
-140.0	+ 5.92	-70.0	+12.80	0.0	-14.33	+ 70.0	-7.40
-137.5	+ 6.26	-67.5	+12.87	+ 2.5	-16.28	+ 72.5	-6.77
-135.0	+ 6.60	-65.0	+12.92	+ 5.0	-17.96	+ 75.0	-6.15
-132.5	+ 6.93	-62.5	+12.94	+ 7.5	-19.30	+ 77.5	-5.55
-130.0	+ 7.26	-60.0	+12.93	+10.0	-20.30	+ 80.0	-4.96
-127.5	+ 7.58	-57.5	+12.88	+12.5	-20.96	+ 82.5	-4.39
-125.0	+ 7.90	-55.0	+12.80	+15.0	-21.32	+ 85.0	-3.83
-122.5	+ 8.21	-52.5	+12.67	+17.5	-21.40	+ 87.5	-3.28
-120.0	+ 8.51	-50.0	+12.50	+20.0	-21.26	+ 90.0	-2.75
-117.5	+ 8.81	-47.5	+12.27	+22.5	-20.94	+ 92.5	-2.23
-115.0	+ 9.10	-45.0	+11.98	+25.0	-20.47	+ 95.0	-1.72
-112.5	+ 9.38	-42.5	+11.62	+27.5	-19.90	+ 97.5	-1.22
-110.0	+ 9.66	-40.0	+11.17	+30.0	-19.24	+100.0	-0.74
-107.5	+ 9.93	-37.5	+10.63	+32.5	-18.53	+102.5	-0.27
-105.0	+10.19	-35.0	+ 9.99	+35.0	-17.77	+105.0	+0.20
-102.5	+10.44	-32.5	+ 9.23	+37.5	-16.99	+107.5	+0.65
-100.0	+10.69	-30.0	+ 8.33	+40.0	-16.20	+110.0	+1.10
- 97.5	+10.93	-27.5	+ 7.28	+42.5	-15.40	+112.5	+1.53
- 95.0	+11.16	-25.0	+ 6.08	+45.0	-14.61	+115.0	+1.96
- 92.5	+11.38	-22.5	+ 4.70	+47.5	-13.82	+117.5	+2.38
- 90.0	+11.59	-20.0	+ 3.14	+50.0	-13.05	+120.0	+2.79
- 87.5	+11.79	-17.5	+ 1.38	+52.5	-12.29	+122.5	+3.19
- 85.0	+11.98	-15.0	- 0.58	+55.0	-11.54	+125.0	+3.59
- 82.5	+12.16	-12.5	- 2.74	+57.5	-10.81	+127.5	+3.98
- 80.0	+12.32	-10.0	- 5.03	+60.0	-10.09	+130.0	+4.36
- 77.5	+12.46	- 7.5	- 7.41	+62.5	- 9.39	+132.5	+4.73
- 75.0	+12.59	- 5.0	- 9.80	+65.0	- 8.71	+135.0	+5.10
- 72.5	+12.70	- 2.5	-12.14	+67.5	- 8.05	+137.5	+5.46
- 70.0	+12.80	0.0	-14.33	+70.0	- 7.40	+140.0	+5.82

Maximum = +12.94 at $(t-T) = -62^d12$

Minimum = -21.40 at $(t-T) = +17.13$

$\frac{dz}{dt} = 0$ at $\begin{cases} (t-T) = -15^d71 \text{ and} \\ (t-T) = +103.93 \end{cases}$

LICK OBSERVATORY, April 16, 1928.

Issued May 12, 1928.

LICK OBSERVATORY BULLETIN

NUMBER 401

FURTHER PHOTOMETRIC MEASURES OF JUPITER'S SATELLITES
AND URANUS, WITH TESTS OF THE SOLAR CONSTANT

BY

JOEL STEBBINS and T. S. JACOBSEN

In the summer of 1927 a second series of photometric measures of the satellites of *Jupiter* was undertaken in continuation of the work of 1926.¹ Mr. Stebbins arrived at Mount Hamilton on July 18, and after several days of preparation observations were made on every clear night until his departure on August 23, after which the work was continued by Mr. Jacobsen until the last of September. During this season it was convenient also to observe *Uranus* which was within a degree or so of *Jupiter*, the dates of opposition being September 22 for *Jupiter* and September 25 for *Uranus*.

The instrumental conditions were practically the same as in 1926, the only change in the photo-electric photometer being the introduction of bright-wire illumination for the guiding eye-piece. There was also added a diagonal prism for the telescope finder, and a small motor for the slow motion in right ascension. These improvements made the observing go somewhat faster, but otherwise the sensitivity and effectiveness of the installation were about the same. The diameter of the focal diaphragm was erroneously recorded in 1926 as 90" instead of 70" which was used in both seasons at Mount Hamilton. The larger diameter is the one in current use at Madison.

In 1927 it was not possible to find comparison stars quite as good as those of the previous season; the number was reduced to three, and these averaged somewhat fainter than the satellites, but were quite near

the apparent magnitude of *Uranus*. The data from the Harvard catalogues are as follows:

TABLE I

COMPARISON STARS, 1927

	R. A. 1900	Decl. 1900	Visual magnitude	Photo- electric magnitude	Spectrum
20 Piscium	23 ^h 42 ^m 8	-3°19'	5 ^m 60	6 ^m 30	K0
27 Piscium	23 53.6	-4 7	5.07	5.77	K0
44 Piscium	0 20.3	+1 23	5.99	6.54	G5

The tentative photo-electric magnitudes were derived by adding the color-indices to the visual magnitudes, and the mean of the three, magnitude 6.20, was used as standard in subsequent reductions.

After the completion of the present series, it was decided to measure the color of all of the objects, and this was done with the photometer at Madison. By using a suitable pair of glasses, violet and yellow, a potassium cell gives accurate color-indices for stars not fainter than the sixth magnitude. For the colored glasses and cell concerned the following is the calibration from twenty-five stars:

Spectrum	Color-Index
F0	-0 ^m 46
F5	- .32
G0	- .16
G5	+ .01
K0	+ .20

On the basis of this calibration the equivalent spectra for the objects observed in 1927 were determined on five or more nights each as follows:

¹ *Lick Obs. Bull.*, 13, 1, 1927.

Satellite I	G9	} Mean=G5
II	G5	
III	G4	
IV	G3	
Uranus	F9	} Mean=G5
20 Piscium	G6	
27 Piscium	G6	
44 Piscium	G4	

Assuming the calibration to be correct, the probable error of the adopted spectrum for each object is about 1/30 of the interval G0 to K0, except for Satellite I where the probable error is 1/10 of the same interval.

The value of the extinction factor in the expression $0^m20 f \sec z$ was determined again in 1927 from a graph of the individual magnitudes of the comparison stars, 20 and 44 *Piscium*, and the resulting value, $f=1.0$, was used for all of the objects except *Uranus*,

where, because of the slightly earlier spectrum, the factor $f=1.05$ was adopted. In 1926 the mean value used was about $f=0.9$, but an exact value was not necessary; all of the measures were differential and the comparison stars were well distributed about the planet. Since the total extinction at the zenith distance concerned is about 0^m3 , a change of 0.05 in the factor for *Uranus* produces a difference of 0^m015 in the resulting mean magnitude.

In Table II the observations are in the same form as in the preceding year. The day begins at Greenwich midnight. The angle α is the jovicentric elongation of the Earth from the Sun, or what we call the solar phase of the satellites. All magnitudes are referred to the mean of the comparison stars, and have already been corrected for atmospheric extinction.

TABLE II

JOURNAL OF OBSERVATIONS										
		Satellites				Comparison stars				Remarks
1927 G. C. T.	α	I	II	III	IV	Uranus	20 Psc	27 Psc	44 Psc	
July 23.428	-10°59			(5 ^m 991)	(6 ^m 789)	(6 ^m 308)	(6 ^m 325)	(5 ^m 724)	(6 ^m 550)	Poor, reject.
.462	-10. 59			(5. 986)	(6. 737)	6. 331	6. 351	5. 722	6. 528	Large D. C.
24.412	-10. 50				6. 754	6. 351	6. 366	5. 706	6. 530	Large D. C.
.464	-10. 50	6 ^m 187	6 ^m 346	5. 606	6. 736	6. 310	6. 331	5. 732	6. 538	Large D. C.
25.425	-10. 41	6. 263			6. 795	6. 316	6. 333	5. 732	6. 535	
.469	-10. 41	6. 260			6. 768	6. 343	6. 339	5. 719	6. 546	
26.408	-10. 32	6. 068	6. 064	6. 092	6. 786	6. 330	6. 333	5. 742	6. 528	I and II close.
.454	-10. 31	6. 038	6. 040	6. 082		6. 329	6. 339	5. 734	6. 531	III and IV close.
27.399	-10. 22			6. 076		6. 340	6. 328	5. 734	6. 539	
.425	-10. 22			6. 058		6. 313	6. 336	5. 725	6. 536	
.469	-10. 21	6. 238		6. 071		6. 322	6. 336	5. 709	6. 554	
28.401						6. 327	6. 343	5. 736	6. 523	
.417						6. 322	6. 333	5. 736	6. 528	
.446						6. 319	6. 339	5. 732	6. 531	
29.415	-10. 01			5. 916	6. 800	6. 330	6. 344	5. 744	6. 512	
.467	-10. 01			5. 884	6. 783	6. 323	6. 328	5. 724	6. 547	
.501						6. 311	6. 329	5. 742	6. 528	
30.398	-9. 91			5. 930	6. 790	6. 318	6. 345	5. 716	6. 536	
.441	-9. 90			5. 913	6. 793	6. 317	6. 343	5. 726	6. 528	
.475						6. 323	6. 324	5. 725	6. 548	
31.401	-9. 79		6. 292	5. 984	6. 790	6. 319	6. 341	5. 729	6. 529	
.444	-9. 79		6. 315	5. 987	6. 788	6. 314	6. 348	5. 716	6. 534	
.477						6. 315	6. 341	5. 723	6. 532	
Aug. 1.394	-9. 67				6. 809	6. 315	6. 325	5. 722	6. 551	
.419	-9. 67				6. 796	6. 316	6. 335	5. 734	6. 533	
.444						6. 320	6. 338	5. 724	6. 542	
2.410	-9. 55	5. 992	5. 986	6. 019	6. 804	6. 307	6. 334	5. 727	6. 537	
.458	-9. 55	6. 014	5. 971	6. 026	6. 805	6. 331	6. 333	5. 724	6. 541	
.490						6. 308	6. 329	5. 722	6. 545	
3.394	-9. 43			6. 013	6. 811	6. 319	6. 340	5. 722	6. 538	
.433	-9. 43			6. 001	6. 801	6. 313	6. 341	5. 719	6. 538	
.465						6. 310	6. 331	5. 711	6. 560	
5.390	-9. 18	6. 097	5. 961			6. 326	6. 335	5. 732	6. 530	I poor.
.458						6. 300	6. 331	5. 725	6. 540	
.481						6. 307	6. 339	5. 714	6. 545	
6.382	-9. 05			5. 854	6. 680	6. 307	6. 331	5. 719	6. 551	
7.401	-8. 91		6. 225	5. 924	6. 639	6. 311	6. 338	5. 722	6. 543	
.450	-8. 91		6. 230	5. 916	6. 619	6. 303	6. 332	5. 718	6. 547	
.483						6. 314	6. 333	5. 726	6. 540	
8.380	-8. 78				6. 592	6. 308	6. 334	5. 726	6. 538	
.416	-8. 77				6. 597	6. 298	6. 326	5. 734	6. 538	
.440						6. 308	6. 323	5. 732	6. 547	

TABLE II—(Continued)

1927 G. C. T.	α	Satellites				Uranus	Comparison stars			Remarks
		I	II	III	IV		20 Psc	27 Psc	44 Psc	
Aug. 9.381	-8°64	5 ^m 979	5 ^m 944	5 ^m 963	6 ^m 600	6 ^m 320	6 ^m 337	5 ^m 724	6 ^m 540	
.437	-8.63	6.002	5.927	5.943	6.599	6.302	6.333	5.732	6.534	
.470						6.310	6.332	5.722	6.548	
10.370	-8.49	6.058		5.954	6.598	6.307	6.324	5.709	6.567	
.431	-8.48			5.937	6.587	6.298	6.323	5.722	6.554	
.464						6.301	6.337	5.713	6.550	
11.379	-8.34	5.932	6.199	5.918	6.636	6.304	6.329	5.712	6.555	IV poor.
.434	-8.33	5.907		5.890	6.587	6.282	6.323	5.724	6.554	
.469						6.300	6.329	5.722	6.548	
14.412	-7.88		6.135	5.835	6.609	6.289	6.340	5.732	6.529	Bright Moon.
.457						6.282	6.332	5.717	6.548	Bright Moon.
15.353	-7.73			5.854	6.592	6.282	6.327	5.729	6.543	Bright Moon.
.409	-7.72				6.644	6.290	6.332	5.732	6.537	Bright Moon.
.444						6.279	6.337	5.725	6.533	Bright Moon.
16.338	-7.57		5.853	5.895	6.629	6.298	6.340	5.706	6.556	
.384	-7.56		5.886	5.912	6.637	6.303	6.344	5.719	6.536	
.415						6.286	6.330	5.716	6.554	
17.331	-7.40			5.880	6.627	6.292	6.341	5.716	6.543	
.399	-7.39	6.015		5.878	6.641	6.306	6.342	5.719	6.539	
.429						6.305	6.334	5.723	6.545	
18.340	-7.23	5.836	6.131	5.842	6.636	6.281	6.330	5.724	6.545	
.393	7.22	5.825	6.144	5.841	6.615	6.288	6.333	5.716	6.548	
.427						6.282	6.328	5.721	6.548	
19.267	-7.07				(6.676)	(6.295)	(6.356)	(5.722)	(6.524)	Extinction test.
.294	-7.07				6.656	6.287	6.337	5.719	6.544	
.330	-7.06	6.001			6.634	6.290	6.340	5.716	6.546	
.372	-7.06	6.054	5.869		6.622	6.282	6.330	5.726	6.544	
.401						6.294	6.337	5.716	6.548	
20.329	-6.89	5.832	5.859	5.715	6.621	6.277	6.337	5.719	6.546	I and II close.
.379	-6.88		5.826	5.704	6.580	6.270	6.326	5.719	6.553	I and II close.
.411						6.292	6.337	5.715	6.544	
21.358	-6.71	6.005	6.074	5.774	6.562	6.299	6.339	5.730	6.531	I poor.
.400	-6.70		6.059	5.762	6.526	6.284	6.329	5.729	6.540	
.427						6.296	6.343	5.715	6.543	
22.326	-6.53			5.818		6.280	6.327	5.719	6.551	
.375	-6.52			5.820		6.284	6.336	5.706	6.555	
.403						6.288	6.333	5.719	6.547	
23.323	-6.35		5.822	5.833	6.488	6.276	6.330	5.729	6.539	
.364	-6.34		5.816	5.854	6.490	6.283	6.329	5.729	6.545	
.397						6.294	6.336	5.712	6.551	
24.317	-6.17			5.838	6.473	6.281	6.328	5.724	6.546	
.352	-6.17			5.810	6.445	6.264	6.338	5.726	6.535	
.378						6.270	6.330	5.736	6.534	
26.328	-5.81	5.946			6.448	6.274	6.338	5.719	6.539	
.365	-5.80	5.957			6.448	6.296	6.331	5.719	6.553	
.392						6.278	6.338	5.722	6.544	
29.344	-5.22			5.752	(6.481)	6.272	6.333	5.716	6.552	IV reject.
Sept. 2.299	-4.42				6.414	6.256	6.330	5.726	6.546	
.353	-4.41	5.816			6.416	6.269	6.323	5.743	6.538	
.385						6.280	6.324	5.721	6.552	
5.290	-3.80	5.654	6.000	5.686	6.426	6.271	6.327	5.716	6.559	
.347	-3.79		5.976	5.691	6.422	6.285	6.338	5.719	6.547	
.392						6.272	6.333	5.725	6.544	
6.278	-3.59	5.751	5.740		6.392	6.268	6.331	5.719	6.550	I incomplete.
10.296	-2.73	5.633	5.711	5.563	6.262	6.266	6.320	5.734	6.545	
.350	-2.72	5.635			6.261	6.272	6.343	5.732	6.529	
.387						6.259	6.330	5.726	6.546	
19.265	-0.80	(5.577)	5.935	5.548	6.138	6.269	6.346	5.704	6.551	I reject.
.326	-0.78	5.504	5.926	5.537	6.126	6.262	6.351	5.704	6.545	
.365						6.249	6.339	5.714	6.545	
20.237	-0.61	5.666			6.103	6.262	6.336	5.716	6.550	
.278	-0.60	5.649			6.111	6.268	6.335	5.709	6.552	
.310						6.269	6.340	5.716	6.542	
21.228	-0.44		5.652	5.617	6.103	6.261	6.330	5.722	6.551	

TABLE II--(Continued)

1927 G. C. T.	α	Satellites				Comparison stars				Remarks
		I	II	III	IV	Uranus	20 Psc	27 Psc	44 Psc	
Sept. 21.281	-0°43		5 ^m 635	5 ^m 600	6 ^m 103	6 ^m 271	6 ^m 331	5 ^m 714	6 ^m 555	
.318						6.263	6.328	5.714	6.559	
22.281	-0.33		5.819	5.573	6.079	6.266	6.352	5.714	6.534	
.327						6.272	6.341	5.705	6.550	
23.232	+0.36			5.556	6.067	6.265	6.339	5.716	6.545	
.291	+0.37			5.552	6.087	6.268	6.333	5.719	6.551	
.324						6.267	6.335	5.713	6.549	
24.217	+0.51		5.577			6.256	6.341	5.716	6.540	
.250	+0.51		5.636			6.266	6.336	5.714	6.549	
.278	+0.52		5.641			6.251	6.330	5.719	6.549	
25.215	+0.67			5.474	6.131	6.258	6.333	5.706	6.557	
.269	+0.70			5.472	6.100	6.264	6.323	5.712	6.568	
.304						6.263	6.331	5.722	6.550	
26.224	+0.90	5 ^m 562	5.955	5.536	6.164	6.271	6.343	5.719	6.540	
.282	+0.91	5.551	5.929	5.532	6.130	6.276	6.333	5.726	6.538	
.319						6.280	6.334	5.715	6.548	
27.224	+1.11	5.686			6.172	6.254	6.330	5.719	6.554	IV poor.
.260						6.265	6.327	5.724	6.549	U. incomplete.
28.225						6.258	6.339	5.704	6.558	
29.214	+1.54			5.643	6.200	6.262	6.343	5.706	6.547	
.260	+1.55		5.849	5.645	6.191	6.267	6.337	5.712	6.552	
.297						6.264	6.328	5.722	6.549	

THE SATELLITES

The reduction to mean opposition of *Jupiter* was computed as before.

TABLE III

REDUCTION TO MEAN OPPOSITION, JUPITER

1927	July	20	-0 ^m 041
		28	+ .014
	Aug.	5	+ .065
		13	+ .112
		21	+ .154
		29	+ .188
	Sept.	6	+ .213
		14	+ .229
		22	+ .235
		30	+ .230

In Table IV the results for each satellite are brought together; the date in the first column identifies the corresponding observation in Table II. The second column contains the orbital phase computed from the time of superior conjunction. Each magnitude in the third column is obtained from the one in Table II by applying the proper correction from Table III; for Satellite III, the correction of 0^m472 has been applied for the sector used. The fourth column contains the correction for solar phase as it was determined in 1927. The reduced magnitudes in the fifth column are derived from the two preceding. The residuals in the sixth column are based upon the final light-curves, and in the seventh column these residuals are averaged for each date.

TABLE IV
REDUCED MAGNITUDES

Satellite I							
1927	Orbital phase	At mean opposition	Reduction for α	Reduced magni- tude	Residual	Mean residual	
July 24	46°8	6 ^m 177	-0 ^m 372	5 ^m 805	+0 ^m 036	+0 ^m 036	
25	242.4	6.259	- .370	5.889	+ .023		
25	251.3	6.257	- .370	5.887	+ .009	+ .016	
26	82.4	6.071	- .368	5.703	- .010		
26	91.8	6.041	- .368	5.673	- .035	- .022	
27	298.3	6.248	- .365	5.883	+ .005	+ .005	
Aug. 2	67.4	6.041	- .348	5.693	- .039		
2	77.1	6.063	- .348	5.715	- .003	- .021	
5	313.8	6.165	- .338	5.827	- .037	- .037	
9	46.2	6.071	- .323	5.748	- .023		
9	57.6	6.094	- .322	5.772	+ .024	.000	
10	247.5	6.155	- .318	5.837	- .037	- .037	
11	92.9	6.035	- .314	5.721	+ .014		
11	104.0	6.011	- .314	5.697	- .007	+ .004	
17	238.3	6.151	- .285	5.866	+ .006	+ .006	
18	69.8	5.977	- .280	5.697	- .029		
18	80.6	5.966	- .280	5.686	- .028	- .028	
19	271.4	6.147	- .275	5.872	- .019		
19	280.0	6.200	- .275	5.925	+ .034	+ .008	
20	114.8	5.982	- .269	5.713	+ .006	+ .006	
21	324.4	6.160	- .263	5.897	+ .044	+ .044	
26	256.2	6.123	- .233	5.890	+ .007		
26	263.7	6.134	- .233	5.901	+ .013	+ .010	
Sept. 2	246.4	6.019	- .183	5.836	- .036	- .036	
5	124.4	5.865	- .160	5.705	- .007	- .007	
6	321.9	5.965	- .152	5.813	- .043	- .043	
10	63.7	5.856	- .118	5.738	+ .002		
10	74.7	5.858	- .118	5.740	+ .020	+ .011	
19	90.0	5.811	- .036	(5.775)	(+ .067)		
19	102.4	5.738	- .035	5.703	- .001	- .001	
20	287.9	5.900	- .027	5.873	- .014		
20	296.3	5.883	- .027	5.856	- .024	- .019	
26	67.0	5.795	- .041	5.754	+ .023		
26	78.8	5.784	- .041	5.743	+ .027	+ .025	
27	270.6	5.919	- .050	5.869	- .022	- .022	

TABLE IV—(Continued)

Satellite II						
1927	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
July 24	288°4	6 ^m 336	—0 ^m 190	6 ^m 146	+0 ^m 005	+0 ^m 005
26	125.5	6.067	— .189	5.878	+ .024	
26	130.2	6.043	— .189	5.854	— .004	+ .010
31	271.9	6.328	— .186	6.142	+ .003	
31	276.2	6.351	— .186	6.165	+ .023	+ .013
Aug. 2	115.6	6.035	— .184	5.851	+ .005	
2	120.5	6.020	— .184	5.836	— .012	— .004
5	57.9	6.029	— .181	5.848	+ .001	+ .001
7	261.8	6.305	— .179	6.126	+ .002	
7	266.8	6.310	— .179	6.131	— .002	.000
9	102.7	6.036	— .176	5.860	+ .020	
9	108.4	6.019	— .176	5.843	+ .001	+ .010
11	305.4	6.302	— .174	6.128	+ .010	+ .010
14	253.1	6.255	— .169	6.086	— .014	— .014
16	88.5	5.983	— .164	5.819	— .017	
16	93.1	6.017	— .164	5.853	+ .017	.000
18	291.6	6.272	— .160	6.112	— .025	
18	297.0	6.285	— .160	6.125	— .006	— .016
19	36.3	6.015	— .158	5.857	— .011	— .011
20	133.5	6.009	— .155	5.854	— .007	
20	138.5	5.977	— .155	5.822	— .044	— .026
21	237.9	6.229	— .153	6.076	+ .019	
21	242.2	6.215	— .153	6.062	— .008	+ .006
23	77.3	5.986	— .147	5.839	+ .002	
23	81.5	5.981	— .147	5.834	— .002	.000
Sept. 5	313.4	6.211	— .100	6.111	.000	
5	319.2	6.187	— .100	6.087	.000	.000
6	52.7	5.954	— .096	5.858	+ .008	+ .008
10	101.5	5.934	— .074	5.860	+ .021	+ .021
19	292.0	6.169	— .024	6.145	+ .007	
19	298.2	6.160	— .023	6.137	+ .008	+ .008
21	131.3	5.887	— .013	5.874	+ .015	
21	136.6	5.870	— .013	5.857	— .008	+ .004
22	238.2	6.054	— .010	6.044	— .015	— .015
24	74.7	5.811	— .015	5.796	— .041	
24	78.0	5.870	— .015	5.855	+ .018	
24	80.9	5.875	— .016	5.859	+ .023	.000
26	278.4	6.188	— .027	6.161	+ .018	
26	284.3	6.162	— .027	6.135	— .008	+ .005
29	226.6	6.079	— .045	6.034	+ .006	+ .006

Satellite III

July 23	86°7	5 ^m 502	—0 ^m 268 (5 ^m 234) (—0 ^m 000)	
23	88.4	5.497	— .268 (5.229) (— .008)	(—0 ^m 004)
24	138.8	5.596	— .266	5.330 + .019 + .019
26	236.6	5.623	— .263	5.360 + .004
26	239.0	5.613	— .263	5.350 — .006 — .001
27	286.5	5.614	— .261	5.353 + .016
27	287.8	5.596	— .261	5.335 .000
27	290.0	5.609	— .261	5.348 + .014 + .010
29	28.0	5.467	— .257	5.210 + .007
29	30.6	5.435	— .257	5.178 — .023 — .008
30	77.5	5.488	— .255	5.233 + .009
30	79.6	5.471	— .255	5.216 — .011 — .001
31	128.0	5.548	— .253	5.295 .000
31	130.1	5.551	— .253	5.298 .000
Aug. 2	229.1	5.596	— .248	5.348 — .009
2	231.5	5.603	— .248	5.355 — .002 — .006
3	278.6	5.596	— .246	5.350 + .008
3	280.6	5.585	— .246	5.339 — .001 + .004
6	69.1	5.456	— .238	5.218 + .002 + .002

TABLE IV—(Continued)

1927	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Aug. 7	120°4	5 ^m 532	—0 ^m 235	5 ^m 297	+0 ^m 014	
7	122.9	5.524	— .235	5.289	+ .002	+0 ^m 008
9	220.2	5.583	— .229	5.354	— .004	
9	223.0	5.563	— .229	5.334	— .024	— .014
10	270.0	5.579	— .226	5.353	+ .007	
10	273.1	5.563	— .226	5.337	— .008	.000
11	320.8	5.549	— .223	5.326	+ .020	
11	323.6	5.522	— .223	5.299	— .005	+ .008
14	113.6	5.483	— .213	5.270	.000	.000
15	161.1	5.507	— .210	5.297	— .037	— .037
16	210.7	5.553	— .207	5.346	— .011	
16	213.0	5.571	— .206	5.365	+ .007	— .002
17	260.7	5.543	— .203	5.340	— .010	
17	264.2	5.542	— .203	5.339	— .009	— .010
18	311.6	5.511	— .199	5.312	— .003	
18	314.2	5.510	— .199	5.311	— .001	— .002
20	51.8	5.393	— .191	5.202	.000	
20	54.3	5.383	— .191	5.192	— .012	— .006
21	103.7	5.457	— .187	5.270	+ .014	
21	105.8	5.446	— .187	5.259	.000	+ .007
22	152.5	5.506	— .183	5.323	— .003	
22	154.9	5.508	— .182	5.326	— .002	— .002
23	202.7	5.525	— .178	5.347	— .009	
23	204.8	5.547	— .178	5.369	+ .013	+ .002
24	252.8	5.535	— .174	5.361	+ .009	
24	254.6	5.507	— .174	5.333	— .019	— .005
29	146.3	5.469	— .151	5.318	.000	.000
Sept. 5	136.6	5.425	— .113	5.312	+ .005	
5	139.5	5.430	— .113	5.317	+ .006	+ .006
10	29.1	5.314	— .083	5.231	+ .029	+ .029
19	121.5	5.310	— .026	5.284	.000	
19	124.6	5.299	— .025	5.274	— .014	— .007
21	220.6	5.380	— .014	5.366	+ .008	
21	223.3	5.363	— .014	5.349	— .009	.000
22	273.7	5.336	— .011	5.325	— .019	— .019
23	321.7	5.319	— .011	5.308	+ .002	
23	324.7	5.315	— .012	5.303	+ .001	+ .002
25	61.7	5.236	— .022	5.214	+ .005	
25	64.5	5.234	— .022	5.212	.000	+ .002
26	112.7	5.297	— .029	5.268	— .001	
26	115.6	5.293	— .029	5.264	— .010	— .006
29	263.5	5.402	— .049	5.353	+ .004	
29	265.8	5.403	— .050	5.353	+ .005	+ .004

Satellite IV

July 23	262°9	6 ^m 792	—0 ^m 534 (6 ^m 238) (+0 ^m 026)	
23	263.6	6.720	— .534 (6.186) (— .026)	(0 ^m 000)
24	284.1	6.744	— .532	6.212 — .003
24	285.2	6.726	— .532	6.194 — .022 — .012
25	305.9	6.791	— .530	6.261 + .038
25	306.9	6.765	— .530	6.235 + .011 + .024
26	327.1	6.789	— .528	6.261 + .027 + .027
29	32.0	6.823	— .540	6.283 + .022
29	33.1	6.806	— .540	6.266 + .005 + .014
30	53.3	6.820	— .538	6.282 + .005
30	54.2	6.823	— .538	6.285 + .007 + .006
31	74.9	6.826	— .536	6.290 — .006
31	75.9	6.824	— .536	6.288 — .008 — .007
Aug. 1	96.4	6.852	— .533	6.319 + .004
1	96.9	6.839	— .533	6.306 — .009 — .002
2	118.4	6.853	— .531	6.322 — .007
2	119.4	6.854	— .531	6.323 — .007 — .007

TABLE IV—(Continued)

Satellite IV						
1927	Orbital phase	At mean opposition	Reduction for α	Reduced magnitude	Residual	Mean residual
Aug. 3	139.6	6.866	-0.528	6.338	+0.012	
3	140.5	6.857	-.528	6.329	+ .004	+0.008
6	204.2	6.754	-.499	6.255	+ .024	+ .024
7	226.2	6.719	-.495	6.224	+ .005	
7	227.3	6.699	-.495	6.204	- .014	- .004
8	247.4	6.678	-.492	6.186	- .027	
8	248.1	6.683	-.492	6.191	- .022	- .024
9	269.0	6.692	-.488	6.204	- .008	
9	270.2	6.691	-.488	6.203	- .009	- .008
10	290.4	6.695	-.484	6.211	- .007	
10	291.7	6.685	-.484	6.201	- .017	- .012
11	312.2	6.739	-.480	6.259	+ .033	
11	313.4	6.691	-.480	6.211	- .016	+ .008
14	17.8	6.729	-.487	6.242	- .011	- .011
15	38.1	6.717	-.481	6.236	- .029	
15	39.4	6.769	-.481	6.288	+ .020	- .004
16	59.5	6.759	-.475	6.284	+ .001	
16	60.5	6.768	-.475	6.293	+ .010	+ .006
17	81.0	6.762	-.469	6.293	- .008	
17	82.5	6.777	-.469	6.308	+ .006	- .001
18	102.8	6.777	-.463	6.314	- .006	
18	104.0	6.756	-.463	6.293	- .027	- .016
19	122.9	6.821	-.457	(6.364)	(+ .033)	
19	123.5	6.801	-.457	6.344	+ .012	
19	124.3	6.780	-.457	6.323	- .009	
19	125.2	6.768	-.457	6.311	- .021	- .006
20	145.9	6.771	-.449	6.322	+ .007	
20	147.0	6.731	-.449	6.282	- .030	- .012
21	168.2	6.717	-.442	6.275	+ .006	
21	169.1	6.682	-.442	6.240	- .028	- .011
23	210.8	6.652	-.417	6.235	+ .009	
23	211.7	6.655	-.416	6.239	+ .013	+ .011
24	232.3	6.642	-.410	6.232	+ .016	
24	233.1	6.614	-.410	6.204	- .012	+ .002
26	275.8	6.625	-.396	6.229	+ .016	
26	276.7	6.625	-.396	6.229	+ .016	+ .016
29	341.2	6.670	-.374	(6.296)		
Sept. 2	66.9	6.617	-.327	6.290	+ .001	
2	68.1	6.619	-.327	6.292	+ .002	+ .002
5	131.8	6.637	-.290	6.347	+ .015	
5	133.1	6.633	-.289	6.344	+ .013	+ .014
6	153.3	6.606	-.277	6.329	+ .031	+ .031
10	240.4	6.485	-.262	6.223	+ .009	
10	241.6	6.484	-.261	6.223	+ .009	+ .009
19	75.0	6.372	-.069	6.303	+ .007	
19	76.3	6.360	-.068	6.292	- .005	+ .001
20	96.1	6.337	-.053	6.284	- .030	
20	97.0	6.345	-.052	6.293	- .022	- .026
21	117.6	6.338	-.038	6.300	- .029	
21	118.8	6.338	-.038	6.300	- .030	- .030
22	140.5	6.314	-.030	6.284	- .041	- .041
23	161.1	6.302	-.031	6.271	- .011	
23	162.4	6.322	-.033	6.289	+ .010	.000
25	204.1	6.365	-.152	6.213	- .018	
25	205.3	6.334	-.153	6.181	- .050	- .034
26	226.0	6.397	-.164	6.233	+ .015	
26	227.3	6.363	-.165	6.198	- .020	- .002
27	247.7	6.405	-.176	6.229	+ .016	+ .016
29	290.9	6.431	-.200	6.231	+ .014	
29	291.9	6.421	-.200	6.221	+ .003	+ .008

As before, we begin with the discussion of the comparison stars. Dividing the measures into two series, and taking the means, there is found:

TABLE V				
MAGNITUDES OF COMPARISON STARS				
1927	20 Piscium	27 Piscium	44 Piscium	
July 23–August 23	6 ^m 335	5 ^m 723	6 ^m 542	
P. E. one observation	±0.005	±0.006	±0.006	
August 24–September 29	6.334	5.718	6.547	
P. E. one observation	±0.005	±0.005	±0.005	

There seems to be no difference in the quality of the earlier and later measures, and the average probable error of about 0.005 may be compared with the values ±0.008 and ±0.012 for the two series of the preceding year. The second series of 1926 was taken under poorer conditions, but there is no apparent reason for the improvement of the present work over the first series of 1926 except that the stars were higher in the sky. As before, the mean of the comparison stars is a satisfactory standard of reference.

In the discussion of the variations of each satellite with orbital and solar phase it seems best to redetermine these light-curves on the basis of the present season only, and then to compare the two sets of results, rather than to attempt to make a uniform reduction of both seasons together. However, as a first guess the solar phase effects of 1927 were assumed to be the same as in 1926, and these were then corrected without prejudice in favor of keeping the changes small. After several approximations the following expressions were found, the phase being taken without regard to sign.

$$\begin{aligned}
 \text{Satellite I,} \quad m &= m_0 + 0.046\alpha - 0.0010\alpha^2 \\
 \text{II,} \quad m &= m_0 + 0.0312\alpha - 0.00125\alpha^2 \\
 \text{III,} \quad m &= m_0 + 0.0323\alpha - 0.00066\alpha^2 \\
 \text{IV}_p, \quad m &= m_0 + 0.090\alpha - 0.0036\alpha^2 \\
 \text{IV}_t, \quad m &= m_0 + 0.112 + 0.060\alpha - 0.0019\alpha^2
 \end{aligned} \tag{A}$$

The expression in (A) for Satellite I was previously incomplete, as that satellite had not been measured near opposition; the expressions for II' and III are about as before; but it is in the case of IV that the new measures have shown a complication. There is a difference between the solar phase effects on the two sides of this satellite, given in (A) for the preceding and following sides in the orbital motion as IV_p and IV_t respectively. At opposition Satellite IV brightens up much more on the front side, the difference being over a tenth of a magnitude. On consideration it is evident that for each position of a satellite in its orbit there will be a different solar phase effect. We always see the same side of our Moon when it is full, but it is not so simple for the satellites of *Jupiter*. In fact, with the combination of orbital- and solar phase variations the wonder is that we can get mean light-curves at all.

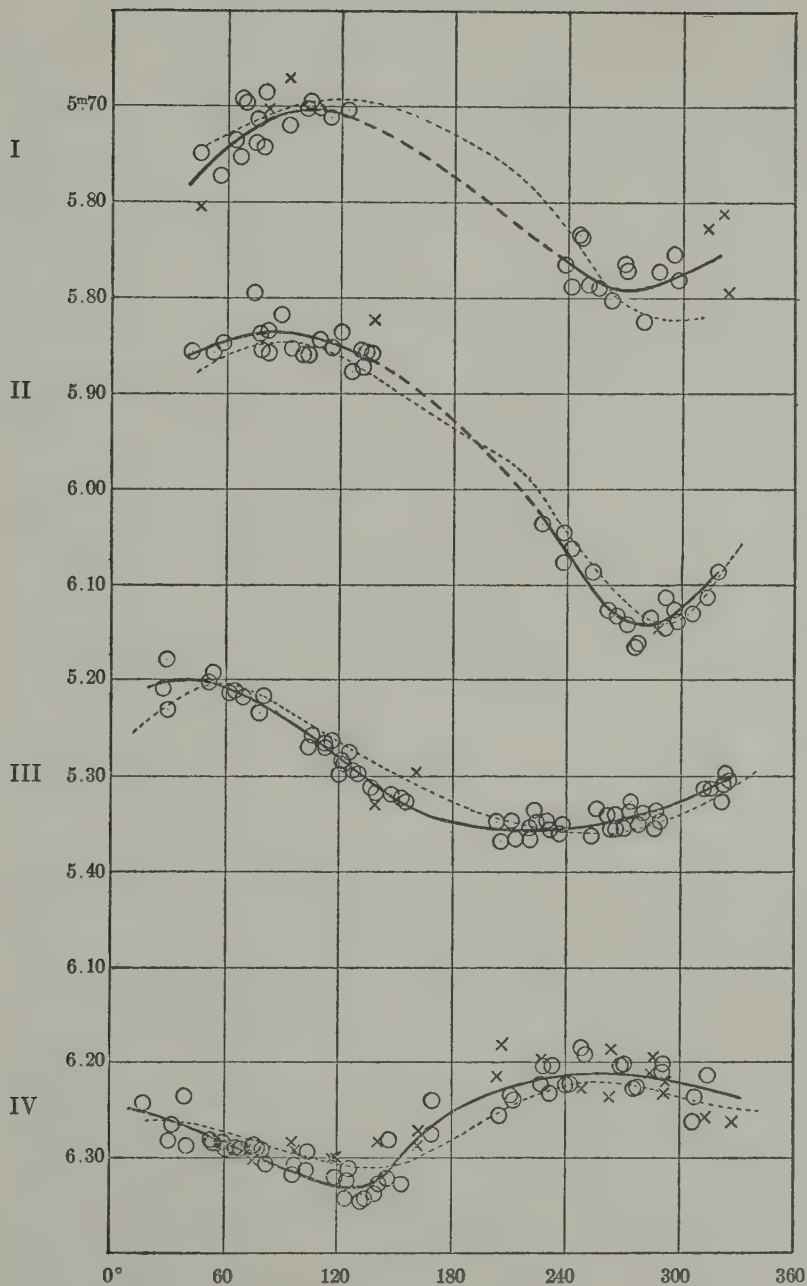


Fig. 1. Variations of Jupiter's Satellites with Orbital Phase.

Full curves, 1927; dotted curves, 1926. Crosses indicate poor measures, or those near opposition for Satellite IV.

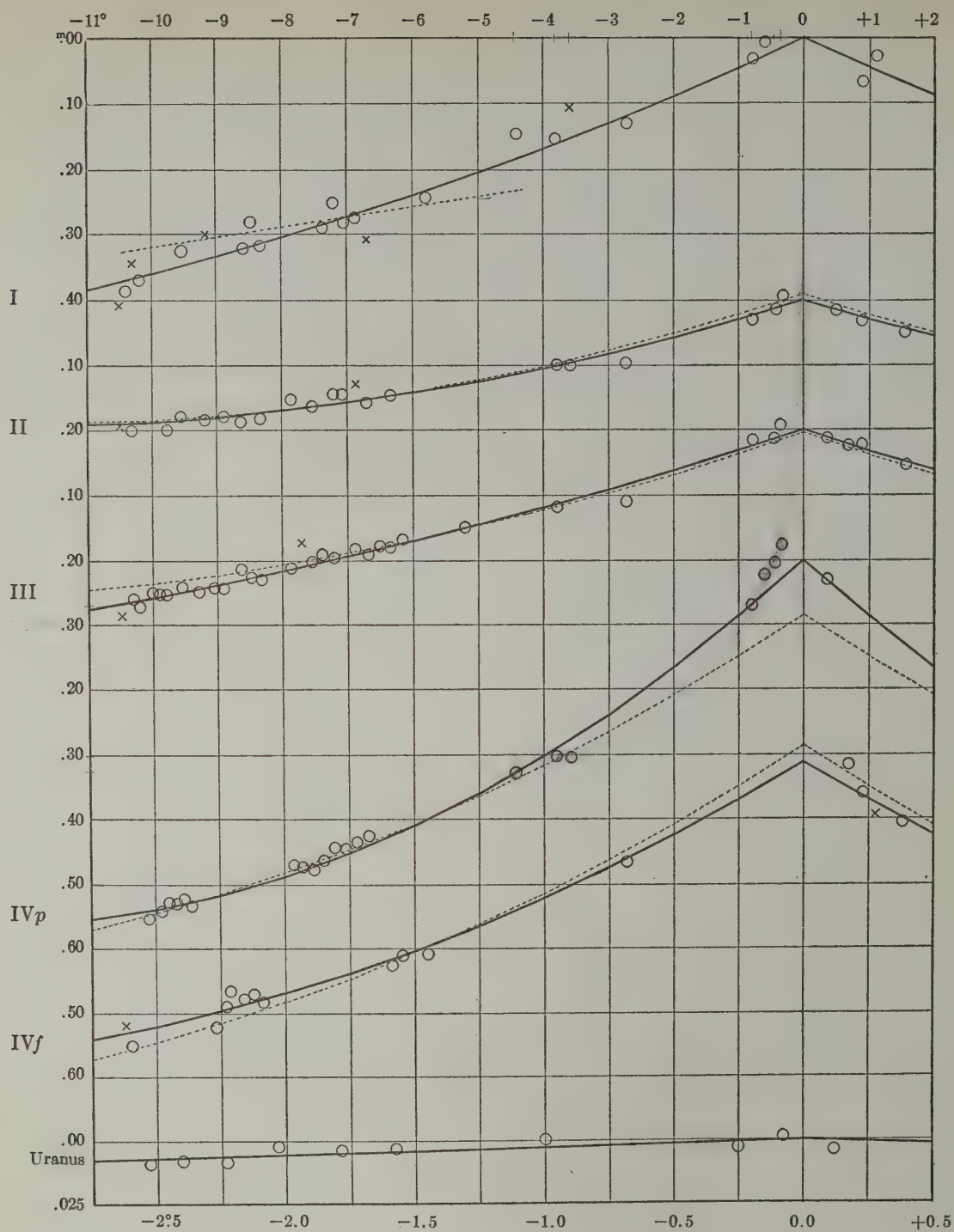


Fig. 2. Variations of Jupiter's Satellites and of Uranus with Solar Phase.
 Full curves, 1927; dotted curves, 1926. Crosses indicate poor measures. The scale for Uranus
 is four times that for the satellites.

Fortunately this difference between two sides is much less for the inner three satellites and the expressions in (A) are sufficient. A test is given by the average of the residuals for small solar phase, say less than 1° , from which we obtain the following:

	I	II	III	IV
Preceding side brightens more than following side.....	-0 ^m .037	-0 ^m .004	+0 ^m .002	+0 ^m .123

It is seen that the differences are negligible for II and III, while the effect for I is opposite in sign to that for IV. It is hopeless to attempt to distinguish between presentations of the satellites other than for the two sides as adopted. This dependence of the solar phase variation upon the orbital position of a satellite was missed in 1926 when there were no good observations near opposition.

The variations with solar phase are shown in Fig. 2, where the curves are the graphs of equations (A), while the plotted magnitudes are derived from the mean residuals of Table IV. For comparison the curves of 1926 are also indicated, the phase α being reversed

in sign. The curves for the two seasons are made to intersect at $\alpha=7^\circ$ for Satellite I, and at 6° for the others. Since the geocentric latitude of *Jupiter* was not zero at opposition, the minimum value of α was $0^\circ.32$, and in all strictness the solar phase cannot be represented by a light-curve in two dimensions. The agreement between the two seasons is satisfactory, and there is apparently not much difference in the light of any satellite at the same angular phase before or after opposition.

The orbital light-curves were derived from the reduced magnitudes in the fifth column of Table IV, and are given in Table VI. The mean magnitude for each satellite was determined by taking the averages of the magnitudes in Table VI for which the differences are printed, these being at symmetrical phases where the curves were pretty well determined in both seasons. To compare the two seasons, we must allow for the difference in the solar phase corrections, and this is best done by selecting as before a phase near the middle of both series, $\alpha=7^\circ$ for Satellite I and 6° for the others. The comparison is then made as in Table VII, which gives also the ranges of variation with orbital phase.

TABLE VI
ORBITAL LIGHT-CURVES OF JUPITER'S SATELLITES

Phase	I			II			III			IV		
	1926	1927	Diff.	1926	1927	Diff.	1926	1927	Diff.	1926	1927	Diff.
20°							5 ^m .257			6 ^m .348	6 ^m .254	
30							5.241	5 ^m .202		6.349	6.259	-0 ^m .090
40					5 ^m .864		5.228	5.200	-0 ^m .028	6.350	6.267	-.083
50	5 ^m .910	5 ^m .764		5 ^m .876	5.853	-0 ^m .023	5.220	5.202	-.018	6.353	6.275	-.078
60	5.900	5.744	-0 ^m .156	5.863	5.845	-.018	5.219	5.208	-.011	6.358	6.284	-.074
70	5.890	5.726	-.164	5.854	5.840	-.014	5.224	5.216	-.008	6.364	6.292	-.072
80	5.882	5.715	-.167	5.850	5.836	-.014	5.233	5.228	-.005	6.370	6.300	-.070
90	5.874	5.708	-.166	5.848	5.836	-.012	5.244	5.237	-.007	6.376	6.309	-.067
100	5.868	5.704	-.164	5.850	5.839	-.011	5.256	5.252	-.004	6.382	6.318	-.064
110	5.864	5.705	-.159	5.854	5.843	-.011	5.266	5.264	-.002	6.388	6.326	-.062
120	5.862	5.709	-.153	5.862	5.848	-.014	5.279	5.284	+ .005	6.393	6.331	-.062
130	5.863			5.874	5.856	-.018	5.291	5.298	+ .007	6.396	6.332	-.064
140				5.888	5.869		5.302	5.312	+ .010	6.396	6.325	-.071
150							5.312	5.323		6.394	6.304	-.090
160							5.323	5.334		6.387	6.284	
170							5.332	5.342		6.378	6.266	
180							5.341	5.348		6.367	6.252	
190							5.348	5.353		6.356	6.242	
200							5.355	5.356		6.344	6.234	
210							5.360	5.357		6.332	6.227	-.105
220				5.992			5.365	5.358	-.007	6.322	6.221	-.101
230				6.016	6.036	+ .020	5.369	5.357	-.012	6.314	6.217	-.097
240	5.994	5.864	-.130	6.048	6.064	+ .016	5.372	5.356	-.016	6.309	6.214	-.095
250	6.024	5.876	-.148	6.075	6.096	+ .021	5.373	5.353	-.020	6.307	6.212	-.095
260	6.052	5.886	-.166	6.098	6.120	+ .022	5.373	5.350	-.023	6.307	6.212	-.095
270	6.073	5.891	-.182	6.120	6.137	+ .017	5.372	5.346	-.026	6.309	6.212	-.097
280	6.084	5.891	-.193	6.136	6.144	+ .008	5.368	5.341	-.027	6.312	6.214	-.098
290	6.090	5.885	-.205	6.142	6.140	-.002	5.362	5.334	-.028	6.317	6.217	-.100
300	6.091	5.877	-.214	6.139	6.127	-.012	5.355	5.326	-.029	6.323	6.221	-.102
310	6.089	5.868		6.120	6.108	-.012	5.346	5.317	-.029	6.328	6.226	-.102
320		5.858		6.092	6.086		5.334	5.307	-.027	6.333	6.231	-.102
330				6.060			5.323			6.336	6.236	-.100
340							5.310			6.339		

TABLE VII

COMPARISON OF MEAN MAGNITUDES

	I	II	III	IV
Range, 1926.....	0 ^m 229	0 ^m 294	0 ^m 155	0 ^m 090
Range, 1927.....	0. 187	0. 308	0. 158	0. 120
Mean magnitude, 1926.....	5.967	5.979	5.307	6.347
Correction for α	-0. 165	+0.009	-0.004	-0.084
Corrected magnitude, 1926.....	5.802	5.988	5.303	6.263
Mean magnitude, 1927.....	5.798	5.976	5.293	6.261
1927-1926.....	-0.004	-0.012	-0.010	-0.002
Residual.....	+0.003	-0.005	-0.003	+0.005

The small average difference of -0^m007 between seasons represents simply the chance agreement of the adopted magnitudes of the two sets of comparison stars based upon the Harvard visual magnitudes. It did not seem worth while to inter-compare these stars photo-electrically as they are some thirty degrees apart in the sky, and, to eliminate the extinction, measures must be made with both groups at a low altitude. A single measure of two of the stars, one in each group, made the satellites 0^m015 brighter in 1927 than in 1926, but this is of no significance.

The orbital light-curves are shown in Fig. 2 where the dotted curves of 1926 are reduced to the same mean magnitudes as those of 1927. The curves of the second season were drawn quite independently of the previous curves, and no effort was made to find either agreement or disagreement between the two seasons. The observations of Satellite I are now numerous enough to show the inferior quality of measures close to the planet, and the curves for that satellite are in poorest agreement. No real differences seem to be established between the curves for each satellite, either in the orbital variations in Fig 1, or in the solar phase variations in Fig. 2. Our photo-electric results therefore do not give weight to any assumption of meteorological changes on the surfaces of these bodies.

The method of reduction of the observations of 1926 has been severely criticized by Guthnick,² and the following paragraphs have been written by Mr. Stebbins in explanation of the points on which there is disagreement.

The chief criticism by Guthnick is that my allowance for atmospheric extinction was too small; that the factor by which the visual extinction was multiplied instead of being about 0.9, which I used for Mount Hamilton, should have been about 1.5, which he uses at Babelsberg. He says there is no reason to anticipate that a mountain station would have a clearer sky than a station near sea level. As it is incredible that anyone could advance this idea seriously it would seem that there must be some misunderstanding, so I quote the

original.³ " auch die Meereshöhe des Beobachtungsortes wird, wenn sie nicht sehr bedeutend ist, keinen wesentlichen Einfluss auf die Extinktionsfaktoren haben. Ob die Extinktionsfaktoren mit zunehmender Meereshöhe ab- oder zunehmen, ist von vornherein nicht zu sagen." The only way I can make sense out of this is to assume that he means that a high-altitude station may be no better than a lower station several thousand miles away. However, one glance at the Mount Hamilton sky by day or by night would convince Guthnick that he is mistaken.

The amount of the atmospheric extinction at the Lick Observatory is not a matter of opinion or conjecture; it can be measured and stated in figures. Adopting the expression $0^m20 f \sec z$, for the total extinction, the factor f varies with the photo-electric cell, with the spectral class of the stars measured, and, in all strictness, with the nightly or even hourly change in the transparency of the air. The data upon which I base my adoption of a factor smaller than Guthnick's are as follows:

(1) The determination in 1926 from two comparison stars taken on many nights, $f=0.88$ for spectrum G0. Guthnick's rejection of this value will be discussed in the sequel.

(2) A similar determination in 1927, $f=1.0$ for spectrum G5. This is the value used in the present paper.

(3) A determination made in 1915 on fifteen nights,⁴ $f=1.0\pm0.024$ (p.e.) for stars of spectrum A0 measured with a rubidium cell.

(4) The long series of measures by Abbot⁵ at Mount Wilson and Washington, D. C. Taking his results for $\lambda=4600$ A, the approximate region of maximum sensitivity of a potassium cell, there follows:

	Mount Wilson	Washington
Transmission at zenith.....	0.84	0.65
Corresponding extinction in magnitude	0.19	0.47
Factor, f	0.95	2.35

The factor for Washington is two and one-half times that at Mount Wilson, a quantitative measure of the superiority of the California sky. Anyone familiar with both Mount Wilson and Mount Hamilton knows that a good sky at either station is better than a fair sky at the other, and it is not stretching matters to put Babelsberg in a class with Washington rather than with Mount Hamilton.

Despite the foregoing evidence, Guthnick has reduced my 1926 observations of *Jupiter's* satellites all over again on the basis of the extinction at Babelsberg.

² *loc. cit.*, p. 116.

⁴ *Lick Obs. Bull.*, 8, 187, 1916.

⁵ *The Sun*, p. 297.

² *Sitzungsberichte der Preussischen Akademie*, 1927, p. 112.

To begin with, he objects to my determination of the factors because the extreme range of zenith distance of the stars used was only from 51° to 64°, giving a range of extinction of 0.122 magnitude. Here he overlooks the fact that if the range of the extinction of the comparison stars is small, that of the satellites is still smaller, and any errors in the differential extinction must practically vanish. Nevertheless, he carries the work through and concludes on the basis of his improved extinction corrections that he has diminished the average deviations of the observations of the satellites from the light-curves by as much as 30 per cent. I shall show that this is an illusion.

The magnitude of a satellite as used for a light-curve is dependent not only upon the correction for extinction but also upon the large correction for solar phase. If we compare the first observation of Satellite I in my Table IV and his Table I, my reduced magnitude is 5^m.921 against his 5^m.858. The discrepancy is 0^m.063, but the difference in our corrections for solar phase is 0^m.069, leaving a remainder of 0^m.006 due to extinction. The average of all the similar values for Satellite I is 0^m.005, and it is only deviations from this constant difference that affect the accordance of the results. The net difference of 0^m.001, due to extinction, applied with the proper sign to my first residual changes it from +0^m.013 to +0^m.014. In the same way, applying these net differences as corrections to the residuals in my Table IV, there follow:

	I	II	III	IV
Average deviation one observation, extinction S.....	±0 ^m .0064	±0 ^m .0128	±0 ^m .0113	±0 ^m .0110
Average net difference of extinction, S-G.....	±0.0014	±0.0026	±0.0028	±0.0034
Average deviation one observation, extinction G.....	±0.0060	±0.0132	±0.0122	±0.0114

Instead of diminishing my residuals, the effect of changing to Guthnick's extinction is to increase them.

The improvement which Guthnick claims, was secured not by his revised extinction corrections but by his use of free-hand curves to represent the solar phase effect, together with his injection of spurious dips (Einsenkungen) in the maxima of the orbital curves of three satellites. I, too, could have made the residuals smaller, but I was anxious not to introduce bias in testing the solar constant, and I represented the solar phase effect by a second-degree expression, leaving some outstanding errors, which, however, did not affect the orbital curves or the tests of the solar constant. The extra minima which Guthnick puts into my curves are based largely upon measures which I had labelled as of inferior precision.

Another test of the extinction is given by the comparison stars, where there is no question of light-curves or of solar phase effects; the residuals are formed from

simple means. Taking the series 1926 August 14 to September 14 in my Table V, upon which were based my light-curves, we have for the four comparison stars:

	(1)	(2)	(3)	(4)
Probable error one observation, extinction S	±0 ^m .0093	±0 ^m .0066	±0 ^m .0078	±0 ^m .0070
Probable error one observation, extinction G	±0.0100	±0.0070	±0.0079	±0.0075

Here again there is no improvement with the use of Guthnick's extinction.

A minor point which Guthnick makes is that it is not permissible to use the same extinction factor for all four satellites, especially as he can see in the telescope the progressive difference in color from Satellites I to IV, ranging from reddish (rötlich) to blue-gray (blau-grau) respectively. I simply bunched the lot and called them all G0 like the Sun, whereas Guthnick guesses a spectrum for Satellite II and determines the others differentially from their photo-electric and visual magnitudes, including also a manufactured solar phase correction for Satellite I. In this way he gets colors equivalent to spectra of F1 to G5, a much larger range than I have measured in 1927, namely G3 to G9. My preference is naturally for the latter values.

If Guthnick's criticisms of my photo-electric results are demonstrably in error, he is still more difficult to follow in his conclusions from his own visual observations. He exhibits⁶ a set of wavy curves for the satellites, which he contrasts with the relatively smooth photo-electric curves. The accuracy which he secures is indicated by the residuals of his normal points, and from the four graphs I find the following:

	I	II	III	IV
Average deviation one normal.....	±0 ^m .008	±0 ^m .013	±0 ^m .006	±0 ^m .019

Anyone familiar with stellar photometry will recognize this as an extraordinary precision in visual work, especially for Satellites I and III. But why are the results so discordant for Satellite IV? Guthnick's procedure is to combine groups of several observations into normal magnitudes and then to draw curves through the normals. The reason for the larger residuals for Satellite IV is simply that he has drawn a smoother curve than for the other satellites. By making the curve for IV about as wavy as that for III the residuals would all be zero.

We can test the accordance to be expected in these normals by going back to his original measures,⁷ where he gives the mean error of an observation as derived from the individual residuals. I assume that his mean error (m.F.) is the same as what I call the average deviation, since the mean error in its technical sense cannot be determined from the residuals from a free-hand curve. Collecting his data we have:

⁶ *loc. cit.*, pp. 131-134.

⁷ *Sitzungsberichte der Preussischen Akademie*, 1907, p. 344.

	I	II	III	IV
Number of single observations.....	155	210	113	107
Number of normals.....	43	39	30	24
Mean error of one observation, according to Guthnick.....	$\pm 0^m.081$	$\pm 0^m.085$	$\pm 0^m.075$	$\pm 0^m.095$
Mean error of a normal, computed.....	± 0.043	± 0.037	± 0.039	± 0.045
Average deviation of a normal, scaled from curve.....	± 0.008	± 0.013	± 0.006	± 0.019

Comparing the last two quantities in each column it is evident that by drawing wavy curves Guthnick has cut down the average deviation of a normal, and thus has increased the apparent weight of his results up to as much as forty-fold. This fictitious accuracy is likely to be misleading even to the observer himself, and when he compares two of his curves of the same satellite, derived in different seasons, he seems to think that the differences are real and due to changes on the surface of the satellite. There is no need of dodging the issue: most of the fluctuations in Guthnick's curves, and the differences which he gets between seasons, look to me like errors of observation.

To summarize, I am unable to accept a single one of Guthnick's improvements in the reduction of my photo-electric measures. His claim that the sky in the neighborhood of Berlin will serve as a standard of excellence the world over certainly needs modification, and in my judgment his visual observations of the satellites of *Jupiter*, taken twenty years ago, are not of an accuracy on which to base many conclusions.

Perhaps this is the place to state also that I fear that not much can be got from the photographic series by Schütte in *Astronomische Nachrichten*, 218, 273, 1923. He found a range in the orbital period of more than a whole magnitude for every one of the satellites, or up to twelve times the variation shown by the photo-electric observations.

URANUS

The measures of *Uranus* in Table II have been reduced in the same manner as those of the satellites of *Jupiter*. The reduction to mean opposition is given in Table VIII.

TABLE VIII

REDUCTION TO MEAN OPPOSITION, URANUS

1927 July 20	$-0^m.268$
28	$-.255$
Aug. 5	$-.243$
13	$-.232$
21	$-.222$
29	$-.214$
Sept. 6	$-.208$
14	$-.204$
22	$-.202$
30	$-.202$

In Table IX are the magnitudes of *Uranus*. The date in the first column identifies the observation in Table II. In the second column is the solar phase α . The rotational phase in the third column is computed from the elements:

$$t_0 = 1928 \text{ July } 23.000 \text{ G.C.T.} + 0^d.451 \times E.$$

The magnitude in the fourth column is derived from the magnitude in Table II by applying the correction in Table VIII. The residuals in the fifth column are based upon the magnitudes in the preceding column and equation (B); and the mean residual for each night is given in the last column.

TABLE IX

MAGNITUDES OF URANUS

1927	α	Rotational phase	At mean opposition	Residual	Mean residual
July 23	$-2^s.60$	0.428	(6 ^m .046)	(+0 ^m .022)	
23	-2.60	0.011	6.069	+ .001	+0 ^m .001
24	-2.58	0.059	6.090	+ .022	
24	-2.58	0.111	6.049	- .019	+ .002
25	-2.56	0.170	6.057	- .011	
25	-2.56	0.214	6.084	+ .016	+ .002
26	-2.53	0.251	6.073	+ .005	
26	-2.53	0.297	6.072	+ .004	+ .004
27	-2.51	0.340	6.084	+ .016	
27	-2.51	0.366	6.057	- .011	
27	-2.51	0.410	6.066	- .002	+ .001
28	-2.48	0.440	6.073	+ .005	
28	-2.48	0.005	6.068	.000	
28	-2.48	0.034	6.065	- .003	+ .001
29	-2.46	0.101	6.077	+ .009	
29	-2.45	0.153	6.070	+ .002	
29	-2.45	0.187	6.059	- .009	+ .001
30	-2.43	0.182	6.067	- .001	
30	-2.43	0.225	6.066	- .002	
30	-2.43	0.259	6.072	+ .004	.000
31	-2.40	0.283	6.069	+ .002	
31	-2.40	0.326	6.065	- .002	
31	-2.40	0.359	6.066	- .001	.000
Aug. 1	-2.38	0.374	6.067	.000	
1	-2.37	0.399	6.068	+ .001	
1	-2.37	0.424	6.072	+ .005	+ .002
2	-2.35	0.037	6.061	- .006	
2	-2.34	0.085	6.085	+ .018	
2	-2.34	0.117	6.062	- .005	+ .002
3	-2.32	0.119	6.074	+ .007	
3	-2.32	0.158	6.068	+ .001	
3	-2.31	0.190	6.065	- .002	+ .002
5	-2.25	0.311	6.084	+ .017	
5	-2.25	0.379	6.058	- .009	
5	-2.25	0.402	6.065	- .002	+ .002
6	-2.22	0.401	6.067	.000	.000
7	-2.19	0.067	6.072	+ .005	
7	-2.19	0.116	6.064	- .003	
7	-2.19	0.149	6.075	+ .008	+ .003
8	-2.16	0.144	6.070	+ .003	
8	-2.16	0.180	6.060	- .007	
8	-2.16	0.204	6.070	+ .003	.000
9	-2.13	0.243	6.084	+ .017	
9	-2.12	0.299	6.066	- .001	
9	-2.12	0.332	6.074	+ .007	+ .008
10	-2.09	0.330	6.072	+ .005	
10	-2.09	0.391	6.063	- .004	

TABLE IX—(Continued)

1927	α	Rotational phase	At mean opposition	Residual	Mean residual
Aug. 10	-2°09	0.424	6.066	-0.001	0.000
11	-2.06	0.437	6.070	+ .004	
11	-2.05	0.041	6.048	- .018	
11	-2.05	0.076	6.067	+ .001	- .004
14	-1.95	0.313	6.059	- .007	
14	-1.94	0.358	6.052	- .014	- .010
15	-1.91	0.352	6.053	- .013	
15	-1.91	0.408	6.061	- .005	
15	-1.91	0.443	6.051	- .015	- .011
16	-1.87	0.435	6.071	+ .005	
16	-1.87	0.030	6.076	+ .010	
16	-1.87	0.061	6.059	- .007	+ .003
17	-1.84	0.075	6.066		
17	-1.83	0.143	6.080	+ .014	
17	-1.83	0.173	6.079	+ .013	+ .009
18	-1.80	0.182	6.056	- .010	
18	-1.79	0.235	6.063	- .003	
18	-1.79	0.269	6.057	- .009	- .007
19	-1.76	0.207	(6.071)	(+ .005)	
19	-1.76	0.234	6.063	- .003	
19	-1.76	0.270	6.066		
19	-1.75	0.312	6.058	- .008	
19	-1.75	0.341	6.070	+ .004	- .002
20	-1.72	0.367	6.054	- .012	
20	-1.71	0.417	6.047	- .018	
20	-1.71	0.449	6.069	+ .004	- .009
21	-1.67	0.043	6.077	+ .012	
21	-1.67	0.085	6.062	- .003	
21	-1.67	0.112	6.074	+ .009	+ .006
22	-1.63	0.109	6.059	- .006	
22	-1.63	0.158	6.064	- .001	
22	-1.63	0.186	6.068	+ .003	- .001
23	-1.59	0.204	6.057	- .008	
23	-1.59	0.245	6.064	- .001	
23	-1.59	0.278	6.075	+ .010	.000
24	-1.55	0.296	6.063	- .002	
24	-1.55	0.331	6.046	- .019	
24	-1.55	0.357	6.052	- .013	- .011
26	-1.46	0.052	6.058	- .007	
26	-1.46	0.089	6.080	+ .015	
26	-1.46	0.116	6.062	- .003	+ .002
29	-1.33	0.362	6.058	- .006	- .006
Sept. 2	-1.15	0.258	6.046	- .018	
2	-1.15	0.312	6.059	- .005	
2	-1.14	0.344	6.070	+ .006	- .006
5	-1.01	0.092	6.063	- .001	
5	-1.00	0.149	6.077	+ .013	
5	-1.00	0.194	6.064		+ .004
6	-0.96	0.178	6.060	- .003	- .003
10	-0.77	0.137	6.060	- .003	
10	-0.76	0.191	6.066	+ .003	
10	-0.76	0.228	6.053	- .010	- .003
19	-0.32	0.086	6.067	+ .005	
19	-0.32	0.147	6.060	- .002	
19	-0.31	0.186	6.047	- .015	- .004
20	-0.27	0.156	6.060	- .001	
20	-0.27	0.197	6.066	+ .005	
20	-0.27	0.229	6.067	+ .006	+ .003
21	-0.22	0.245	6.059	- .002	
21	-0.22	0.298	6.069	+ .008	
21	-0.22	0.335	6.061		+ .002
22	-0.17	0.396	6.064	+ .003	
22	-0.17	0.442	6.070	+ .009	+ .006
23	-0.12	0.445	6.063	+ .002	

TABLE IX—(Continued)

1927	α	Rotational phase	At mean opposition	Residual	Mean residual
Sept. 23	-0°12	0.053	6.066	+0.005	
23	-0.12	0.086	6.065	+ .004	+0.004
24	-0.08	0.077	6.054	- .007	
24	-0.08	0.110	6.064	+ .003	
24	-0.08	0.138	6.049	- .012	- .005
25	-0.04	0.173	6.056	- .005	
25	-0.04	0.227	6.062	+ .001	
25	-0.04	0.262	6.061		- .001
26	+0.05	0.280	6.069	+ .008	
26	+0.05	0.338	6.074	+ .013	
26	+0.05	0.375	6.078	+ .017	+ .013
27	+0.09	0.378	6.052	- .009	
27	+0.09	0.414	6.063	+ .002	- .004
28	+0.14	0.026	6.056	- .005	- .005
29	+0.19	0.113	6.060	- .001	
29	+0.20	0.159	6.065	+ .004	
29	+0.20	0.196	6.062	+ .002	+ .002

To determine the solar phase effect for *Uranus*, the normal magnitudes were formed as in Table X. The first column gives the solar phase, the second, the mean magnitude from Table IX, the third, the final residual, from equation (B), and the fourth, the number of observations.

TABLE X

NORMAL MAGNITUDES OF URANUS

α	Magnitude	Residual	No. Obs.
-2°53	6.070	+0.002	13
-2.40	6.068	+ .001	15
-2.23	6.069	+ .002	13
-2.03	6.063	- .003	14
-1.79	6.065	- .001	16
-1.58	6.064	- .001	15
-1.00	6.061	- .003	11
-0.25	6.063	+ .002	11
-0.08	6.060	- .001	9
+0.12	6.064	+ .003	9

From the magnitudes of Table X was derived the following expression, where as before α is taken without regard to sign:

$$m = 6.0607 + 0.0028\alpha \quad (\text{B})$$

$$\pm 12 \quad \pm 7$$

The change of magnitude with solar phase is almost negligible, being only 0.007 for the 2°6-range of phase covered. This solar phase variation, with the residuals from Table X, is shown in Fig. 2.

When the observations of *Uranus* were begun in 1927 it was supposed that we would find the same variation as announced by Leon Campbell,⁸ who, with a visual photometer, determined a light-range of 0.15 magnitude in 0.451 days, presumably the rotation period of the planet. This may be tested by grouping the observations of Table IX into normals on the basis of Campbell's period, as is done in Table XI. Each normal comprises six observations, taking the mean of the phases and of the corresponding residuals.

⁸ *Harvard Circular*, No. 200, 1917.

TABLE XI

TEST OF URANUS FOR ROTATIONAL VARIATION

Phase	Mean residual	Phase	Mean residual
0.024	0.000	0.242	+0.002
0.053	+ .001	0.266	- .002
0.078	+ .002	0.292	+ .003
0.094	+ .004	0.317	.000
0.113	- .002	0.336	+ .004
0.133	+ .001	0.355	- .007
0.152	+ .003	0.373	- .004
0.168	.000	0.400	- .001
0.184	- .006	0.421	- .002
0.195	+ .002	0.443	+ .002
0.221	.000		

From the residuals of Table XI there is found,

Probable error of one normal = ± 0.0020 ,

which is small enough to exclude the possibility of variation during the interval of observation. The individual measures by Campbell have not been published, so it is not possible to judge of their accuracy, but on the basis of the photo-electric observations we shall have to call *Uranus* as nearly constant in light as any object in the sky.

THE SOLAR CONSTANT

The results on *Jupiter's* satellites and on *Uranus* may be used for a test of the constancy of the solar radiation in the same manner as in 1926. For each object the mean residual on a night presumably includes the reflected effect of any change in the Sun. The mean results of the satellites are of such different weights that the residuals of two or more satellites were combined according to the following scheme.

Weight 1—One poor observation.

“ 2—One good or two poor observations.

“ 3—Two or more good observations.

In Table XII the weight assigned to the mean residual is the sum of the individual weights, plus one unit for each satellite after the first, all divided by 3 and rounded off to indicate the quality of the mean. Because of the complicated solar phase effect the residuals for Satellite IV from September 19 to 29 were not used; also the measures of II and III were discarded on September 22, the date of opposition, when all the satellites were measured slightly bright. For *Uranus* there were a number of nights with fewer measures but these do not happen to give the large residuals.

TABLE XII
MEAN RESIDUALS BY NIGHTS

	II		III		IV		Mean		Uranus	
	Residual	Wt.	Residual	Wt.	Residual	Wt.	Residual	Wt.	Residual	Wt.
1927										
July 23		..	(-0.004)	0	(0.000)	0		..	+0.001	1
24	+0.005	1	+ .019	1	- .012	1	+0.004	2	+ .002	2
25		..		..	+ .024	3	+ .024	1	+ .002	2
26	+ .010	3	- .001	3	+ .027	1	+ .008	3	+ .004	2
27		..	+ .010	3		..	+ .010	1	+ .001	3
28		..		..		..		..	+ .001	3
29		..	- .007	3	+ .014	3	+ .004	2	+ .001	3
30		..	- .001	3	+ .006	3	+ .002	2	.000	3
31	+ .013	3	.000	3	- .007	3	+ .002	3	.000	3
Aug. 1		..		..	- .002	3	- .002	1	+ .002	3
2	- .004	3	- .006	3	- .007	3	- .006	3	+ .002	3
3		..	+ .004	3	+ .008	3	+ .006	2	+ .002	3
5	+ .001	2		..		..	+ .001	1	+ .002	3
6		..	+ .002	2	+ .024	2	+ .013	2	.000	1
7	.000	3	+ .008	3	- .004	3	+ .001	3	+ .003	3
8		..		..	- .024	3	- .024	1	.000	3
9	+ .010	3	- .014	3	- .008	3	- .004	3	+ .008	3
10		..	.000	3	- .011	3	- .006	2	.000	3
11	+ .010	2	+ .008	3	+ .008	2	+ .009	3	- .004	3
14	- .014	2	.000	2	- .011	2	- .008	3	- .010	2
15		..	(- .037)	0	- .004	3	- .004	1	- .011	3
16	.000	3	- .002	3	+ .006	3	+ .001	3	+ .003	3
17		..	- .010	3	- .001	3	- .006	2	+ .009	3
18	- .016	3	- .002	3	- .016	3	- .011	3	- .007	3
19	- .011	2		..	- .009	3	- .010	2	- .002	3
20	- .026	1	- .006	3	- .012	3	- .011	3	- .009	3
21	+ .006	3	+ .007	3	- .011	3	+ .001	3	+ .006	3
22		..	- .002	3		..	- .002	1	- .001	3
23	.000	3	+ .002	3	+ .011	3	+ .004	3	.000	3
24		..	- .005	3	+ .002	3	- .002	2	- .011	3
26		..		..	+ .016	3	+ .016	1	+ .002	3
29		..	.000	2		..	.000	1	- .006	1

TABLE XII—(Continued)

1927 Sept.	II		III		IV		Mean		Uranus	
	Residual	Wt.	Residual	Wt.	Residual	Wt.	Residual	Wt.	Residual	Wt.
2		..		..	+ .002	3	+ .002	1	— .006	3
5	+ .000	3	+ .006	3	+ .014	3	+ .007	3	+ .004	3
6	+ .008	2		..	+ .031	2	+ .020	2	— .003	1
10	+ .014	2	+ .029	2	+ .009	3	+ .016	3	— .003	3
19	+ .008	3	— .007	3	(+ .001)	3	.000	2	— .004	3
20		..		..	(— .026)	3		..	+ .003	3
21	+ .004	3	.000	3	(— .030)	3	+ .002	2	+ .002	3
22	(— .015)	2	(— .019)	2	(— .041)	2	(— .017)	..	+ .006	2
23		..	+ .002	3	(.000)	3	+ .002	1	+ .004	3
24	.000	3		..		..	.000	1	— .005	3
25		..	+ .002	3	(— .034)	3	+ .002	1	— .001	3
26	+ .005	3	— .006	3	(— .002)	3	.000	2	+ .013	3
27		..		..	(+ .016)	1		..	— .004	2
28		..		..		..		..	— .005	1
29	+ .006	2	+ .004	3	(+ .008)	3	+ .005	2	+ .002	3

Disregarding the weights in Table XII, the average of the forty-one mean residuals for the satellites is $\pm 0^m.006$, and rejecting three nights of small weight, July 25, August 8, and 26, this is reduced to $\pm 0^m.005$. The corresponding value from twenty nights in 1926 was $\pm 0^m.004$, so there is not much difference between the two seasons.

For *Uranus* the accordance is somewhat better. Forty-seven nights give an average deviation of $\pm 0^m.0036$, and as the residuals depend upon the two constants in (B), the probable error of a night's result is $\pm 0^m.0031$. Quite apart from the final accordance, our feeling during the course of the observations was that one good measure of *Uranus* was worth all that could be got from the satellites. The sky correction was much smaller than that for the satellites, and the planet was nearer the average brightness of the comparison stars.

If there was any marked solar variation during the interval of 1927 it should show in a parallel run of the residuals for the mean and *Uranus* in Table XII, but an inspection shows that there is practically no connection between them. The only possibility of such an effect is in the ten days following August 14, but the agreement there is probably a coincidence. An outstanding discordance is on September 26, one day after the opposition of *Uranus*, when the planet was measured faint by $0^m.013$. To all appearances the observations were reliable on that date, and the internal agreement was satisfactory. The satellite measures at the same time give a residual of $0^m.000$, so we can scarcely suspect a sudden jump in the solar radiation.

The results on the satellites in 1926 were compared by Abbot with his solar measures without finding any connection between the two series, though his measures showed no marked change in the Sun during the short interval of comparison. From the data communicated by Dr. Abbot in a private letter we find the average deviation of a day's result on the solar constant to be

± 0.37 per cent. The corresponding average deviation from the observations of *Jupiter's* satellites on the same twenty dates was ± 0.0040 magnitude or ± 0.37 per cent, by coincidence exactly the same accordance as that of the Smithsonian measures. The only conclusion to be drawn from this comparison is that the Sun did not change much during the interval, and the same result will probably follow for 1927.

After an experience of two seasons we are not very optimistic over the use of observations of *Jupiter's* satellites for testing the solar constant over long periods. The oppositions of *Jupiter* are now running into the winter season for the northern hemisphere, and nightly observations are not possible at any observatory possessing a photo-electric photometer. Due to the solar phase variations the satellites are not as good reflectors as was anticipated, though they should still serve to detect any change in the Sun as great as say two or three per cent.

With *Uranus*, however, the case is more favorable. If the planet itself should show no more variation than in 1927, it would make an excellent object for continued tests of the Sun. The photometric measures of *Uranus* are much more satisfactory than those of the satellites, and, because of the slower motion of the planet in the sky, the same comparison stars may be used for a longer time. Of course with observations of a single object it is not possible to distinguish between sudden variations of the planet and of the Sun, and confirmation of such variations would have to come from elsewhere.

SUMMARY

The photometric measures of *Jupiter's* satellites in 1927 confirm in all essential details the results of the previous season, and give improved light-curves for the orbital and solar phase variations, especially near opposition. No change whatever has been found in

the mean magnitude or in the character of variation of any of the satellites. It is shown that the criticisms of the methods of reduction used in this work are without foundation.

The planet *Uranus* was also observed in 1927 and found to be remarkably constant in light, there being no short-period variation as previously announced from visual observations.

The test of the solar constant given by the results on the satellites and on *Uranus* shows that on forty-seven nights there was no change in the Sun large

enough to be detected; the average deviation of a result from one night being ± 0.006 magnitude for the satellites and ± 0.0036 magnitude for *Uranus*.

We are indebted to Associate Director Aitken for his support of this investigation and to various members of the Observatory staff for assistance in recording the observations.

MADISON AND MOUNT HAMILTON,

March, 1928,

Issued May 12, 1928.

END OF VOLUME XIII.

INDEX TO VOLUME XIII

	PAGE		PAGE
Apparent Diameters of South Polar Cap. Observations of <i>Mars</i> at the Opposition of 1924. Robert J. Trumpler.....	44	Jacobsen, T. S. On The Radial Velocity Curve of S <i>Sagittae</i>	112
Applegate, Dorothy. The Spectroscopic Binary β <i>Doradus</i>	12	See also Stebbins, Joel and.	
Berman, L. and F. L. Whipple. Elements and Ephemeris of Comet <i>j</i> 1927 (Schwassmann-Wachmann).....	117	<i>Jupiter</i> , Photographs of <i>Mars</i> and of, Taken by Light of Different Colors During 1926. William H. Wright....	50
Elements and Ephemeris of Comet <i>a</i> 1928 (Reinmuth)....	173	<i>Jupiter's</i> Satellites and <i>Uranus</i> With Tests of the Solar Constant, Further Photometric Measures of. Joel Stebbins and T. S. Jacobsen.....	180
Binary 112 <i>Herculis</i> . Note on the Orbit of the Spectro- scopic. W. F. Meyer.....	49	<i>Jupiter</i> , Satellites of, Light Variations of the, and Their Application to Measures of the Solar Constant. Joel Stebbins.....	1
Binary, The Spectroscopic, β <i>Doradus</i> . Dorothy Applegate.....	12	McNally, P. A., S. J., and Elizabeth E. Sternberg, Ele- ments and Ephemeris of Object Comas Solà (1927 AA).....	48
Bobrovnikoff, N. T. Motion of Matter in the Tail of Comet 1908 III (Morehouse).....	161	McNally, P. A., see also Robert S. Richardson and.	
Campbell, W. W. and R. J. Trumpler. Observations Made with a Pair of Five-foot Cameras on the Light- Deflections in the Sun's Gravitational Field at the Total Solar Eclipse of September 21, 1922.....	130	<i>Mars</i> . Determination of the Diameter of, from Measures of the Limb. Robert J. Trumpler.....	23
Carbon Stars, The Spectra of the. C. D. Shane.....	123	Observations of, at the Opposition of 1924. Robert J. Trumpler.....	19
Cepheids. The Spectroscopic Binary β <i>Doradus</i> . Dorothy Applegate.....	12	Photographs of, and of <i>Jupiter</i> taken by Light of Differ- ent Colors During 1926. William H. Wright.....	50
Comas Solà Object (1927AA), Preliminary Elements of. Robert S. Richardson, P. A. McNally, S. J., and Elizabeth E. Sternberg.....	47	Maxwell, Allan D. Space Distribution of the Stars of Different Spectral Classes as Determined From Studies in the <i>Cygnus</i> Region of the Milky Way, The.....	68
Elements and Ephemeris of. P. A. McNally, S. J., and Elizabeth E. Sternberg.....	48	Mayall, N. U., H. G. Miles, and F. L. Whipple. Elements and Ephemeris of Comet <i>k</i> 1927 (Skjellerup).....	120
Comet <i>d</i> 1927 (Stearns), Elements and Ephemeris of. H. Thiele.....	46	Meridian Circle Observations of Eros Stars. R. H. Tucker.....	92, 104
Comet <i>j</i> 1927 (Schwassmann-Wachmann), Elements and Ephemeris of. L. Berman and F. L. Whipple.....	117	Meyer, W. F. Note on the Orbit of the Spectroscopic Binary 112 <i>Herculis</i>	49
Comet <i>k</i> 1927 (Skjellerup). Elements and Ephemeris of. N. U. Mayall, H. G. Miles, and F. L. Whipple.....	120	Miles, H. G. See Mayall, N. U. and.	
Comet <i>a</i> 1928 (Reinmuth), Elements and Ephemeris of. L. Berman and F. L. Whipple.....	173	Motion of Matter in the Tail of Comet 1908 III (More- house). N. T. Bobrovnikoff.....	161
Comet 1908 III (Morehouse), Motion of Matter in the Tail of. N. T. Bobrovnikoff.....	161	Note on the Orbit of the Spectroscopic Binary 112 <i>Herculis</i> . W. F. Meyer.....	49
Courvoisier's Yearly Refraction: See Campbell and Trumpler.		Observations Made With a Pair of Five-foot Cameras on the Light Deflections in the Sun's Gravitational Field at the Total Solar Eclipse of September 21, 1922. W. W. Campbell and R. J. Trumpler.....	130
Crawford, R. T. Radial Velocity Orbit of χ <i>Draconis</i>	176	Observations of <i>Mars</i> at the Opposition of 1924. Robert J. Trumpler.....	19
<i>Cygnus</i> Region of the Milky Way, The Space Distribution of the Stars of Different Spectral Classes as Deter- mined from Studies in the. Allan D. Maxwell.....	68	On the Radial Velocity Curve of S <i>Sagittae</i> . T. S. Jacobsen..	112
<i>Doradus</i> , β , The Spectroscopic Binary. Dorothy Applegate.....	12	Photographs of <i>Mars</i> and of <i>Jupiter</i> Taken by Light of Different Colors During 1926. William H. Wright....	50
<i>Draconis</i> , χ , Radial Velocity Orbit of.....	176	Radial Velocity Curve of S <i>Sagittae</i> , On the. T. S. Jacobsen	112
Einstein's Theory of Relativity. See Campbell and Trumpler.		Radial Velocity Orbit of χ <i>Draconis</i> . R. T. Crawford.....	176
Eros Stars, Meridian Circle Observations of. R. H. Tucker.....	92, 104	Reinmuth, Elements and Ephemeris of Comet <i>a</i> 1928. L. Berman and F. L. Whipple.....	173
Further Photometric Measures of <i>Jupiter's</i> Satellites and <i>Uranus</i> With Tests of the Solar Constant. Joel Stebbins and T. S. Jacobsen.....	180		
<i>Herculis</i> , 112. Note on the Orbit of the Spectroscopic Binary. W. F. Meyer.....	49		

	PAGE		PAGE
Richardson, Robert S., P. A. McNally, S. J., and Elizabeth E. Sternberg. Preliminary Elements of Object Comas Solà (1927 AA).....	47	Stebbins, Joel and T. S. Jacobsen. Further Photometric Measures of <i>Jupiter's</i> Satellites and <i>Uranus</i> With Tests of the Solar Constant.....	180
<i>Sagittae</i> , S. On the Radial Velocity Curve of. T. S. Jacobsen.....	12	Sternberg, Elizabeth E., See Richardson, R. S., and; McNally, P. A. and.	
Satellites: See Jupiter.		Trumpler, Robert J. Observations of Mars at the Opposition of 1924.....	19
Shane, C. D. The Spectra of the Carbon Stars.....	123	Trumpler, R. J., W. W. Campbell and. Observations Made With a Pair of Five-foot Cameras on the Light-Deflections in the Sun's Gravitational Field at the Total Solar Eclipse of September 21, 1922.....	130
Space Distribution of the Stars of Different Spectral Classes as Determined From Studies in the <i>Cygnus</i> Region of the Milky Way, Allan D. Maxwell.....	68	Tucker, R. H. Meridian Circle Observations of Eros Stars.....	92, 104
Spectra of the Carbon Stars, The. C. D. Shane.....	123	<i>Uranus</i> , With Tests of the Solar Constant, Further Photometric Measures of <i>Jupiter's</i> Satellites and. Joel Stebbins and T. S. Jacobsen.....	180
Spectroscopic Binary 112, <i>Herculis</i> , Note on the Orbit of the. W. F. Meyer.....	49	Whipple, F. L., See Berman, L. and. See also Mayall, N. U., H. G. Miles, and.	
β <i>Doradus</i> , The. Dorothy Applegate.....	12	Wright, William H. Photographs of <i>Mars</i> and of <i>Jupiter</i> Taken by Light of Different Colors During 1926.....	50
Stebbins, Joel. The Light-Variations of the Satellites of <i>Jupiter</i> and their Application to Measures of the Solar Constant.....	1		

522 19 1618 V17 NO 385 401
LICK OBSERVATORY LICK OBSERVATORY BUL

INSERT BOOK
MASTER CARD
FACE UP IN
FRONT SLOT
OF G.R. PUNCH

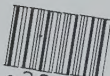


MASTER CARD

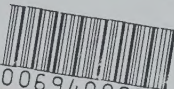
UNIVERSITY OF ARIZONA
LIBRARY

GILOBE 50114-0

522.19 L61B V13 N0 385 >



a39001



006940087b

